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Relay Assignment in Cooperative Networks

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To my wife,

Maryam

and to my parents,

Ziba and Yahya

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Contents

GLOSSARY

Glossary

AF	Amplify-and-forward
BER	Bit error rate
BP	Binary integer programming
BPSK	Binary phase shift keying
CDF	Cumulative density function
CF	Compress-and-forward
CSI	Channel state information
CVRP	Capacitated vehicle routing problem
DF	Decode-and-forward
DLBCR	Distributed linear block code relaying
E2E	End-to-end
GF	Galois field
GPS	Global positioning system
i.i.d.	Independent and identically distributed
i.n.d.	Independent but not identically distributed
LP	Linear programming
MBFSK	Modified Bessel function of the second kind
MGF	Moment generating function
ML	Maximum likelihood
MMR	Mobile Multihop Relay
PDF	Probability density function
PEP	Pairwise error probability
PSD	Power spectral density
RS	Reed-Solomon
SER	Symbol-error-rate
SNR	Signal to noise ration
VRP	Vehicle routing problem

1

Introduction

1.1 General Ideal of Cooperative Communications

The study of cooperative systems goes back to the work of Van der Meulen [1] and Cover and El Gamal [2]. In cooperative networks, a set of single-antenna nodes work together to achieve their individual goals or fulfill a common goal. These networks utilize the broadcast nature of wireless signals by observing that a source signal intended for a particular destination can be overheard at neighboring nodes. These nodes, called relays, process the signals they overhear and transmit towards the destination. In this way, the destination receives multiple versions of the message from the source and one or more relays and combines them to obtain a more reliable estimate of the transmitted signal. In other words, in these networks, the nodes share their resources using short-range communications and interact to form a distributed multi-antenna system and achieve spatial diversity. [3, 4, 5].

The basic network studied in [1, 2] consists of a source, a destination, and a relay node, where the channels are characterized by constant links and additive white Gaussian noise, i.e. other channel effects are not taken into account. In the late 1990s, the cooperative *wireless* communication came to the center of attention and since then, it has been developed in many directions to battle fading, pathloss and shadowing.

User cooperation is a key method to realize the potential throughput and coverage of wireless networks [4, 6]. Owing to its significant advantages, cooperative communications has proved itself as a strong candidate for the underlying technology for most future wireless applications, including 4G cellular networks, wireless sensor networks (IEEE 802.15.4), and fixed broadband wireless systems (WiMax, IEEE 802.16j).

The contents of this chapter are as follows. First, in Section 1.2, we describe the destructive effects in the wireless communication channels and then, in Section 1.3, we briefly review the main research issues in cooperative networks. Section 1.4 describes different relay assignment criteria which covers almost all of this dissertation. The remaining section briefly describe network coding, fairness in cooperative networks and cross-layer design.

1.2 Destructive Effects in Wireless Fading Channels

1.2.1 Fading

The study of any topic related to wireless channels, is strictly related to the effects of fading as being one of the most important performance-limiting phenomena that occur in wireless radio channels. This effect was first observed and analyzed in troposcatter systems. In any wireless communication channel, there could be more than one path over which the signal propagates between the transmitter and receiver. The presence of multiple paths is due to atmospheric scattering and refraction, or reflections from surrounding objects such as hills, buildings and other facilities. At the receiver, these multipath waves with randomly distributed amplitudes and phases are combined together to give a resultant signal that continuously changes in time and space. Therefore, a receiver at one location may have a signal that is totally different from the signal at another location, only a short distance away, because of the change in the phase relationship among the incoming radio waves. This causes significant fluctuations in the signal amplitude. This phenomenon of random fluctuations in the received signal level is termed as fading. The short-term variation in the signal amplitude caused by the local multipath is called small-scale fading, and is observed over distances of about half a wavelength. On the other hand, long-term variation in the mean signal level is called large-scale fading. This effect is a result of movement over distances large enough to cause gross variations in the overall path between the transmitter and the receiver. Large-scale fading is also known as shadowing, because these fluctuations in the mean signal level are caused by the mobile terminal moving into the shadow of surrounding objects. Due to the effect of multipath, a moving receiver terminal can experience several fades in a very short distance, or the vehicle may stop at a location where the signal is in deep fade. In such a situation, maintaining good communication becomes an issue of great importance.

1.2.2 Pathloss

Pathloss normally includes propagation losses caused by the natural expansion of the radio wave in free space, absorption losses, and losses caused by other phenomena. This loss is calculated by averaging the received power at a particular distance over a sufficiently large area. It is usually expressed in dB and can be represented by the path loss exponent, whose value is normally in the range of 2 to 4

$$L = 10n \log_{10}(d) + C$$

Here, L is the path loss in decibels, n represents the path loss exponent and d is the distance between the transmitter and the receiver. Hence, it behaves linearly in decibels. From one point of view, pathloss is a useful phenomenon, because it limits interference and makes frequency reuse possible, but it rapidly diminishes the useful signal power. Hence it is an important component in the design and analysis of the link budget.

1.2.3 Shadowing

Shadowing is one kind of deviation of the attenuation that a signal experiences over certain propagation media and it is due to shadowing from obstacles. Shadowing is calculated by averaging the received power at a particular distance over an area of radius of approximately shadowing coherence distance. This yields a variation in the received power around the pathloss. Shadowing is normally modeled as Gaussian in decibels, that is log-normal in linear scale. Since shadowing cannot be absorbed by suitable channel codes, other techniques should be employed to combat its effect.

1.3 Research Issues in Cooperative Networks

1.3.1 Cellular Wireless Networks

In a very rough classification, we can consider the cooperation scenarios for cellular wireless networks in three different contexts [7].

– Base-station cooperation

– Dedicated wireless relays

– Mobile relays

One defining element of a cellular system is the base station that is connected to an infrastructure known as the backhaul. This backhaul has a much higher capacity and better reliability than the wireless links. Other elements of the system are mobiles that operate subject to energy constraints (battery) as well as constraints on computational complexity and the number of antennas. In each cell, there are multiple mobiles as well as frequency reuse, which leads to intracell interference. The path-loss leads to significant variations in signal power at various points in the cell. The cooperative radio communication may engage one or more of these defining elements. Within the context of cellular radio, cooperative communication may be used to enhance capacity, improve reliability, or increase coverage. It is usually used in the downlink.

In the communication between a base station and a mobile, there are three forms for the cooperating entity: 1) another base station; 2) another mobile; 3) a dedicated (often stationary) wireless relay node. This cooperating entity may have various amounts of information about the source data and channel state information.

Base station cooperation: Among different forms of base station cooperation, the simplest way involves the exchange of information among neighboring cells regarding their cell-edge nodes. It means that each of the base stations can change the frequency of the nodes that generate and/or are harmed by the most cochannel interference. This kind of cooperation and its similar scenarios are in the realm of interference management.

Dedicated wireless relays: In the traditional cellular networks, usually a basic voice service is provided for all of the subscribers. Unlike such networks, broadband wireless cellular networks promise a high data rate throughout the coverage area. The fulfillment of this promise is difficult for the subscribers at the cell-edge. One way to satisfy the required data rate is to decrease the cell-size; however, it requires the installation of additional base stations and hence, it is a costly solution. A relay station can be used to improve throughput and capacity, to extend the coverage area of a base station, or to provide coverage in so-called holes. Due to above issues, the IEEE 802.16 Working Group has developed the IEEE 802.16j standard with techniques that are compatible with the WiMAX standard.

Mobile relays: Mobile relays have been a hot topic for research in the last few years, however, compared to base station cooperation and dedicated relays, their implementation is much more difficult. Here, we try to list the distinguishing characteristics of mobile relays. Maybe the most important difference between mobile relays and fixed relays is the limitation on the power and energy of mobile relays. Usually, fixed relays can be connected to the power network, while mobile relays rely on battery power. Considering the present day technologies, this factor is a severe limitation. This limitation seems to be a stable one, because on the technological horizon, no energy storage device with much higher energy densities is visible. Another distinguishing factor of most mobile nodes is the size limitation. Usually, this factor limits the number of antennas. Another distinguishing factor which is fairly related to size and energy, is the computational complexity. Considering the above issues, we can assume that the future technologies in mobiles will become more computationally complex, while the power available to them will grow at a much slower pace. The limited resources of a mobile station brings forward a fundamental question on the tradeoff between the needs of the node itself and the relaying for the other nodes.

1.4 Different Approaches to Relay Assignment

The classical relay channel is usually modeled as a single-source multiple-relay single-destination network. The majority of previous literature on relay networks focuses on this scenario. However, more general cases with multiple sources and multiple relays still lack consideration. In these networks, when the number of relays is large, it becomes a challenge to design the network architecture. In other words, which nodes should play the role of relays for each transmitting node? To this end, several relay assignment approaches have been developed in the literature. These techniques can be classified based on their optimization criteria. Each approach has its own pros and cons. Normally, the best criterion for one specific network configuration is not necessarily the best for other network configurations. In the following subsections, we give an overview of the existing relay assignment techniques. Some of these techniques are further explored throughout this thesis.

1.4.1 Relay Assignment Based on Max-min Criterion

The max-min criterion is an interesting criterion where the minimum signal-to-noise (SNR) of all possible permutations are compared, and the one whose minimum SNR is the maximum is selected [8]. This method achieves full diversity for all nodes in a network consisting of N source-destination pairs and N relays [9, 10, 11]. However, it fails to achieve diversity for a network consisting of N source nodes, N relay nodes and a single destination. This result stems from the fact that, in the second hop (relay-destination), there are only N available channels. Hence, when one of those channels is in deep fade, the corresponding source experiences deep fading and there is no way to escape from this situation. This fact dominates the performance of this scheme and results in an overall diversity of one.

1.4.2 Relay Assignment Based on Sum-SNR or Sum-rate Criteria

The authors in [12, 13, 14, 15] select the relay-assignment permutation that has the maximum sum of the SNR/rate values or the maximum weighted sum of SNR/rate values among all permutations. The mentioned weights can impose some practical constraints, such as limitation on the average and maximum consumed power in the relay nodes. The optimum permutation can be found through an exhaustive search over all possible relay-assignment permutations. However, by formulating the problem in a canonical form, it can be solved by using linear programming methods. For example, Jianwei et al. [12] propose a solution based on graph theory and simplex algorithm. Li et al. [13] and Danhua et al. [14] propose a solution based on Binary Integer Programming (BIP). Also, Yi et al. [15] provide a heuristic search algorithm to find a close-to-optimal solution. However, in these papers, there is no analysis of the diversity order or the end-to-end (E2E) bit error rate (BER) in the face of relay assignment.

1.4.3 Sequential Relaying

In this method, for each realization of the channels, the sources sequentially choose their relays from the available relay nodes [16]. The priority of the source-nodes for relay selection is according to a predefined order. For instance, the priority can be given to the source that has the weakest direct channel to the destination. The diversity order achieved by the i-th source through cooperation is $M - i + 1$, where M is the number of relays. Since each source benefits from both its direct channel to the destination and cooperation, this method brings a balance among different sources and offers the same diversity to all of the sources.

1.4.4 Geographical Approach

Other techniques involve using the geographical information of the nodes rather than the quality of the channels between different nodes in the assignment process. This approach, which is related to routing techniques in ad-hoc networks and cross-layer optimization, is well studied in the literature [17, 18, 19, 20]. In these papers, it is assumed that the channels do not change fast and the resulting relay assignment is valid for a relatively long time. In [20], Global Positioning System (GPS) information is employed to select the closest decoding relay to the destination for forwarding parity information.

1.4.5 Space-Time Relaying

Space-time code design criteria for relaying channels are presented in [21, 22, 23]. In this approach, there is no need to know the channel state information for relay assignment. It is shown in [24] that this system achieves full diversity. However the drawback here is that all of the relay nodes need to simultaneously receive and retransmit the information of all the sources in parallel channels. Hence this approach needs more complex hardware. Furthermore a strict synchronization among all nodes is required in order to simultaneously receive the corresponding signals of various nodes at the destination.

1.5 Network Coding

Network coding is a relatively young field of study. It goes back to the paper of Ahlswede, Cai, Li, and Yeung [25] in 2000. This field is a relevant topic which deals with the quality improvement of wired and wireless communication networks in multiple ways [25, 26, 27]. In classical relay networks, each relay node transmits a copy of its received message. In contrast, in network coding each node is allowed to perform some computations. From another point of view, the classical cooperative communication protocols keep information of different users in separate orthogonal channels, whereas, network coding, combines the information of different sources in a very smart way.

The classical example of network coding is the butterfly network (Fig. 1.1). In this network there are two source nodes (S_1 and S_2 at the top of the Fig. 1.1), where each one has one bit of information denoted by A and B. There are two destination nodes (D_1 and D_2). The goal is to send A and B to both destinations. Each edge in Fig. 1.1 can carry only a single bit. In classical networks, the central link is able to carry A or B, but not both. Suppose we send A through the central link; then D_1 would receive A twice and not know B. Sending B poses a similar problem for D_2. In this case, it is said that *routing is insufficient* because no routing scheme can transmit both A and B simultaneously to both destinations. By sending the sum of the bits through the center, we can send both A and B to both destinations simultaneously. In other words, we encode A and B using the formula "$A + B$". D_1 receives A and $A + B$, and extracts B from these two values. This is a linear code because the encoding and decoding schemes are linear operations.

Various theoretical studies suggest that significant gains can be obtained by using network coding, specially in multi-hop wireless networks and for serving multicast sessions, which are examples of fast-emerging technologies and services. The main advantages of network coding are the smart use of resources, robustness and energy efficiency.

1.6 Fairness in Cooperative Networks

Wireless communications is facing the scarcity of radio resources, such as time slots, subcarriers, codes, energy or power, and so on. Due to this reason, optimal use of the

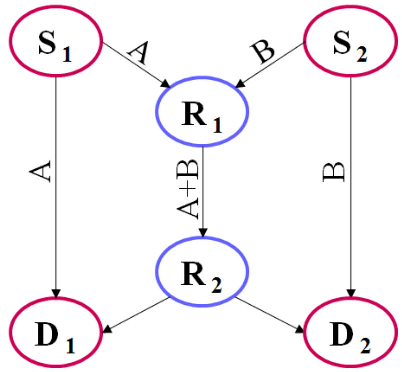

Figure 1.1: Butterfly Network

resources becomes mandatory. In this context, many questions rise about the management of resources. One common objective is to achieve fairness among nodes. For transmitting nodes, fairness is usually measured by achieving the same quality of service among all nodes. For relaying nodes, fairness is normally measured by equally distributing the load among them.

The above questions imply that, any mobile relay must balance its own needs with relaying for other nodes. In [28], this fundamental tradeoff is addressed and it is shown that it does not constitute a zero-sum game. This includes not only the power consumption and the computational burden, but also the total spectral efficiency available to a node. Hence, there are fundamental questions of the motivation of a relay node to use local resources for other nodes. In addition some nodes may have more chances to act as relays, or consume more power in cooperative transmissions so that their energy may be used up rapidly [29]. In this scenario, not only will the heavily-used nodes suffer from a short lifetime, but also the other nodes will not be able to achieve the expected cooperative gain due to the lack of available relays. Besides, selfish users or heavily-loaded terminals may refuse to cooperate in order to save their energy.

From another point of view, the same question rises about resource allocation strategies (Section 1.4): an important group of relay assignment strategies in the literature are the opportunistic strategies. The term "opportunistic" means that the resources will be dynamically allocated based on users' instantaneous channel state information (CSI).

The key idea here is to allocate more resources to the nodes with better conditions, which in turn leads to more efficient resource utilization. However, an opportunistic strategy benefits those users with better conditions, it can cause starvation of users with worse channel conditions. This unfair resource allocation can severely degrade the quality of service for some users. On the other hand, those schemes that provide absolute fairness penalize users with better conditions and reduce overall network efficiency. From a network operator perspective, the first choice (more efficient resource utilization) is preferable, but from the users' point of view, it is important to have a minimum guaranteed quality of service. The question is, how can the network operator manage this trade-off?

1.7 Cross-Layer Design

Relay assignment in cooperative networks is inherently a network problem, as shown in [3, 5]. Therefore, there are some efforts towards considering additional higher layer network issues in the relay assignment such as combining node cooperation with automatic repeat request (ARQ) in the link layer [30] or resource allocation in the MAC layer [31]. The key idea behind this approach is that the optimization across different layers can be incorporated into a unified framework. The authors in [32] present a good survey of the literature in this area. Consideration of the instantaneous channel quality in the routing protocols can also optimize system performance [33]. Dynamic routing protocols avoid links in deep fades and propose alternative reliable routes from source to destination. The relay assignment problem discussed in this work lies in the category of cross-layer optimization since it tries to find the best E2E route for all of the nodes in a network.

2

Simple Two-Hop Relaying Channel

2.1 Introduction

This Chapter introduces the basic models for user cooperation and achieves some results which are used in the remaining parts of this thesis. Basically, there are three different relaying modes in the literature [34]: amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF).

- In amplify-and-forward strategy, the relay station amplifies the received signal from the source node and forwards it to the destination
- In decode-and-forward strategy, the relay station decodes the received signal from the source node, re-encodes it and forwards it to the destination
- In compress-and-forward strategy, the relay station compresses the received signal from the source node and forwards it to the destination without decoding the signal where Wyner-Ziv coding can be used for optimal compression.

2.2 Analysis of Two-Hop Networks with AF Relaying

In this section, we revisit a two-hop network with AF relaying. Through this study, we derive some results that will be used in the following chapters to analyze the performance of some more complex relaying schemes.

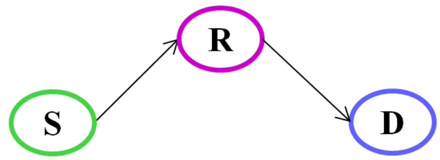

Figure 2.1: Simple two-hop cooperative network

The network under consideration comprises a single source S, a single relay R and a single destination D (see Fig. 2.1). The objective here is to derive an exact expression for the probability density function (PDF) for the E2E SNR. Assume that terminal S is transmitting a signal $x(t)$, with an average power normalized to one. The received signal at terminal R can be written as

$$r_R(t) = h_{S,R}x(t) + n_{S,R}(t) \tag{2.1}$$

where $h_{S,R}$ is the fading amplitude of the channel between terminals S and R, and $n_{S,R}(t)$ is additive white Gaussian noise (AWGN) with one sided power spectral density (PSD) N_0. The received signal at the relay is then multiplied by the gain G of the relay and then retransmitted to terminal D. The received signal at terminal D can be written as

$$r_D(t) = h_{R,D}G(h_{S,R}x(t) + n_{S,R}(t)) + n_{R,D}(t) \tag{2.2}$$

where $h_{R,D}$ is the fading amplitude of the channel between terminals R and D, and $n_{R,D}(t)$ is an AWGN with one sided PSD N_0.

The overall SNR at the receiving end can then be written as

$$\Gamma_{S,R,D} = \frac{|h_{R,D}Gh_{S,R}|^2}{\left[|h_{R,D}G|^2 + 1\right] N_0} = \frac{\frac{|h_{S,R}|^2}{N_0}\frac{|h_{R,D}|^2}{N_0}}{\frac{|h_{R,D}|^2}{N_0} + \frac{1}{G^2 N_0}} \tag{2.3}$$

The equivalent SNR of the two channels is a function of the relay gain. One choice for the gain was given in [5] to be

$$G^2 = \frac{1}{|h_{S,R}|^2 + N_0} \tag{2.4}$$

In this case, substituting (2.4) in (2.3) leads to an equivalent SNR, $\Gamma_{S,R,D}$ given by

$$\Gamma_{S,R,D} = \frac{\Gamma_{S,R}\Gamma_{R,D}}{\Gamma_{S,R} + \Gamma_{R,D} + 1}, \tag{2.5}$$

where $\Gamma_{S,R}$ and $\Gamma_{R,D}$ are the per-hop SNRs defined as $\Gamma_{S,R} \triangleq |h_{S,R}|^2/N_0$ and $\Gamma_{R,D} \triangleq |h_{R,D}|^2/N_0$. We assume the channels are subject to Rayleigh fading and the averages of $\Gamma_{S,R}$ and $\Gamma_{R,D}$ are $1/(k\lambda)$ and $1/\lambda$, respectively. Therefore $\Gamma_{S,R}$ and $\Gamma_{R,D}$ follow exponential distribution, that is $\Gamma_{S,R} \sim k\lambda e^{-k\lambda\gamma}$ and $\Gamma_{R,D} \sim \lambda e^{-\lambda\gamma}$.

This network is well studied in the literature [35, 36] however, previous works on this subject normally neglect the 1 in the denominator of (2.5) in calculating the PDF of $\Gamma_{S,R,D}$. Here, we give the exact cumulative density function (CDF), which is needed in the analysis in Chapter 3.

Theorem 1. *The CDF of the E2E SNR for a two-hop Rayleigh channel with AF relaying is*

$$F_{S,R,D}(\gamma) = 1 - 2\lambda e^{-(1+k)\lambda\gamma}\sqrt{k\left(\gamma^2+\gamma\right)}K_1\left(2\lambda\sqrt{k\left(\gamma^2+\gamma\right)}\right), \tag{2.6}$$

where $K_i(x)$ is the i^{th} order modified Bessel function of the second kind and γ denotes the instantaneous SNR value.[1]

Proof. See appendix 2.4.1. □

Reviewing the proof of (2.6) leads us to a simple and useful result which is used later in this thesis. In this proof, (2.6) is achieved by integrating the PDFs of the SNRs of the two channels ($S \to R$ and $R \to S$) over the region specified by $\Gamma_{S,R,D} \leq \gamma$

$$F_{S,R,D}(\gamma) = \iint_D f_X(x) f_Y(y)\,dx\,dy,$$

where X and Y are used for Γ_{SR} and Γ_{RD}, respectively. The integration surface D (shown in Fig. 2.2), can be divided into two regions, namely, D_1 and D_2, where D_1 denotes the region $\{X < \gamma\} \cup \{Y < \gamma\}$ and the remaining region is denoted by D_2. Denoting the result of the integral over D_1 and D_2 by $F_1(\gamma)$ and $F_2(\gamma)$, respectively, the following theorem results.

Theorem 2. *At high SNR, the CDF in (2.6) can be well approximated by the integration over only D_1. Thus, we have*

$$F_{S,R,D}(\gamma) \approx F_1(\gamma) = 1 - \exp(-(1+k)\lambda\gamma), \tag{2.7}$$

1. We found that this result is also calculated in a parallel work by Louie et al.[37].

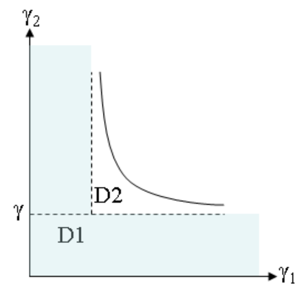

Figure 2.2: Integration surface $\Gamma_{S,R,D} \leq \gamma$

because the contribution of $F_2(\gamma)$ to the average error probability and diversity order is negligible compared to that of $F_1(\gamma)$.

Proof. See appendix 2.4.2. □

2.3 Introducing the Basic Model for DF Relaying

DF relaying uses relays that demodulate and decode the transmitted signal from the source before re-encoding and retransmitting it toward the destination. In this thesis, we follow the decode and forward relaying model proposed in [38, 39]. In this model, similar to AF scheme, there are two time-slots. In the first time-slot, the source broadcasts its signal to the relay node and the destination node. During the following time slot, if the channel between the source and the relay node is sufficiently good to allow for successful decoding, the relay first decodes and then forwards the source information to the destination, otherwise, it stays silent. In this case, the PDF of SNR for the equivalent channel between source and the destination is

$$f_{eq}(\gamma) = f_{(2,D)|\text{link down}}(\gamma) Pr[\text{link down}] + f_{(2,D)|\text{link active}}(\gamma) Pr[\text{link active}], \quad (2.8)$$

where $f_{(2,D)}(\gamma)$ represents the PDF of the channel SNR between the second terminal and the destination. If the channel between the source and the relay is not sufficiently good to allow for successful decoding, the conditional PDF $f_{(2,D)|\text{link down}}(\gamma)$ is $\delta(\gamma)$.

We denote Pr[link down] by α. So, (2.8) becomes

$$f_{eq}(\gamma) = \alpha\delta(\gamma) + (1-\alpha) f_{(2,D)|\text{link active}}(\gamma). \tag{2.9}$$

2.4 Appendices

2.4.1 Proof of Theorem 1

The CDF of $\Gamma_{S,R,D}$ can be calculated by the following integral over the two-dimensional region $\Gamma_{S,R,D} \leq \gamma$

$$F_{S,R,D}(\gamma) = \iint_D f_X(x) f_Y(y) dx dy. \tag{2.10}$$

The integration region D in (2.10) can be divided into two regions, namely, D_1 and D_2, where D_1 denotes the region $\{X < \gamma\} \cup \{Y < \gamma\}$ and the remaining region is denoted by D_2. We denote the result of the integral over D_1 and D_2 by $F_1(\gamma)$ and $F_2(\gamma)$, respectively. Since $\Pr\{X \leq \gamma\} = 1 - e^{-k\lambda\gamma}$ and $\Pr\{Y \leq \gamma\} = 1 - e^{-\lambda\gamma}$ for D_1, we have

$$\begin{aligned} F_1(\gamma) &= \Pr\{X \leq \gamma\} + \Pr\{Y \leq \gamma\} - \Pr\{X \leq \gamma, Y \leq \gamma\} \\ &= 1 - e^{-k\lambda\gamma} + 1 - e^{-\lambda\gamma} - (1 - e^{-k\lambda\gamma})(1 - e^{-\lambda\gamma}) \\ &= 1 - e^{-(k+1)\lambda\gamma}. \end{aligned}$$

We now introduce two new variables $X_1 \triangleq X - \gamma$ and $Y_1 \triangleq Y - \gamma$. First let us find the boundary of D_2 in terms of X_1 and Y_1. From (2.5) we have

$$\gamma = \frac{XY}{X+Y+1} = \frac{(\gamma + X_1)(\gamma + Y_1)}{(\gamma + X_1) + (\gamma + Y_1) + 1}$$

which yields $X_1Y_1 = \gamma^2 + \gamma$. Then,

$$\begin{aligned}
F_2(\gamma) &= \int_0^\infty \int_0^{\frac{\gamma^2+\gamma}{y_1}} \lambda e^{-\lambda(\gamma+x_1)} k\lambda e^{-k\lambda(\gamma+y_1)} dx_1 dy_1 \\
&= e^{-(1+k)\lambda\gamma} \int_0^\infty \int_0^{\frac{\gamma^2+\gamma}{y_1}} \lambda e^{-\lambda x_1} k\lambda e^{-k\lambda y_1} dx_1 dy_1 \\
&= e^{-(1+k)\lambda\gamma} \int_0^\infty \left(-e^{-\lambda\frac{\gamma^2+\gamma}{y_1}} + 1\right) k\lambda e^{-k\lambda y_1} dy_1 \\
&= e^{-(1+k)\lambda\gamma} \int_0^\infty -e^{-\lambda\frac{\gamma^2+\gamma}{y_1}} k\lambda e^{-k\lambda y_1} dy_1 + e^{-(1+k)\lambda\gamma} \left[-e^{-k\lambda y_1}\right]_0^\infty \\
&= -e^{-(1+k)\lambda\gamma} \int_0^\infty k\lambda e^{-\lambda\frac{\gamma^2+\gamma}{y_1} - k\lambda y_1} dy_1 + e^{-(1+k)\lambda\gamma}
\end{aligned}$$

The remaining integral has the following form.

$$\Upsilon(a,b) = \int_0^\infty \exp\left(-\frac{a}{y_1} - by_1\right) dy_1 = \sqrt{\frac{a}{b}} \int_0^\infty \exp\left(-\sqrt{ab}\left(\frac{1}{z} + z\right)\right) dz.$$

By assuming $z = e^t$, we have

$$\Upsilon(a,b) = \int_{-\infty}^\infty e^{-\sqrt{ab}2\cosh t} e^t dt.$$

After some manipulations, we obtain

$$\Upsilon(a,b) = \int_0^\infty e^{-2\sqrt{ab}\cosh t} 2\cosh t dt.$$

This result is the integral form of $K_1(x)$ ([40], Chapter 6.22 , Eq. (5)). Therefore,

$$\Upsilon(a,b) = \int_0^\infty e^{\left(-\frac{a}{y_1} - by_1\right)} dy_1 = \sqrt{\frac{4a}{b}} K_1\left(\sqrt{4ab}\right),$$

where $a = \lambda\left(\gamma^2 + \gamma\right)$ and $b = k\lambda$. By using this result, we can rewrite $F_2(\gamma)$ as

$$F_2(\gamma) = -2\lambda e^{-(1+k)\lambda\gamma} \sqrt{k\left(\gamma^2+\gamma\right)} K_1\left(2\lambda\sqrt{k\left(\gamma^2+\gamma\right)}\right) + e^{-(1+k)\lambda\gamma}. \tag{2.11}$$

Then $F(\gamma) = F_1(\gamma) + F_2(\gamma) = 1 - 2\lambda e^{-(1+k)\lambda\gamma} \sqrt{k\left(\gamma^2+\gamma\right)} K_1\left(2\lambda\sqrt{k\left(\gamma^2+\gamma\right)}\right)$, which proves (2.6).

2.4.2 Proof of Theorem 2

In Appendix 2.4.1, $F_1(\gamma)$ and $F_2(\gamma)$ are calculated. Let us denote the average error probability resulting from $F_i(\gamma)$ by P_i. We start with the definition of average error probability, that is,

$$\begin{aligned} P_E &= \int_0^\infty cQ(\sqrt{M\gamma}) f(\gamma) d\gamma \\ &= \int_0^\infty \int_{\sqrt{M\gamma}}^\infty \frac{c}{\sqrt{2\pi}} e^{-x^2/2} f(\gamma) dx d\gamma \end{aligned}$$

where c and M are scalars that depend on the modulation scheme employed. By changing the order of integration, we have

$$\begin{aligned} P_E &= \int_0^\infty \int_0^{x^2/M} \frac{c}{\sqrt{2\pi}} e^{-x^2/2} f(\gamma) d\gamma dx \\ &= \int_0^\infty \frac{c}{\sqrt{2\pi}} e^{-x^2/2} F(x^2/M) dx. \end{aligned} \tag{2.12}$$

By replacing $F(\gamma)$ with $F_1(\gamma)$, we have

$$\begin{aligned} P_1 &= \int_0^\infty \frac{c}{\sqrt{2\pi}} e^{-x^2/2} \left(1 - e^{-(k+1)\lambda x^2/M}\right) dx \\ &= \frac{c}{2} - \int_0^\infty \frac{c}{\sqrt{2\pi}} e^{-\frac{x^2}{2}\left(1+\frac{2(k+1)\lambda}{M}\right)} dx \\ &= \frac{c}{2} - \frac{c}{2\sqrt{1+\frac{2(k+1)\lambda}{M}}}. \end{aligned} \tag{2.13}$$

For high average SNR values ($\lambda \to 0$) the first term in the Taylor expansion of (3.30) is $\frac{c(k+1)\lambda}{2M}$. By using Proposition 1 in [41], this result implies that the system has a diversity order of 1. For $F_2(\gamma)$, since the first order terms cancel each other in its Taylor expansion, the first term in the Taylor expansion of (2.11) in terms of λ is at least 2. Therefore, at high SNR, the contribution of $F_2(\gamma)$ to the average error probability is negligible compared to $\frac{c(k+1)\lambda}{2M}$.

3

Order-Statistical AF Relaying

3.1 introduction

In statistics, the r^{th} order statistic of a statistical sample is equal to its r^{th} smallest value. In this chapter, we address the analysis of order statistical relaying channels, i,e, we consider different scenarios where order statistics appear in the PDF of the relaying channel. Those scenarios involve: order statistical one-hop relaying channel; two-hop relaying channel where the PDF in one hop follows order statistics; two-hop relaying channel where the PDF in both hops follow different order statistics; two-hop relaying channel where the E2E SNR follows order statistics. These scenarios are generalizations of the best-relay selection scheme and correspond to the case when the best relay is unavailable due to some reasons (for example, the best relay is used by other users). We derive a closed form expression for the PDF and the BER in each scenario. We present several numerical results that validate the analytical results.

3.1.1 General Assumptions

In this chapter, we make the following assumptions.

– Throughout this chapter, we will frequently refer to the *clustering* concept, i.e. we assume some terminals are clustered relatively close to each other (location-based clustering) such that the channels between each of them and a terminal out of the cluster have the same average SNRs. This clustered structure is a common model in the literature and has been selected by a long-term routing process [42]. The mentioned routing scheme can track and take into account variations in path-loss and shadowing, hence guarantees equivalent average SNRs for the terminals in one cluster. Therefore, our cooperative scheme should combat the effects of small scale fading.

- All of the channels in one cluster are assumed to be independent and identically distributed (i.i.d.) Rayleigh-fading channels. We also assume that the channels in different clusters are independent.
- The channels are slow fading and remain constant during the resource allocation process.
- For cooperation, the traditional relay mode is employed (as in [5]). In the first time slot, the source nodes transmit and the relays and the destination receive. In the second time slot, the relay nodes transmit and the destination receives. Simultaneous transmitting nodes use orthogonal channels.
- All nodes are assumed to work in half-duplex mode, i.e. they cannot transmit and receive at the same time.

Throughout the chapter, the PDF and the CDF of a random variable Γ are denoted by $f_\Gamma(\gamma)$ and $F_\Gamma(\gamma)$, respectively.

3.1.2 Outline and Contributions of This Chapter

The main contributions of this chapter are as follows.

1. A two-hop Rayleigh channel is considered where the PDF of SNR in one of the hops follows order statistics. The PDF and BER of the equivalent E2E channel in AF mode are calculated.

2. A two-hop Rayleigh channel is considered where the PDF of SNR in both hops follows different order statistics. Again, the PDF and BER of the equivalent E2E channel in AF mode are calculated. This scenario and the scenario in Step 1 represent generalizations of the work in [43].

3. A new approximation for the modified Bessel function of the second hop is proposed. This approximation is more accurate compared to the classical approximation and it is easy to handle. It can have several applications in the statistical expressions of AF relaying links.

4. A two-hop Rayleigh channel is considered where the E2E PDF of SNR in AF mode follows order statistics. The BER of the equivalent E2E channel is calculated. We remark that a similar expression was derived in [44]. However, the expression derived in [44] is based on some approximations, and thus is accurate only at high SNRs, whereas our approach is exact and valid for all range of SNRs.

5. The results of Step 4 are useful for the analysis of other relaying schemes. As an example, they are necessary to analyze the performance of max-min relay assignment [11]. We discuss the application of the proposed formula in the corresponding sections.

The rest of this chapter is organized as follows. Section 3.1.3 presents a short review of order statistics. Section 3.1.4 reviews a proposition by Zhengdao and Giannakis which quantifies the diversity order and the coding gain in fading channels. The performance of the r^{th} weakest channel among a set of N i.i.d. Rayleigh fading channels is studied in Section 3.2. The performance of the two-hop relaying channel where the PDF in one hop or both hops follow order statistics are analyzed in Sections 3.3 and 3.4, respectively. Finally, the performance of the r^{th} weakest E2E two-hop relaying channel is analyzed in Section 3.4. Fig. 3.1 shows a comprehensive overview of different scenarios analyzed in this chapter.

3.1.3 Preliminary: Order Statistics

If random variables $X_1, X_2, ..., X_N$ are sorted and then written as $X_{1:N} \leq X_{2:N} \leq ... \leq X_{N:N}$, then $X_{r:N}$ is called the r^{th} order statistic ($r = 1, ..., N$). Although random variables X_i are assumed to be i.i.d., $X_{r:N}$ are necessarily dependent because of the inequality relations among them. If $F_{r:N}(x)$ denotes the CDF of the r^{th} order statistic, then we have

$$\begin{aligned} F_{r:N}(x) &= Pr\{X_{r:N} \leq x\} \\ &= Pr\{\text{at least } r \text{ of the } X_i \text{ are less than or equal to } x\} \\ &= \sum_{i=r}^{N} \binom{N}{i} [F_X(x)]^i [1 - F_X(x)]^{N-i}. \end{aligned} \tag{3.1}$$

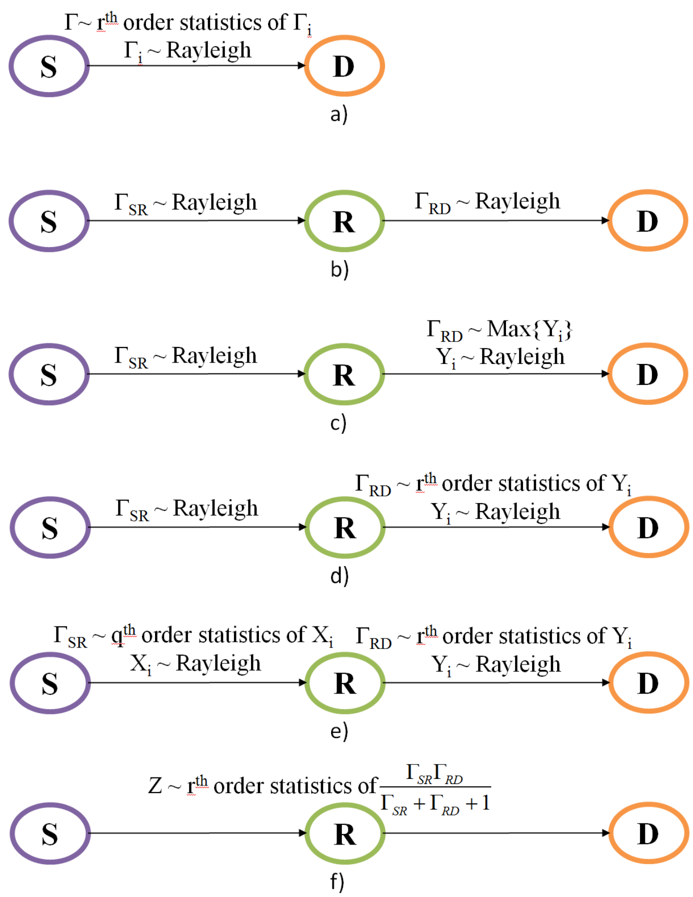

Figure 3.1: Chapter outline: a) r^{th} weakest channel among a set of i.i.d. Rayleigh fading channels, studied in Section 3.2; b) Simple two-hop AF cooperation studied in [35] and revisited in Chapter 2; c) Channel in one of the hops is the maximum among N i.i.d. Rayleigh fading channels, studied in [43]; d) Channel in one of the hops follows the r^{th} order-statistics of the Rayleigh fading, studied in Section 3.3; e) Channels in both hops follow different order-statistics of the Rayleigh fading, studied in Section 3.4; f) The E2E channel is the r^{th} order-statistics of N two-hop cooperation channels where each individual channel is Rayleigh, studied in Section 3.5.

Let us replace $F_X(x)$ in (3.1) by the exponential PDF (which is the distribution of SNR for a Rayleigh fading channel) to derive the CDF of the r^{th} order statistic.

$$F_{r:N}(x) = \sum_{i=r}^{N} \binom{N}{i} (1 - e^{-\lambda x})^i (e^{-\lambda x})^{N-i}. \tag{3.2}$$

A useful formula to calculate the order statistics comes from the well-known relation between binomial sums and the incomplete beta function [45]:

$$F_{r:N}(x) = I_{F(x)}(r, N - r + 1). \tag{3.3}$$

where $I_p(a, b)$ is the incomplete beta function. By expanding and calculating the derivative of (3.3) we have the following simpler formula:

$$\begin{aligned} f_{r:N}(x) &= \frac{(1 - e^{-\lambda x})^{r-1} (e^{-\lambda x})^{N-r} \lambda e^{-\lambda x}}{\beta(r, N - r + 1)} \\ &= \frac{(1 - e^{-\lambda x})^{r-1} e^{-\lambda x(N-r+1)} \lambda}{\beta} \\ &= \sum_{i=0}^{r-1} \frac{1}{B} \binom{r-1}{i} (-1)^i \lambda e^{-\lambda x(N-r+1+i)} \\ &= \sum_{i=0}^{r-1} \Lambda_i e^{-\lambda_i x}. \end{aligned} \tag{3.4}$$

where the constant $\beta(r, N - r + 1)$ is the beta function which is replaced by B for simplicity. Also $\frac{1}{B}\binom{r-1}{i}(-1)^i\lambda$ is denoted by Λ_i and $(N - r + 1 + i)\lambda$ is denoted by λ_i.

3.1.4 Preliminary: Relation Between Diversity Order and The PDF

At high SNR, the average probability of error of a transmission system in a fading channel is usually represented by

$$P_E \approx (G_c \overline{\gamma})^{-G_d} \tag{3.5}$$

where G_c is termed the *coding gain*, G_d is referred to as the *diversity order*, and $\overline{\gamma}$ denotes the average SNR value. Throughout this dissertation, we will widely use proposition 1 in [41]. According to this proposition when all the derivatives up to order $(t - 1)$ of the PDF of SNR are null at zero, but the t^{th} order derivative is not zero, the system has a diversity order of $(t+1)$ and the coding gain can be approximated with:

$$P_E(\gamma) = \frac{2^{t+\frac{1}{2}} a\Gamma(t+\frac{3}{2})}{\sqrt{2\pi}(t+1)} k^{-(t+1)}$$

where t is the smallest power of γ in the Taylor expansion of $f_\Gamma(\gamma)$ and a is a constant: $f_\Gamma(\gamma) = a\gamma^t + O(\gamma^{t+\epsilon})$.

3.2 Order Statistical One-hop Channel

In this section, the r^{th} weakest channel among a set of i.i.d. Rayleigh fading channels is studied to find the average probability of error. The results are necessary to analyze the performance of various relay assignment methods in the remaining parts of this dissertation. By using the PDF of the equivalent SNR in 3.2, we can calculate the symbol error rate of the transmission.

Theorem 1. *The diversity order offered by $\Gamma_{r:N}$, $1 \leq r \leq N$ is equal to r and the average BER is*

$$\begin{aligned} P_E &= \int_0^\infty cQ(\sqrt{M\gamma}) f_{r:N}(\gamma) d\gamma \\ &= c \sum_{i=0}^{r-1} \Lambda_i \frac{1}{2\sqrt{(1+\lambda_i \frac{2}{M})}}, \end{aligned} \tag{3.6}$$

where c and M are constants specifying the type of modulation. For example, for BPSK transmission $c = 1$ and $M = 2$. Here Λ_i and λ_i are defined the same as in (3.4).

Proof. See appendix 3.6.1. □

Theorem 2. *At high SNR, (3.6) simplifies to*

$$P_E \approx \sum_{i=r}^{N} \binom{N}{i} \frac{\lambda^i}{2^{i+1}} \frac{(2i-1)(2i-3)...(1)}{[(N-i)\lambda+1]^{i+\frac{1}{2}}}. \tag{3.7}$$

Proof. See appendix 3.6.2. □

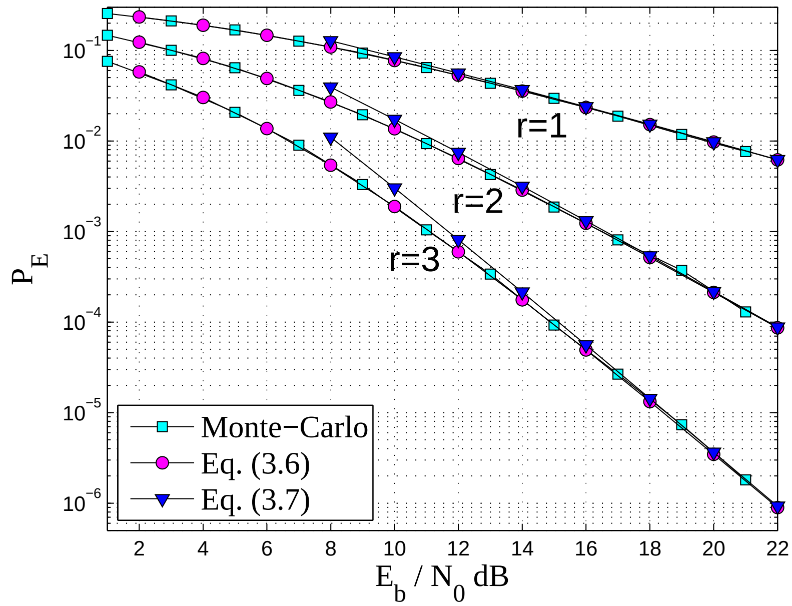

Figure 3.2: BER analysis of different order-statistical one-hop channels when $N = 4$

The advantage of equation (3.7) over (3.6) is that the diversity order can be evaluated more easily from this equation. At high SNR the diversity order is r, because $j\lambda$ in $(j\lambda + 1)$ is small compared to one. Fig. 3.2 shows this result compared with the Monte-Carlo simulation for $N = 4$ and $r = 1, 2, 3$. As we see in this figure, (3.6) is always a perfect match for the performance of the link, and (3.7) has a good match at high SNR.

3.3 Order-Statistics in One Hop

Assume a simple relay configuration of one source S, one destination D and N relays R_i, $i = 1, ..., N$ (Fig. 3.3). The source has no direct link to the destination and the transmission is performed only via relays. We assume that the relays are clustered close to each other and the channels in each cluster are assumed to be i.i.d Rayleigh fading. The considered links have an average SNR equal to $1/\delta$ for the links $S \to R_i$ and $1/\lambda$ for the links $R_i \to D$.

The resource allocator continuously monitors the quality of relay-destinations channels. Due to this information, the best available relay link among $R_i \to D$ $(i = 1, ..., N)$

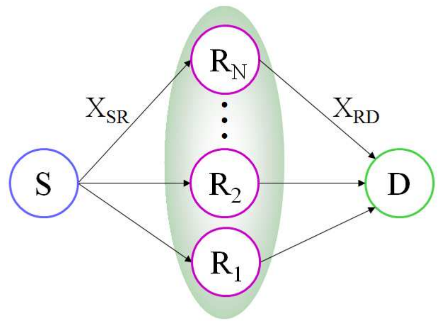

Figure 3.3: A set of one source, N relays and one destination.

is assigned to the source node. Without any loss of generality, we assume that the relays are sorted in order of their SNR magnitude and R_i represents the relay with the i^{th} smallest received SNR at the destination. We denote the index of the best available relay by r, i.e. the PDF of the SNR in the second hop follows r^{th} order-statistic of exponential distribution ($F_{r:N}(x)$). This scenario is interesting in practical mobile and ad-hoc systems where only neighboring (1 hop) channel information is available to the nodes [46]. Another application for this scenario is the analysis of relay assignment based on max-min criterion (Chapter 5).

3.3.1 Statistical expressions

Theorem 3. *The CDF of the equivalent E2E SNR received at the destination for the relaying link under consideration ($S \to R_r \to D$) can be approximated as*

$$F_X(x) \approx 1 - \sum_{i=0}^{r-1} 2x\Lambda_i \sqrt{\frac{\delta}{\lambda_i}} e^{-x(\delta+\lambda_i)} K_1\left(2x\sqrt{\delta\lambda_i}\right) U(x) \tag{3.8}$$

where, same as before $\frac{1}{B}\binom{r-1}{i}(-1)^i\lambda$ *and* $(N-r+i+1)\lambda$ *are respectively denoted by* Λ_i *and* λ_i *and* $U(\cdot)$ *is the unit step function.* $K_i(x)$ *denotes the* i^{th} *order modified Bessel function of the second kind.*

Proof. See appendix 3.6.3. □

Theorem 4. *The PDF of the equivalent E2E SNR for the relaying link under consideration ($S \to R_r \to D$) can be approximated as*

$$f_X(x) \approx \sum_{i=0}^{r-1} 2x\delta\Lambda_i e^{-x(\delta+\lambda_i)} \left[\frac{(\delta+\lambda_i)}{\sqrt{\delta\lambda_i}} K_1\left(2x\sqrt{\delta\lambda_i}\right) + 2K_0\left(2x\sqrt{\delta\lambda_i}\right) \right] U(x). \quad (3.9)$$

Proof. See appendix 3.6.4. □

3.3.2 Average probability of error

Theorem 5. *The average probability of error for the relaying link under consideration ($S \to R_r \to D$) can be approximated as*

$$P_E \approx \frac{c}{2} - \sum_{i=0}^{r-1} \frac{c}{B} \binom{r-1}{i} \frac{(-1)^i}{(N-r+1+i)} \frac{1}{2\sqrt{1+2(\delta+\lambda_i)/M}} \quad (3.10)$$

Again, c and M are constants specifying the type of modulation, and $Q(\sqrt{M\gamma})$ represents the bit error probability of this modulation for Gaussian channel.

Proof. See appendix 3.6.5. □

Although equation (3.10) does not reveal the diversity order explicitly, the diversity order of the equivalent link is one. We will express the diversity order offered by (3.10) in theorem 9 in the next section.

3.3.3 Simulations and discussions

Computer simulations are performed in order to validate the proposed analytical expressions. Fig. (3.4) compares (3.8) with Monte-Carlo simulation. The perfect match between our result and the Monte-Carlo simulation is obvious from this figure. In this simulation, we have assumed SNR$= 20\ dB$, $N = 4$, and $r = 4$. Fig. (3.5) shows the average probability of error for this scenario. It is assumed that $N = 7$ and both hops have the same average SNR value $1/\delta = 1/\lambda$. The simulation results refer to BPSK modulation and to different values of r. This shows that for high SNRs, there is a significant difference between $r = 1$ and $r = 2$. This is because the first hop for $r = 2$ achieves diversity 2, however this diversity is dumped since the second hop

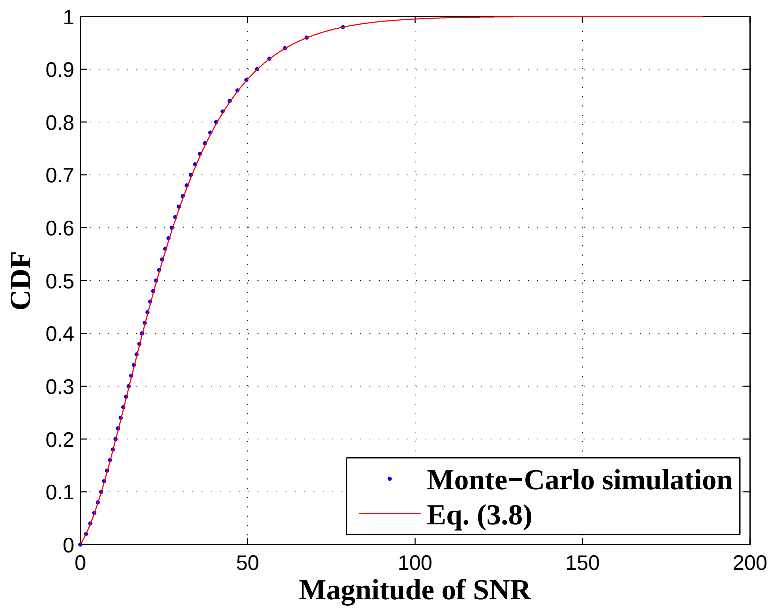

Figure 3.4: Comparison of Equation (3.8) with Monte-Carlo simulation for $N = 4$ and $r = 2$.

(presenting a first order diversity) plays the role of a bottleneck, but still shows itself as an improvement in coding gain (a horizontal shift in the BER curve). Another interesting observation is that for good signal to noise ratios, the performance of the relaying link for $r = 2$ converges to that of $r = N$. It means that for high average SNR values, we can choose the best between two randomly selected relays and the performance will be almost the same as selection of the best relay among all the available relays.

3.4 Order-Statistics in Both Hops

In Section 3.3, we have assumed the SNR of the channel in the first hop to be exponentially distributed, because the relay selection was only for the second hop. In this section, we assume that there are N orthogonal channels available for the first hop and each source has the possibility of using the best available channel to its corresponding relay. We assume that the SNR distribution for this channel ($S \to R_r$) follows the q^{th}

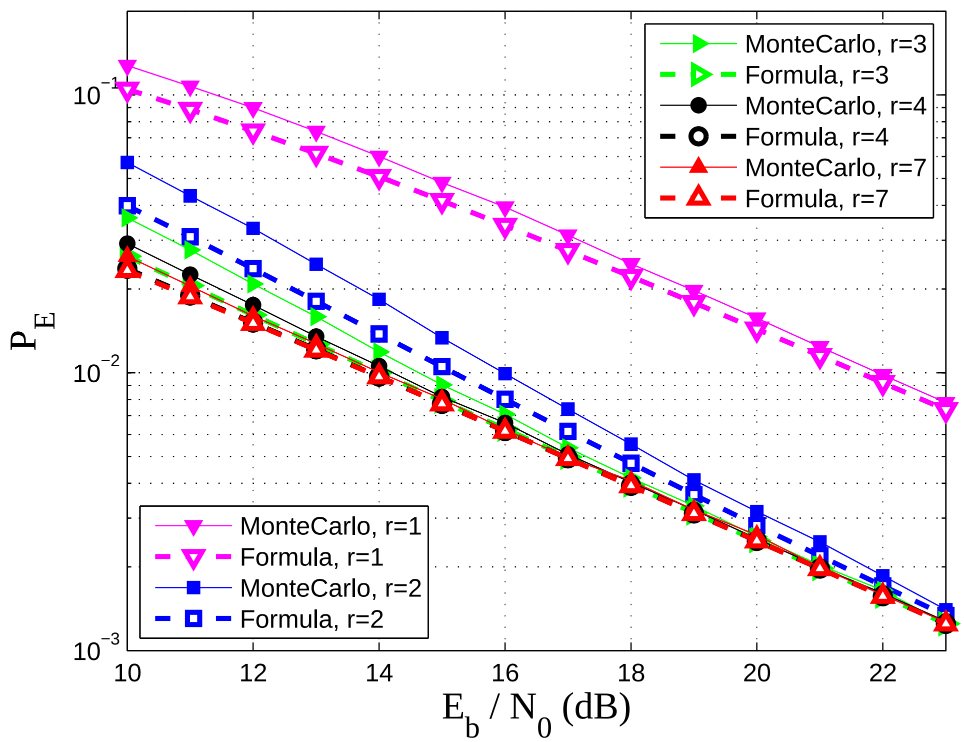

Figure 3.5: P_E results generated by (3.10) compared to Monte-Carlo simulations for $N = 7$ and different values of r.

order-statistic of exponential distribution ($F_{q:N}(x)$). The statistical expressions (PDF and CDF), the average error probability and the diversity order for this scenario are calculated in this section.

3.4.1 Statistical expressions

Theorem 6. *The CDF of the equivalent E2E SNR received at the destination for the relaying link under consideration ($S \to R_r \to D$) can be approximated as*

$$F_X(x) \approx 1 - \sum_{i=0}^{q-1} \sum_{j=0}^{r-1} 2x\Delta_i \Lambda_j \frac{1}{\sqrt{\delta_i \lambda_j}} e^{-x(\delta_i + \lambda_j)} K_1 \left(2x\sqrt{\delta_i \lambda_j} \right) U(x) \tag{3.11}$$

where the constants $\frac{1}{B(q,N-q+1)}\binom{q-1}{i}(-1)^i\delta$ *and* $(N-q+1+i)\delta$ *are respectively denoted by* Δ_i *and* δ_i*.*

Proof. See appendix 3.6.6. □

Theorem 7. *The PDF of the equivalent E2E SNR received at the destination for the relaying link under consideration ($S \to R_r \to D$) can be approximated as*

$$f_X(x) \approx \sum_{i=0}^{q-1} \sum_{j=0}^{r-1} 2x\Delta_i \Lambda_j e^{-x(\delta_i + \lambda_j)} \left[\frac{(\delta_i + \lambda_j)}{\sqrt{\delta_i \lambda_j}} K_1 \left(2x\sqrt{\delta_i \lambda_j} \right) + 2K_0 \left(2x\sqrt{\delta_i \lambda_j} \right) \right] U(x). \tag{3.12}$$

Proof. The proof is straightforwardly similar to that of Theorem 4. □

3.4.2 Average probability of error

Theorem 8. *The average probability of error for the relaying link under consideration ($S \to R_r \to D$) can be approximated as*

$$P_E \approx \frac{c}{2} - \sum_{i=0}^{q-1} \sum_{j=0}^{r-1} \frac{c}{B_1 B_2} \binom{q-1}{i} \binom{r-1}{j} (-1)^{i+j} \frac{1}{\delta_i \lambda_j} \frac{1}{2\sqrt{1 + \delta_i + \lambda_j}} \tag{3.13}$$

where the constants $B(q, N-q+1)$ *and* $B(r, N-r+1)$ *are denoted by* B_1 *and* B_2 *respectively.*

Proof. See appendix 3.6.7. □

Theorem 9. *The diversity order for the relaying link under consideration is* $\min\{q, r\}$

Proof. See appendix 3.6.8. □

3.4.3 Simulations and discussions

Computer simulations are performed in order to validate the proposed analytical expressions. Fig. (3.6) compares (3.11) with Monte-Carlo simulation. The perfect match between this curve and the Monte-Carlo simulation is obvious from this figure. Fig. (3.7) shows the average probability of error when SNR information is available for both hops. Again it is assumed that $1/\delta = 1/\lambda$, $N = 7$, $q = 4$ and the simulation is performed by using BPSK modulation. From this figure, it can be inferred that by increasing r, as far as $r \leq q$ there is an increase in the diversity order. The amount of this diversity order is exactly what we expected by theorem 7. We can not increase the diversity order by increasing r beyond $r = q$, but this produces an improvement in the coding gain. Another interesting observation is that at high SNR values, when $r = q + 1$, the performance of the relaying link converges to that of $r = N$. This result shows that, when there is a bottleneck in one of the hops, (q is fixed), it is almost enough to have $r = q + 1$ in order to achieve the best possible performance.

3.5 Order Statistical E2E Two-hop Channel

Again we consider a network comprising a single source, a cluster of N relays denoted by R_i, $1 \leq i \leq N$ and a single destination. We assume that the SNR information of all links (source-relay and relay-destination) are available to the resource allocator, which can be centralized or semi distributed. We derive the PDF of the E2E SNR corresponding to the r^{th} weakest link, and then use that result to derive the E2E BER. The approach adopted here is different from what has been presented in the literature in the sense that it is based on the exact expression for the PDF of the two hop channels (obtained in Chapter 2).

Since the N relay terminals are clustered relatively neat to each other, the source-relay channels are assumed to have the same average SNR. The same assumption holds

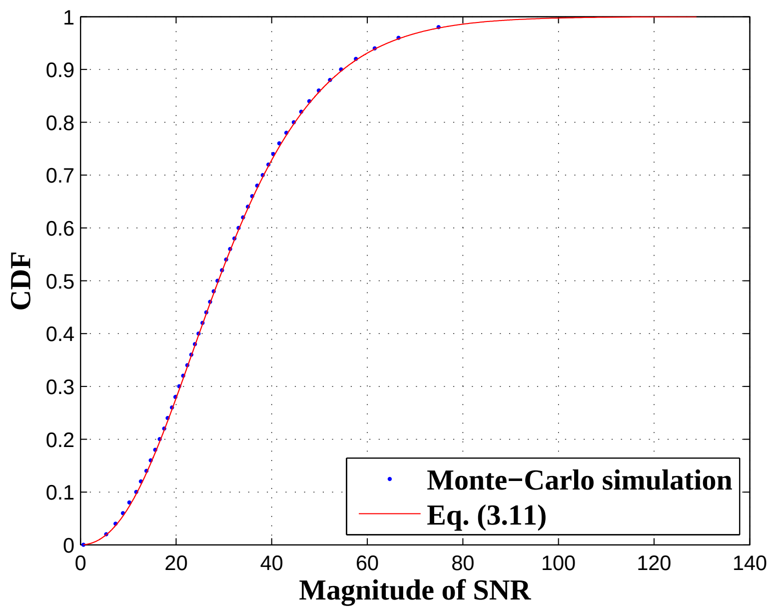

Figure 3.6: Comparison of Equation (3.11) with Monte-Carlo simulation for $N = 4$, $r = 3$ and $q = 2$.

true for the relay-destination channels. The channels are assumed to be i.i.d. Rayleigh distributed. The work in [44] is also devoted to the same problem where the authors approximate each two-hop SNR ($S \rightarrow R_i \rightarrow D$) by $min(\Gamma_{SR_i}, \Gamma_{R_iD})$ for simplicity. Throughout our simulations, we will show that the approximation in [44] fails to achieve accurate results for low average SNR values; furthermore its error increases by augmenting the number of nodes in the system. Here, we use the exact SNR expression given in (2.6), which results in more accurate formulas for all range of SNR values.

Note that the PDF of the E2E SNR through each relay (source-relay-destination) follows the distribution in (2.6). Therefore, the powers of the modified Bessel function $K_1(x)$ appear in the order statistics. Because of the difficulty in dealing with the powers of the modified Bessel function $K_1(x)$, we need to have a good approximation for this function.

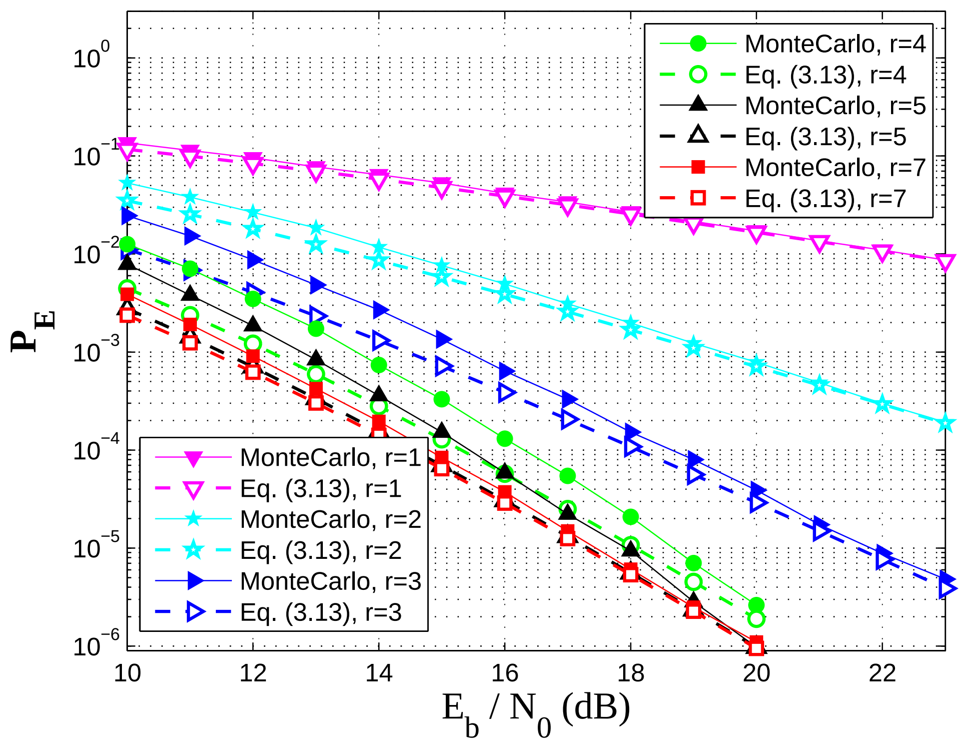

Figure 3.7: P_E of equation (3.13) compared to Monte-Carlo simulations for $N = 7$, $q = 4$ and different values of r.

3.5.1 A New Approximation For $K_1(x)$

Bessel functions widely appear in statistical analysis of many applications related to AF cooperative networks. In order to achieve closed form expressions or simplify the statistical expressions of these applications, some approximations for Bessel function are required. Specially that, there is no closed form answer for the integrals containing the powers of Bessel functions. This approximation comes with the price of inaccuracy in the results, specially at low SNR values. For example, the classic approximation for the first order modified Bessel function of the second kind (MBFSK) is $K_1(x) \approx 1/x$ ([47], Eq. (17.7.2.1.2)), which fails to achieve good results at low average SNR values (big argument values for MBFSK). In this part, we propose a new approximation for the first order MBFSK in order to decrease the drawback of the classical approximation. The proposed approximation is

$$K_1(x) \approx \frac{1}{x} \exp\left(-\frac{x^2}{2}\right) \tag{3.14}$$

Fig. 3.8 compares the classic approximation with the proposed one. As it is evident from this figure, the proposed approximation is always a better approximation where its advantage over the classical one is significant for large values of its argument.

3.5.2 Average Probability of Error

Plugging (3.14) into (2.6) and then plugging the result into (3.1), the order statistic for the two-hop channel is achieved. Therefore, the E2E BER for this system can be obtained, as follows.

Theorem 10. *For $\Gamma_{r:N}$, $1 \leq r \leq N$, the E2E BER for binary phase shift keying (BPSK) transmission can be approximated as*

$$P_E \approx \frac{1}{4\sqrt{\pi}} \sum_{i=r}^{N} \binom{N}{i} \sum_{j=0}^{i} \binom{i}{j} (-1)^j \frac{A_{i,j}}{B_{i,j}} \exp\left(\frac{A_{i,j}^4}{2B_{i,j}^2}\right) k_{\frac{1}{4}}\left(\frac{A_{i,j}^4}{2B_{i,j}^2}\right), \tag{3.15}$$

where

$$B_{i,j}^2 = \lambda^2 k\left(N - i + j - u\right),$$
$$2A_{i,j}^2 = \frac{1}{2} + (1+k)\lambda\frac{1}{2}(N - i + j) - 2\lambda^2 k(N - i + j - u).$$

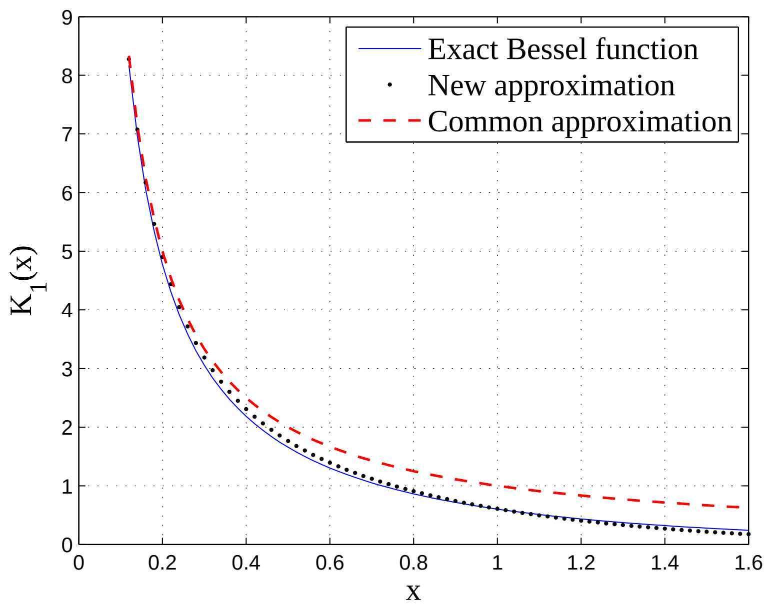

Figure 3.8: The common approximation of $K_1(x)$ compared with the proposed approximation

and $k_{\frac{1}{4}}(.)$ *is the* $\frac{1}{4}^{th}$ *order modified Bessel function of the second kind.*

Proof. See appendix 3.6.9. The proof for other modulation types is similar. $\square$

When the average SNRs for both hops is the same, $A_{i,j}$ and $B_{i,j}$ simplify to $A_{i,j}^2 = \frac{1}{4} + \frac{1}{2}\lambda(1+\lambda)(N-i+j)$ and $B_{i,j}^2 = \frac{1}{2}\lambda^2 (N-i+j)$.

For very high average SNRs, the numerical calculation of (3.15) can become cumbersome, because the argument of $K_{1/4}(\cdot)$ grows and the exponential term becomes very small. Therefore the whole expression becomes indeterminate. To avoid this problem, we propose to use the classical approximation ($K_1(x) \approx 1/x$) instead of that in (3.14), which is accurate enough for very high SNRs. By plugging this expression into (2.6) and plugging the result into (3.1) we obtain

$$f_{r:N}(\gamma) \approx \frac{(k+1)\lambda \left(1 - e^{-\gamma(k+1)\lambda}\right)^{r-1} \left(e^{-\gamma(k+1)\lambda}\right)^{N-r+1}}{\beta(r, N-r+1)}. \tag{3.16}$$

Using this result, the E2E BER for this link turns out to be

$$P_E \approx \sum_{v=0}^{r-1}(-1)^v \binom{r-1}{v} \frac{1}{\beta(r, N-r+1)(v+N-r+1)} \times \left(1 - \frac{1}{\sqrt{(k+1)\lambda(v+N-r+1)+1}}\right). \tag{3.17}$$

Proof. See appendix 3.6.10. □

In order to have a simpler expression that reveals the diversity order, we obtain the E2E BER by just integrating the first term in the Taylor expansion of (3.16) in which the power of $(1-e^{-(k+1)\lambda\gamma})$ is $(r-1)$. This term exists in all of the derivatives up to order $(r-2)$. Noting that this term is null at zero, its Taylor expansion becomes

$$f_{r:N}(\gamma) \approx \frac{((k+1)\lambda)^r}{\beta(r, N-r+1)}\gamma^{r-1} + O(\gamma^{r+\epsilon}). \tag{3.18}$$

Using this result, the E2E BER can be approximated as

$$P_E \approx \frac{((k+1)\lambda)^r \Gamma(r+0.5)}{2\sqrt{\pi} r \beta(N-r+1, r)}. \tag{3.19}$$

Proof. See appendix 3.6.11. □

At high SNR, it is easily inferred from from (3.19) that the diversity order is r. Fig. 3.9 compares P_E for $r = 3, 4$ for different values of the energy-per-bit to the noise power spectral density ratio, i.e. E_b/N_0, using Monte-Carlo simulations and the expressions in (3.15) and (3.17) where $N = 4$. As shown in this figure, there is a perfect match between (3.15) and simulation results. Furthermore, (3.17) matches very well the simulation curves for high average SNRs.

To emphasize the importance of obtaining the exact PDF of the E2E SNR (given in (3.15)), we compare in Fig. 3.10 the E2E BER performance in (3.15) with the solution in [44] in which the E2E SNR is approximated by $min(\Gamma_{SR}, \Gamma_{RD})$, (please note that in [44], this PDF is combined with the PDF of the direct source-destination channel in order to find the overall PDF and the E2E BER is not offered for the relay link explicitly). In this simulation, we assumed $r = N$ and $N = 1, 3, 7$. We also plot in the same figure the Monte-Carlo simulations. As shown in the figure, there is a perfect match between (3.15) and the simulations for all range of SNR and for all values of N. Whereas there is a large gap between the approximation and the simulations for low to

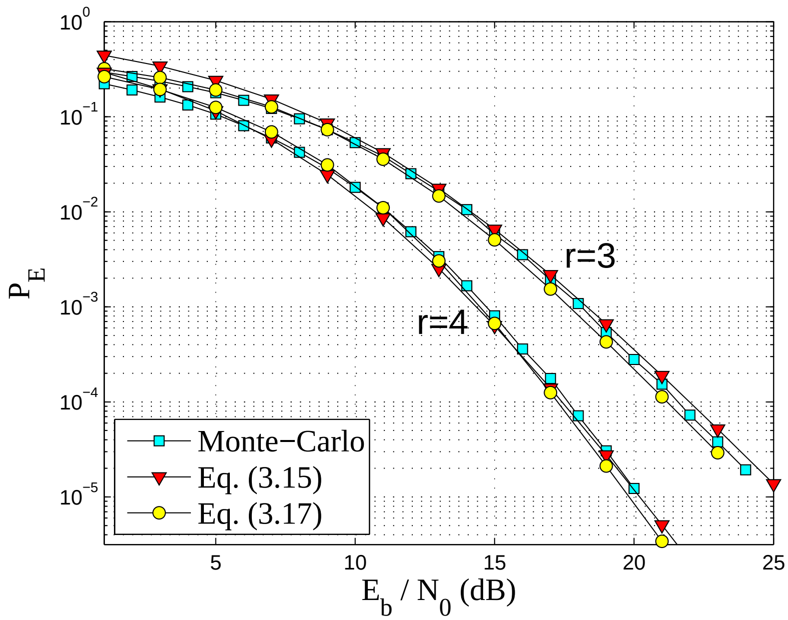

Figure 3.9: Comparison of Monte-Carlo simulation with equations (3.15) and (3.17)

medium SNRs when N is large. This gap, however, becomes smaller for smaller values of N and for higher values of SNR.

3.6 Appendices

3.6.1 Proof of Theorem 1: Average probability of error

Let us start with the definition of the average error probability:

$$\begin{aligned} P_E &= \int_0^\infty cQ(\sqrt{Mx}) f_{r:N}(x)dx \\ &= c\int_0^\infty \int_{\sqrt{Mx}}^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} f_{r:N}(x)dydx \end{aligned}$$

where c and M are constants determined by the type of modulation. By changing the order of integration, we have

$$\begin{aligned} P_E &= c\int_0^\infty \int_0^{y^2/M} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} f_{r:N}(x)dxdy \\ &= c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} F_{r:N}(y^2/M)dy. \end{aligned} \tag{3.20}$$

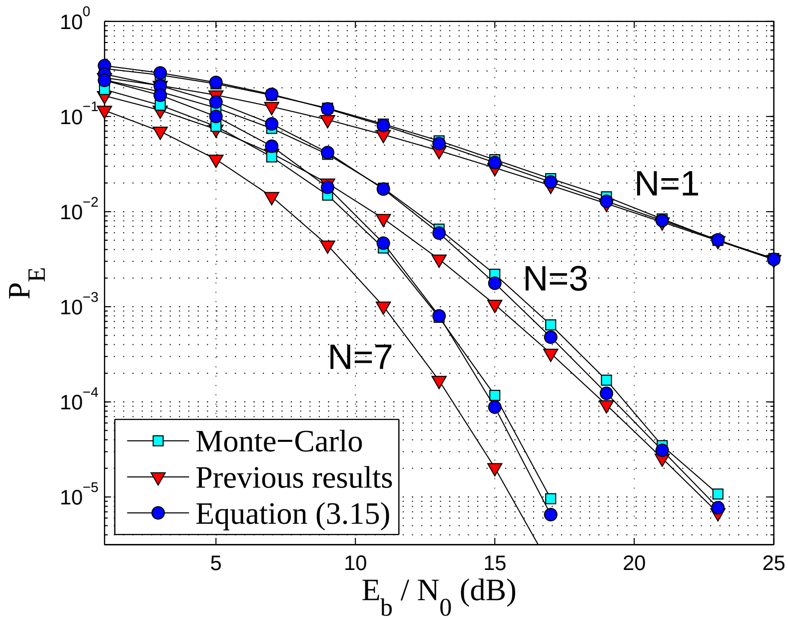

Figure 3.10: Comparison of Monte-Carlo simulation with the previous results ([44]) and Equation (3.15) for $r = N$ and $N = 1, 3, 7$. For the higher values of N, the match between the results of [44] and Monte-Carlo simulation occurs at higher SNR values.

By plugging from (3.4), we have

$$\begin{aligned}
P_E &= c\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}\sum_{i=0}^{r-1}\Lambda_i e^{-\lambda_i\frac{y^2}{M}}dy\\
&= c\sum_{i=0}^{r-1}\Lambda_i\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}(1+\lambda_i\frac{2}{M})}dy\\
&= c\sum_{i=0}^{r-1}\Lambda_i\frac{1}{\sqrt{2(1+\lambda_i\frac{2}{M})}}
\end{aligned}$$

and the proof is complete.

3.6.2 Proof of Theorem 2: Average probability of error

We start by plugging (3.2) to (3.20)

$$\begin{aligned}
P_E &= \int\limits_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\sum_{i=r}^{N}\binom{N}{i}(1-e^{-\lambda\frac{x^2}{2}})^i(e^{-\lambda\frac{x^2}{2}})^{N-i}dx\\
&= \sum_{i=r}^{N}\frac{1}{\sqrt{2\pi}}\binom{N}{i}\int\limits_0^{\infty}e^{-\frac{x^2}{2}}(1-e^{-\lambda\frac{x^2}{2}})^i(e^{-\lambda\frac{x^2}{2}})^{N-i}dx. \qquad (3.21)
\end{aligned}$$

First let us handle the internal integral, let us assume

$$\Delta(\lambda) = \int_0^{\infty}e^{-\frac{x^2}{2}}(1-e^{-\lambda\frac{x^2}{2}})^i(e^{-\lambda\frac{x^2}{2}})^{N-i}dx. \qquad (3.22)$$

By assuming high SNR (small λ), $e^{-\lambda\frac{x^2}{2}}$ in the first parentheses can be approximated by the first two terms in its Taylor expansion $1-\lambda\frac{x^2}{2}$, because the term $e^{-\frac{x^2}{2}}$ for high values of x tends to zero, so we have

$$\Delta(\lambda) = \int_0^{\infty}e^{-\frac{x^2}{2}}(\lambda\frac{x^2}{2})^i(e^{-\lambda\frac{x^2}{2}})^{N-i}dx.$$

Using integration by parts, $\int\alpha d\beta = \alpha\beta - \int\beta d\alpha$ and defining $d\beta = xe^{-\frac{x^2}{2}[(N-i)\lambda+1]}$, the first term ($\alpha\beta$) becomes

$$\alpha\beta = \left[(\frac{\lambda}{2})^i x^{2i-1}\frac{e^{-\frac{x^2}{2}[(N-i)\lambda+1]}}{-[(N-i)\lambda+1]}\right]_0^{\infty} = 0,$$

and the second term ($-\int \beta d\alpha$) becomes

$$-\int \beta d\alpha = \int_0^{\infty} (\frac{\lambda}{2})^i \frac{2i-1}{[(N-i)\lambda+1]} x^{2i-2} e^{-\frac{x^2}{2}[(N-i)\lambda+1]} dx.$$

Thus, after integration by parts, only one term remains. We repeat the integration by parts for i times. Each time the first term ($\alpha\beta$) becomes null and finally we have

$$\Delta(\lambda) = \frac{\lambda^i (2i-1)(2i-3)...(1)}{2^i [(N-i)\lambda+1]^i} \int_0^{\infty} e^{-\frac{x^2}{2}[(N-i)\lambda+1]} dx.$$

This integral is the integral of a Gaussian function, then we have

$$\begin{aligned}\Delta(\lambda) &= (\frac{\lambda}{2})^i \frac{(2i-1)(2i-3)...(1)}{[(N-i)\lambda+1]^i} \frac{1}{2} \sqrt{\frac{2\pi}{(N-i)\lambda+1}} \\ &= (\frac{\lambda}{2})^i \frac{(2i-1)(2i-3)...(1)}{[(N-i)\lambda+1]^{i+\frac{1}{2}}} \frac{\sqrt{2\pi}}{2}.\end{aligned}$$

After substituting the expressions for $\Delta(\lambda)$ and α, (3.7) results.

3.6.3 Proof of Theorem 3: CDF

Let the RV s X_1 and X_2 denote the instantaneous SNR s of the links $S \to R$ and $R \to D$, respectively. Their PDFs are given by

$$\begin{cases} X_1 \sim \delta e^{-\delta x} U(x) \\ X_2 \sim \sum_{i=0}^{r-1} \Lambda_i e^{-\lambda_i x} U(x) \end{cases} \tag{3.23}$$

Then we have

$$\begin{cases} \frac{1}{X_1} \sim \frac{\delta}{x^2} e^{-\delta/x} U(x) \\ \frac{1}{X_2} \sim \frac{1}{x^2} \sum_{i=0}^{r-1} \Lambda_i e^{-\lambda_i/x} U(x) \end{cases} \tag{3.24}$$

Similarly as [35], the moment generating function (MGF) of the variables $1/X_1$ and $1/X_2$ can be evaluated by the help of [48] (eq. (3.471.9)) and using the symmetry property of the modified Bessel function (i.e., $K_{-\nu}(z) = K_{\nu}(z)$) given in [48](eq. 8.486.16):

$$\begin{cases} M_1(s) = 2\sqrt{\delta s} K_1(2\sqrt{\delta s}) \\ M_2(s) = 2\sum_{i=0}^{r-1} \Lambda_i \sqrt{\frac{s}{\lambda_i}} K_1(2\sqrt{\lambda_i s}) \end{cases} \tag{3.25}$$

Therefore the MGF of $X = (1/X_1) + (1/X_2)$ is given by

$$\begin{aligned} M(s) &= \sum_{i=0}^{r-1} 2\sqrt{\delta s} K_1(2\sqrt{\delta s}) 2\Lambda_i \sqrt{\frac{s}{\lambda_i}} K_1(2\sqrt{\lambda_i s}) \\ &= \sum_{i=0}^{r-1} 4\Lambda_i \sqrt{\frac{\delta}{\lambda_i}} s K_1(2\sqrt{\delta s}) K_1(2\sqrt{\lambda_i s}). \end{aligned}$$

By using [49] Eq. (13.2.20) and the differentiation property of the Laplace transform, we can write the CDF of X as

$$\begin{aligned} F_X(x) &= \mathcal{L}^{-1}\{\frac{M(s)}{s}\} \\ &= 1 - \sum_{i=0}^{r-1} 2x\Lambda_i \sqrt{\frac{\delta}{\lambda_i}} e^{-x(\lambda_i+\delta)} K_1\left(2x\sqrt{\lambda_i \delta}\right) U(x). \end{aligned} \tag{3.26}$$

3.6.4 Proof of Theorem 4: PDF

We differentiate the CDF in (3.8) with respect to x. For this purpose we need the derivative of the modified Bessel function. This is given in [48], eq. (8.486.12):

$$\begin{aligned} & u\frac{d}{du}K_1(u) + K_1(u) = -uK_0(u) \\ & \Rightarrow \frac{d}{d(2x\sqrt{\lambda_i\delta})} K_1\left(2x\sqrt{\lambda_i\delta}\right) + \frac{1}{2x\sqrt{\lambda_i\delta}} K_1\left(2x\sqrt{\lambda_i\delta}\right) \\ & = -K_0\left(2x\sqrt{\lambda_i\delta}\right) \\ & \Rightarrow \frac{1}{2\sqrt{\lambda_i\delta}} \frac{d}{dx} K_1\left(2x\sqrt{\lambda_i\delta}\right) + \frac{1}{2x\sqrt{\lambda_i\delta}} K_1\left(2x\sqrt{\lambda_i\delta}\right) \\ & = -K_0\left(2x\sqrt{\lambda_i\delta}\right) \\ & \Rightarrow \frac{d}{dx} K_1\left(2x\sqrt{\delta\lambda_i}\right) + \frac{1}{x} K_1\left(2x\sqrt{\delta\lambda_i}\right) \\ & = -2\sqrt{\delta\lambda_i} K_0\left(2x\sqrt{\delta\lambda_i}\right). \end{aligned}$$

Substituting the last result in the derivative of $F_X(x)$ we have:

$$\begin{aligned} f_X(x) &= \sum_{i=0}^{r-1} 2x\Lambda_i \sqrt{\frac{\delta}{\lambda_i}} (\delta+\lambda_i) e^{-x(\delta+\lambda_i)} K_1\left(2x\sqrt{\delta\lambda_i}\right) U(x) + \sum_{i=0}^{r-1} 4x\Lambda_i \delta e^{-x(\delta+\lambda_i)} K_0\left(2x\sqrt{\delta\lambda_i}\right) U(x) \\ &= \sum_{i=0}^{r-1} 2x\Lambda_i \delta e^{-x(\delta+\lambda_i)} \left[\frac{(\delta+\lambda_i)}{\sqrt{\delta\lambda_i}} K_1\left(2x\sqrt{\delta\lambda_i}\right) U(x) + 2K_0\left(2x\sqrt{\delta\lambda_i}\right)\right] U(x). \end{aligned}$$

3.6.5 Proof of Theorem 5: Average probability of error

We start with (3.20)

$$P_E = c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} F_{r:N}(y^2/M)dy. \tag{3.27}$$

Before plugging the expression for $F_{r:N}(y^2/M)$ in the above formula, we can simplify it for high average SNR values, where $K_1(x)$ can be approximated by $1/x$, (see [50], Eq. (9.6.9)). Substituting in (3.8), the CDF of the considered link can be simplified as follows

$$\begin{aligned} F_{r:N}(x) &= 1 - \sum_{i=0}^{r-1} 2x\Lambda_i \sqrt{\frac{\delta}{\lambda_i}} e^{-x(\lambda_i+\delta)} K_1\left(2x\sqrt{\lambda_i\delta}\right) U(x) \\ &= 1 - \sum_{i=0}^{r-1} \Lambda_i \frac{1}{\lambda_i} e^{-x(\lambda_i+\delta)} U(x) \\ &= 1 - \sum_{i=0}^{r-1} \frac{1}{B}\binom{r-1}{i}(-1)^i \frac{e^{-x(\lambda_i+\delta)}}{(N-r+1+i)} U(x). \end{aligned} \tag{3.28}$$

Hence, by using the last result, the average error probability P_E can be evaluated as:

$$\begin{aligned} P_E &= c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} F_{r:N}(y^2/M)dy \\ &= c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} \left(1 - \sum_{i=0}^{r-1} \frac{1}{B}\binom{r-1}{i}(-1)^i \frac{e^{-\frac{y^2}{M}(\lambda_i+\delta)}}{(N-r+1+i)}\right) dy. \end{aligned}$$

This result leads to:

$$\begin{aligned} P_E &= \frac{c}{2} - \sum_{i=0}^{r-1} \frac{c}{B}\binom{r-1}{i}(-1)^i \frac{1}{(N-r+1+i)} \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-(y^2/2)(1+2(\lambda_i+\delta)/M)} dy \\ &= \frac{c}{2} - \sum_{i=0}^{r-1} \frac{c}{B}\binom{r-1}{i}(-1)^i \frac{1}{(N-r+1+i)} \frac{1}{2\sqrt{1+2(\lambda_i+\delta)/M}}. \end{aligned}$$

3.6.6 Proof of Theorem 6: CDF

The proof of this theorem is straightforwardly similar to that of theorem 3. The MGF of $X = (1/X_1) + (1/X_2)$ is

$$\begin{aligned}
M(s) &= \sum_{i=0}^{q-1} 2\Delta_i \sqrt{\frac{s}{\delta_i}} K_1(2\sqrt{\delta_i s}) \sum_{j=0}^{r-1} 2\Lambda_j \sqrt{\frac{s}{\lambda_j}} K_1(2\sqrt{\lambda_j s}) \\
&= \sum_{i=0}^{q-1}\sum_{j=0}^{r-1} 4\Delta_i\Lambda_j \frac{1}{\sqrt{\delta_i\lambda_j}} s K_1(2\sqrt{\delta_i s}) K_1(2\sqrt{\lambda_j s}).
\end{aligned}$$

Again by using the differentiation property of the Laplace transform, similar to (3.26) we have

$$F_X(x) = 1 - \sum_{i=0}^{q-1}\sum_{j=0}^{r-1} 2x\Delta_i\Lambda_j \frac{1}{\sqrt{\delta_i\lambda_j}} e^{-x(\delta_i+\lambda_j)} K_1\left(2x\sqrt{\delta_i\lambda_j)}\right) U(x).$$

3.6.7 Proof of Theorem 8: Average probability of error

Let us start from equation (3.20), then we have:

$$P_E = c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} F_{q,r:N}(y^2/M) dy.$$

Using the same simplification as in (3.28) for $F_{q,r:N}(y^2/M)$ we have

$$\begin{aligned}
P_E &= c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \left(1 - \sum_{i=0}^{q-1}\sum_{j=0}^{r-1} \frac{\Delta_i\Lambda_j}{\delta_i\lambda_j} e^{-y^2(\delta_i+\lambda_j)/2}\right) dy \\
&= \frac{c}{2} - c\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \sum_{i=0}^{q-1}\sum_{j=0}^{r-1} \frac{1}{B_1}\binom{r-1}{i}(-1)^i \frac{1}{B_2}\binom{q-1}{j}(-1)^j \frac{e^{-(y^2/2)(\lambda_i+\delta_j)}}{\lambda_i\delta_j} dy \\
&= \frac{c}{2} - c\sum_{i=0}^{q-1}\sum_{j=0}^{r-1} \frac{1}{B_1B_2}\binom{q-1}{i}\binom{r-1}{j}(-1)^{i+j}\frac{1}{\delta_i\lambda_j}\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-(y^2/2)(1+\delta_i+\lambda_j)} dy \\
&= \frac{c}{2} - \sum_{i=0}^{q-1}\sum_{j=0}^{r-1} \frac{c}{B_1B_2}\binom{q-1}{i}\binom{r-1}{j}(-1)^{i+j}\frac{1}{\delta_i\lambda_j}\frac{1}{2\sqrt{1+\delta_i+\lambda_j}}.
\end{aligned}$$

3.6.8 Proof of Theorem 9: Diversity order analysis

We introduce a new random variable Υ for the SNR of this link. Then, from (2.5) we have

$$\Upsilon = \frac{X_{SR}X_{RD}}{X_{SR}+X_{RD}+1} = \frac{X_1X_2}{X_1+X_2+1}$$

where X_{SR} and X_{RD} are respectively replaced by X_1 and X_2 for simplicity. CDF of Υ can be calculated by the following integral over the region $\Upsilon \le x$

$$F_{\Upsilon}(x) = \iint_D f_{X_1}(x_1) f_{X_2}(x_2) dx_1 dx_2. \tag{3.29}$$

Similar to Section 2.2, the integration surface D in (3.29) can be divided into two regions, namely D_1 and D_2, where D_1 shows the region $\{x_1 < x\} \cup \{x_2 < x\}$ and the remaining is denoted by D_2 (Fig. (2.2)). Let us denote the result of integral in (3.29) over D_1 and D_2 by $F_{\Upsilon_1}(x)$ and $F_{\Upsilon 2}(x)$, respectively. Then for D_1 we have:

$$F_{\Upsilon_1}(x) = \Pr\{X_1 \le x\} + \Pr\{X_2 \le x\} - \Pr\{X_1 \le x, X_2 \le x\} \tag{3.30}$$

where $\Pr\{X_1 \le x\} = F_{r:N}(x)$ and $\Pr\{X_2 \le x\} = F_{q:N}(x)$. Differentiating (3.30) gives the PDF of Υ_1 around zero. Then, we can write the Taylor expansion of $F_{\Upsilon_1}(x)$ around zero. Using proposition 1 in [41] (Section 3.1.4), this result implies that the system achieves diversity $min(r,q)$. For D_2 we have:

$$\begin{aligned}
\Pr\{\Upsilon_2 \le x\} &= \int_x^{\infty} \int_x^{\frac{x(x_2+1)}{x_2-x}} f_{X_1}(x_1) f_{X_2}(x_2) dx_1 dx_2 \\
&= \int_x^{\infty} f_{X_2}(x_2) \sum_{i=q}^{N} \binom{N}{i} \Bigg[\left(1-\exp\left(-\lambda x \frac{(x_2+1)}{x_2-x}\right)\right)^i \\
&\quad \times \left(\exp\left(-\lambda x \frac{(x_2+1)}{x_2-x}\right)\right)^{N-i} - (1-\exp(-\lambda x))^i (\exp(-\lambda x))^{N-i} \Bigg] dx_2.
\end{aligned}$$

Now, let us consider the average error probability for Υ_2. From (3.20) we have:

$$\begin{aligned}
P_{E_2} =& c\int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} F_{\Upsilon_2}(y^2/2) dy \\
=& c\int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \int_{y^2/2}^{\infty} f_{X_2}(x_2) \sum_{i=q}^{N} \binom{N}{i} \Bigg[\left(1-\exp\left(-\lambda \frac{y^2}{2} \frac{(x_2+1)}{x_2-\frac{y^2}{2}}\right)\right)^i \\
& \left(\exp\left(-\lambda \frac{y^2}{2} \frac{(x_2+1)}{x_2-\frac{y^2}{2}}\right)\right)^{N-i} - \left(1-\exp\left(-\lambda \frac{y^2}{2}\right)\right)^i \left(\exp\left(-\lambda\frac{y^2}{2}\right)\right)^{N-i} \Bigg] dx_2 dy.
\end{aligned}$$

In the above expression, the Taylor expansion of $f_{X_2}(x_2)$ in terms of λ is of order r (because X_2 is the r^{th} order statistic). The Taylor expansion of the terms in brackets in terms of λ is of order q. Hence the whole result of the integral (which is a number between zero and one) is of order $r+q$ at least. Hence, for large average SNR value, this term is negligible compared to the $P_{E_1} = k\lambda^{\min\{r,q\}}$ (resulting from (3.30)).

3.6.9 Proof of Theorem 10

For high SNR values, substituting $K_1(x) = \frac{1}{x}e^{-x^2/2}$ in (2.6), yields

$$F(\gamma) = 1 - \exp\left(-(1+k)\lambda\gamma - 2\lambda^2 k\left(\gamma^2+\gamma\right)\right).$$

Substituting this result in (3.1), gives the CDF

$$\begin{aligned}
F_{r:N}(\gamma) &= \sum_{i=r}^{N}\binom{N}{i}F^i(\gamma)\left(1-F(\gamma)\right)^{N-i} \\
&= \sum_{i=r}^{N}\binom{N}{i}\sum_{j=0}^{i}\binom{i}{j}(-1)^j\left(e^{-(1+k)\lambda\gamma-2\lambda^2k\left(\gamma^2+\gamma\right)}\right)^j\left(e^{-(1+k)\lambda\gamma-2\lambda^2k\left(\gamma^2+\gamma\right)}\right)^{N-i} \\
&= \sum_{i=r}^{N}\binom{N}{i}\sum_{j=0}^{i}\binom{i}{j}(-1)^j\exp\left(-(1+k)\lambda\gamma(N-i+j) - 2\lambda^2k\left(\gamma^2+\gamma\right)(N-i+j)\right).
\end{aligned}$$

For the average error probability, we substitute this result in (3.20). This produces

$$\begin{aligned}
P_E &= \int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-x^2/2}F_{r:N}(x^2/2)dx \\
&= \int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\sum_{i=r}^{N}\binom{N}{i}\sum_{j=0}^{i}\binom{i}{j}(-1)^j \\
&\quad \times \exp\left(-(1+k)\lambda\frac{x^2}{2}(N-i+j) - 2\lambda^2k\left(\frac{x^4}{4}+\frac{x^2}{2}\right)(N-i+j)\right)dx.
\end{aligned}$$

This last integral has the following form.

$$\Phi(A_{i,j}, B_{i,j}) = \int_0^\infty \exp\left(-B_{i,j}^2x^4 - 2A_{i,j}^2x^2\right)dx. \tag{3.31}$$

Since the modified Bessel function of the second kind widely appears in AF relaying [51] and by invoking Equation (2.6) which involves the modified Bessel function of the second kind, we try to express the result of the last integral in terms of the modified Bessel function of the second kind. By calculating the first and the second derivatives of (3.31), it is not so difficult to write their linear combination such that the results would be zero. This results in the following differential equation

$$x^2 y'' + xy' - \left(\frac{1}{16} + x^2\right) y = 0, \tag{3.32}$$

where $y(-\frac{Z^4}{2}) = 2\sqrt{2}z^{-1}\exp(-\frac{Z^4}{2})\Phi(A_{i,j}, B_{i,j})$. This is the general form of the modified Bessel's differential equation, therefore the answer turns out to be

$$\int_0^\infty e^{\left(-B_{i,j}^2 x^4 - 2A_{i,j}^2 x^2\right)} dx = 2^{-3/2} \frac{A_{i,j}}{B_{i,j}} \exp\left(\frac{A_{i,j}^4}{2B_{i,j}^2}\right) k_{\frac{1}{4}}\left(\frac{A_{i,j}^4}{2B_{i,j}^2}\right),$$

where $A_{i,j}^2 = \frac{1}{4} + \frac{1}{4}(1+k)\lambda(N-i+j) + \frac{1}{2}\lambda^2 k(N-i+j)$ and $B_{i,j}^2 = \frac{1}{2}\lambda^2 k\,(N-i+j)$. Substituting this result in P_E yields (3.15).

3.6.10 Proof of Equation (3.17)

We have

$$\begin{aligned} f_{r:N}(\gamma) &\approx \frac{(k+1)\lambda\left(1-e^{-\gamma(k+1)\lambda}\right)^{r-1}\left(e^{-\gamma(k+1)\lambda}\right)^{N-r+1}}{\beta(r, N-r+1)} \\ &= \frac{(k+1)\lambda}{\beta(r, N-r+1)} \sum_{v=0}^{r-1} \binom{r-1}{v} (-1)^v e^{-\gamma(k+1)\lambda(v+N-r+1)}. \end{aligned}$$

In order to use (3.20), $F_{r:N}(\gamma)$ is needed. For this purpose, the inner integral in $F_{r:N}(x^2/2) = \int_0^{x^2/2} f_{r:N}(\gamma)d\gamma$ becomes

$$\Delta(x) = \int_0^{x^2/2} e^{-\gamma(k+1)\lambda(v+N-r+1)} d\gamma = -\frac{1}{(k+1)\lambda(v+N-r+1)}\left(e^{-\frac{x^2}{2}(k+1)\lambda(v+N-r+1)} - 1\right).$$

Substituting this result in (3.20) and integrating over x yields (3.17).

3.6.11 Proof of Equation (3.19)

In order to use (3.20), we need to calculate $F_{r:N}(\frac{x^2}{2}) = \int_0^{x^2/2} f_{(r)}(\gamma)d\gamma$

$$\begin{aligned} F_{r:N}(\frac{x^2}{2}) &= \int_0^{x^2/2} \frac{((k+1)\lambda)^r}{\beta(r, N-r+1)} \gamma^{r-1} d\gamma \\ &= \frac{((k+1)\lambda)^r}{\beta(r, N-r+1)r}\left(\frac{x^2}{2}\right)^r. \end{aligned}$$

If we change the integration variable $w = x^2/2$ and express the result of integration in terms of Gamma function, (3.19) is obtained.

4

Sequential Relaying

4.1 Introduction

In this chapter, we propose a relay assignment scheme for a network comprising a cluster of N source nodes, a cluster of N relay nodes and a single destination. This network configuration falls under the framework of uplink transmission in cellular networks. As for the destination node, it could be thought of as a base-station and the N transmitting nodes as cellular users. This was motivated by the fact that the existing relay assignment schemes fail to achieve diversity for all transmitting nodes in this network configuration.

To address this issue, we propose a new and simple relay assignment scheme where, for each set of channel realizations, the sources sequentially choose their relays among the remaining relays. In the relay assignment process, the priority of the source nodes for relay-selection is based on the quality of the source-destination links, i.e. the source nodes that have weaker source-destination channels, have higher priority in getting assigned relays with stronger relay-destination links. It is assumed that only one relay node is assigned to a single transmitting node, which has been shown to have the capability to maximize the network throughput [8], [52]. As such, the number of relays could be more than N. Since each source benefits from both its direct channel to the destination and that through the assigned relay, the proposed scheme achieves balance among different sources, and therefore all sources achieve the same diversity, as it will be shown. We propose two different versions of the proposed scheme: one for AF relaying and one for DF relaying. We statistically analyze the proposed scheme where we invoke important results that we derived for simple two-hop networks in Chapter 2. Specifically, we derive an *exact* expression for the PDF of the E2E SNR.

4.1.1 System Model

The relay network under consideration is shown in Fig. 4.1 where there are N terminals in each cluster and a single destination. Without loss of generality, for each realization of the channels, S_r and R_r denote the terminal with the r^{th} weakest source-destination channel ($S_r \rightarrow D$) and the relay with the r^{th} weakest relay-destination channel ($R_r \rightarrow D$), respectively. $\Gamma_{i,j,D}$ denotes the equivalent E2E SNR of the $S_i \rightarrow R_j \rightarrow D$ link.

We also make the following assumptions. All of the channels in one cluster are assumed to be independent and identically distributed (i.i.d.) Rayleigh-fading channels. The channels in two clusters are also independent. The channels are slow fading and remain constant during the resource allocation process. For cooperation, a two-slot relay mode is employed (similar to Chapter 2). In the first time slot, the source nodes transmit and the relays and destination receive. In the second time slot, the relay nodes transmit and the destination receives. Simultaneous transmitting nodes use orthogonal channels. All nodes are assumed to work in the half-duplex mode, i.e. they cannot transmit and receive at the same time. We assume that the SNR information of all links ($\Gamma_{S_i,D}$, $\Gamma_{R_j,D}$ and $\Gamma_{i,j}$) are available to the resource allocator, which can be centralized or semi distributed.

Throughout the chapter, the PDF and the CDF of random variable Γ are denoted by $f_\Gamma(\gamma)$ and $F_\Gamma(\gamma)$, respectively. For the joint statistics of multiple random variables, the joint PDF of $\Gamma_1, \Gamma_2, ..., \Gamma_N$ is denoted by $P_{1,2,...,N}(\gamma_1, \gamma_2, ..., \gamma_N)$. If we integrate this function with respect to Γ_1 in order to calculate the CDF, the result is denoted by $P_{1,2,...,N}(\gamma_1 \leq \gamma, \gamma_2, ..., \gamma_N)$.

4.1.2 Contributions of this Chapter

1. We propose a simple relay assignment technique for the mentioned network configuration where it is shown that all transmitting nodes achieve space diversity.

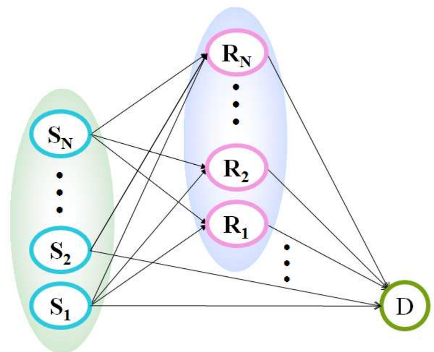

Figure 4.1: System model. $\Gamma_{S_i,D}$: direct link to the destination; $\Gamma_{i,j}$: link between terminals

The proposed technique applies to both AF and DF relaying.

2. We provide rigorous performance analysis of the proposed scheme for AF and DF modes where we derive a closed-form expression for the E2E BER performance.
3. In deriving the E2E BER expression in Step 2, we invoke intermediate results that we derived for a simple two-hop network comprising a source, a relay and a destination (see Chapter 2).

4.2 Proposed Sequential AF Relaying

In this section, we propose a sequential relay-assignment algorithm for AF relaying and analyze its performance while invoking the results obtained in previous sections.

4.2.1 Algorithm Outline

1. By using the SNR information of both $\Gamma_{S_1,j}$ and $\Gamma_{R_j,D}$, terminal S_1 is assigned the best relay among R_j, for $j = 1, 2, \ldots, N$. That is, the relay that maximizes the E2E SNR for the weakest user is selected.
2. Then S_2 is assigned the best relay among the remaining relays. In general S_r is assigned the best relay among the $N - r + 1$ remaining relays.
3. The above steps repeat until all source nodes are assigned relays.

4.2.2 Performance Analysis

For S_r, we assume that the best $r-1$ relays have already been assigned to sources $S_1, S_2, \ldots, S_{r-1}$ (this assumption is not necessarily true because the best relays for two different sources are not necessarily the same). Hence in the worst case, S_r can select the $(N-r+1)$-th best relay.

Theorem 1. *The CDF of the E2E SNR and the BER of the r^{th} weakest source (S_r) are given by (4.2) and (4.3), respectively.*

$$F_{eq(r)}(\gamma) = \sum_{s=1}^{L}\sum_{t=s}^{N}\binom{N}{t}\left(1-e^{-\lambda\gamma}\right)^{t}\left(e^{-\lambda\gamma}\right)^{N-t} e^{-k\lambda\gamma}(1-e^{-k\lambda\gamma})^{L-s} + (1-e^{-k\lambda\gamma})^{L} \tag{4.1}$$

$$= \sum_{s=1}^{L}\sum_{t=s}^{N}\sum_{u=0}^{t}\sum_{v=0}^{L-s}\binom{N}{t}\binom{t}{u}\binom{L-s}{v}(-1)^{u+v}e^{-\lambda\gamma\left(u+N-t+k(1+v)\right)} + (1-e^{-k\lambda\gamma})^{L} \tag{4.2}$$

$$P_E = \sum_{s=1}^{L}\sum_{t=s}^{N}\sum_{u=0}^{t}\sum_{v=0}^{L-s}\binom{N}{t}\binom{t}{u}\binom{L-s}{v}\frac{(-1)^{u+v}}{2\sqrt{1+\frac{2}{M}\lambda\left(u+N-t+k\left(1+v\right)\right)}} + \sum_{v=0}^{L}\binom{L}{v}\frac{(-1)^{v}}{2\sqrt{1+\frac{2}{M}\lambda v}}. \tag{4.3}$$

where $L = N - r + 1$. When $k = 1$, i.e., the two clusters have the same average SNR, (4.2) and (4.3) simplify to

$$F_{eq(r)}(\gamma) = \sum_{s=1}^{L}\sum_{t=s}^{N}\binom{N}{t}\left(1-e^{-\lambda\gamma}\right)^{t+L-s}\left(e^{-\lambda\gamma}\right)^{N-t+1} + (1-e^{-\lambda\gamma})^{L} \tag{4.4}$$

$$P_E = \sum_{s=1}^{L}\sum_{t=s}^{N}\sum_{u=0}^{t+L-s}\binom{N}{t}\binom{t+L-s}{u}\frac{(-1)^{u}}{2\sqrt{1+\frac{2}{M}\lambda(u+N-t+1)}} + \sum_{v=0}^{L}\binom{L}{v}\frac{(-1)^{v}}{2\sqrt{1+\frac{2}{M}\lambda v}}. \tag{4.5}$$

respectively.

Proof. See appendix 4.4. ∎

Using this relay assignment algorithm, S_r achieves a diversity order of $N-r+1$. This result can be easily confirmed by considering the Taylor series expansion of (4.1) and using Proposition 1 in [41]. According to this proposition, when the PDF of the SNR is approximated by a single polynomial term when $\gamma \to 0^+$ ($f_\Gamma(\gamma) = a\gamma^t + O(\gamma^{t+\epsilon})$), the system has a diversity order of $t+1$, where $\epsilon > 0$ and a is a positive constant (see Section 3.1.4).

On the other hand, the direct link $S_r \to D$ provides a diversity order equal to r [16]. Thus, using maximum ratio combining, we can conclude that the diversity order of each terminal S_r is increased to $N+1$ (because $r + (N-r+1) = N+1$). This means that by using this relay assignment technique we can achieve full fairness among all sources.

Fig. 4.2 shows some Monte-Carlo results for the E2E BER referring to the proposed relay assignment method when $N = 4$. The analytical results based on (4.5) are also shown for comparison. A good match is observed for high SNRs. It is also observed that for example, the second weakest user (corresponding to $r = 2$), achieves diversity three and the strongest one achieves diversity one. We remark that the direct channel between each source and the destination is not considered in these simulations, which would otherwise achieve diversity five for all the users. Fig. 4.3 shows some Monte-Carlo simulations for the E2E BER referring to the proposed method along with other relay assignment criteria when $N = 4$. In the figure, the direct channel between each source and the destination is also considered. Therefore, all users achieve the same diversity order, which is $N+1 = 5$. As it is obvious from this figure, this method outperforms other methods of relay assignment in terms of diversity order.

4.3 Proposed Sequential DF Relaying

In this section, we propose a sequential relay-assignment algorithm for DF relaying and analyze its performance while invoking the results obtained in previous sections.

4.3.1 Algorithm Outline

1. First, S_1, the source with the weakest direct channel to the destination, selects its relay. For this purpose, if R_N can successfully decode the message, it will be

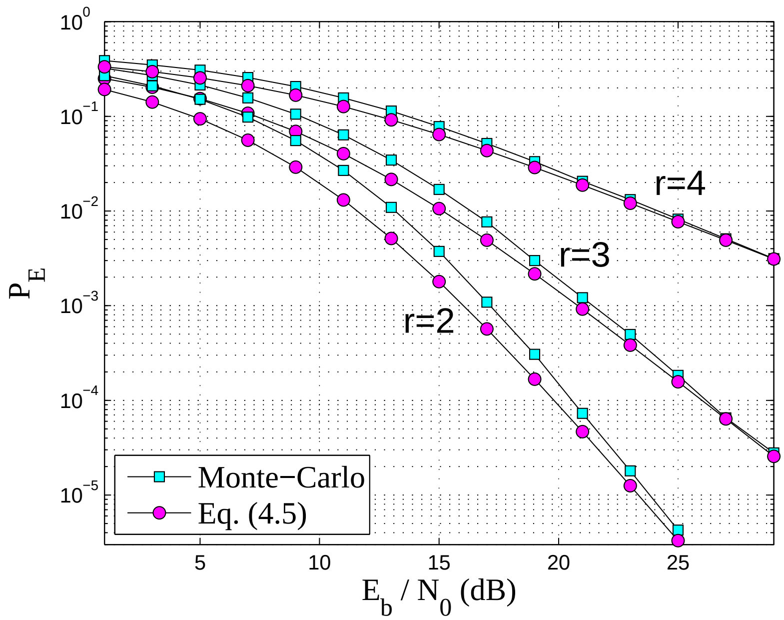

Figure 4.2: Monte-carlo simulation results of P_E referring to different users U_r, $r = 2, 3, 4$ through the indirect link when $2N = 8$ compared with equation (4.5)

selected as the relay. If R_N cannot successfully decode the message, we should look at the next terminal (R_{N-1}). If this terminal can successfully decode the message, it will be selected as the relay, otherwise, we continue in the same manner.

2. After S_1 selected its relay, S_2 selects its relay among the remaining relays by using the same algorithm.
3. The above steps are repeated until all source nodes are assigned relays.

4.3.2 Performance Analysis

In order to find an expression for P_E of S_r, again we assume that the best $r-1$ relays have already been assigned to the previous sources. Hence, the following links are available to S_r: $S_r \to R_j \to D$ where $1 \leq j \leq (N - r + 1)$. Now we are ready to formulate the result:

If R_{N-r+1} can successfully decode the message, this terminal will be selected as the relay. From (2.9), it is inferred the probability of this event is $1 - \alpha$. The average probability of error P_E for the link $R_{N-r+1} \to D$ is given by (3.6) or (3.7). Therefore

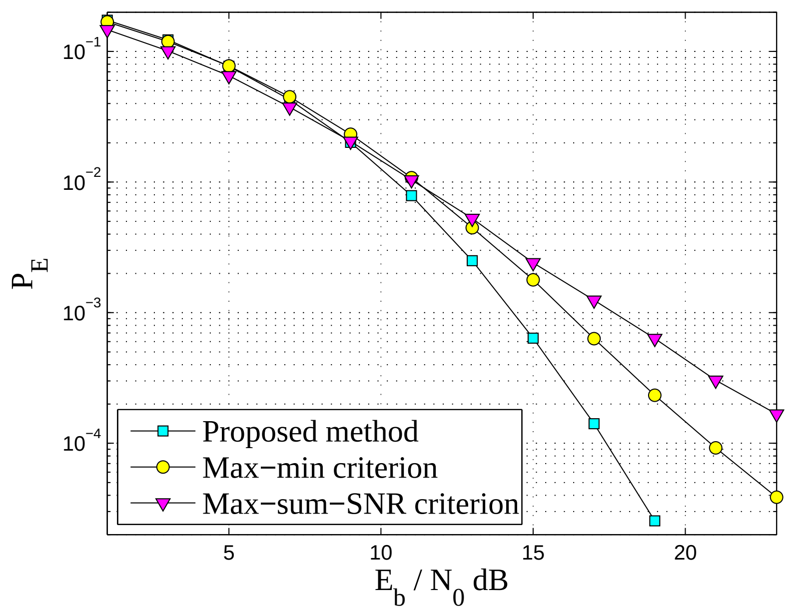

Figure 4.3: Comparison of P_E for different relay-assignment methods for N=4 using Monte-carlo simulation

the total probability of error for this case is $(1-\alpha)P_{e(N-r+1:N)}$. If R_{N-r+1} cannot successfully decode the message, we should look at the next terminal (R_{N-r}). If this terminal can successfully decode the message, it will be selected as the relay, the probability of this event is $\alpha(1-\alpha)$, so the total probability of error for this second case is $\alpha(1-\alpha)P_{e(N-r+1:N)}$. By the same reasoning, when $R_{N-r+1-i}$ is selected as the relay, the total probability of error is $\alpha^i(1-\alpha)P_{e(N-r+1-i:N)}$. Hence the total probability of error is

$$P_E = \frac{1}{2}\alpha^{N-r+1} + \sum_{i=0}^{N-r} \alpha^i(1-\alpha)P_{e(N-r+1-i:N)} \tag{4.6}$$

The first term $\frac{1}{2}\alpha^{N-r+1}$ represents the case when none of the relays can successfully decode the message, because in this case, the probability of error is $\frac{1}{2}$. If we replace (3.7) in (4.6), we have

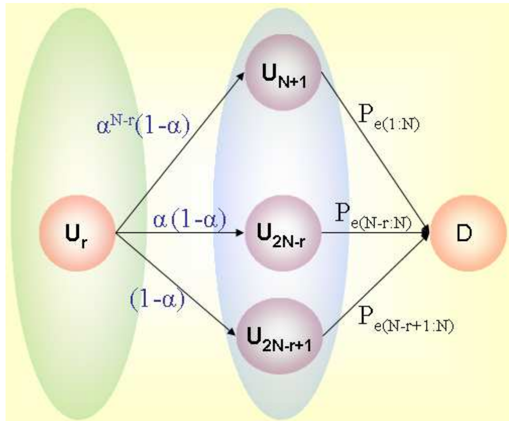

Figure 4.4: Block diagram of DF with complete SNR information showing the selection probability of each relay

$$
\begin{aligned}
P_E =& \frac{1}{2}\left(1-e^{-k\lambda\gamma_{th}}\right)^{N-r+1} + \sum_{i=0}^{N-r}\left(1-e^{-k\lambda\gamma_{th}}\right)^{i} e^{-k\lambda\gamma_{th}} \\
& \times \sum_{j=N-r+1-i}^{N} \binom{N}{j} \frac{\lambda^j}{2^{j+1}} \frac{(2j-1)(2j-3)...(1)}{[(N-j)\lambda+1]^{j+\frac{1}{2}}}
\end{aligned} \tag{4.7}
$$

After some manipulations, it is found that the smallest power of λ in the Taylor series expansion of (4.7) is $N-r+1$. This result implies that the system has a diversity order of $N-r+1$. On the other hand, similar to the AF mode, S_r benefits from its direct channel to the destination. Since the direct link $S_r \rightarrow D$ achieves diversity r, its total diversity order is $N+1$. Figure (4.5) shows the probability of error for the indirect link in a system with $2N = 8$ and $r = 2$. In this simulation, similar to [38], we have assumed $\gamma_{th} = 0dB$. The diversity order 3 is obvious from this picture. This figure also shows that by increasing the SNR, equation (4.7) becomes a better approximation for P_E.

4.4 Appendix: Proof of Equation (4.2)

Let us sort the elements of $\Gamma_{j,D}$ and form their order statistics. Without loss of generality, let us assume that R_j has the j^{th} weakest SNR in the second hop. For the

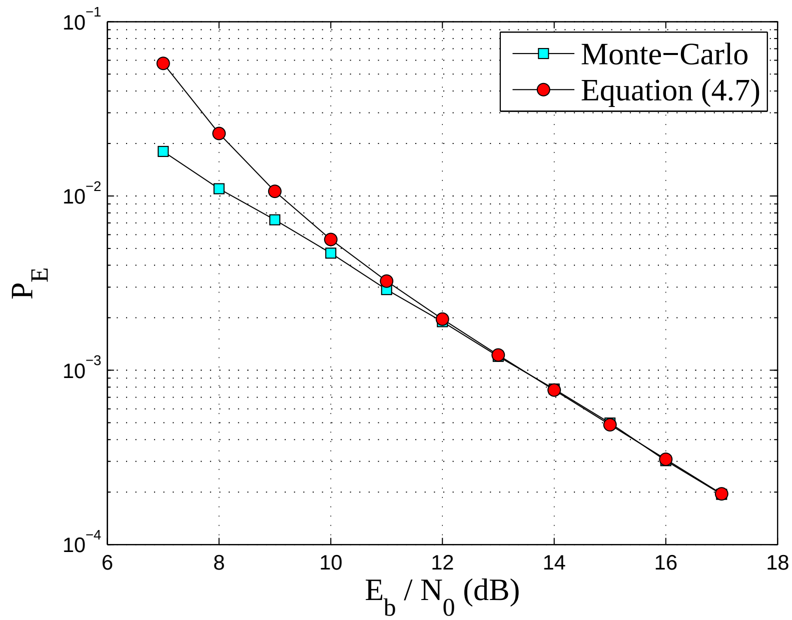

Figure 4.5: Probability of error for two-hop link with DF and complete SNR information ($2N = 8$ and $r = 2$)

sake of abbreviation we denote the SNR of the link $R_j \rightarrow D$ by Y_j (i.e. Y_1 denotes the weakest SNR in the second hop). For the proposed method of relay-assignment, the equivalent CDF of the SNR for S_r is

$$
\begin{aligned}
F_{eq(r)}(\gamma) &= \Pr\{\max_{1\leq i\leq N-r+1} \Gamma_{r,i,D} \leq \gamma\} \\
&= \Pr\{\Gamma_{r,1,D} \leq \gamma, \Gamma_{r,2,D} \leq \gamma, ..., \Gamma_{r,N-r+1,D} \leq \gamma\}.
\end{aligned}
$$

The distributions of the SNR for the second channel of the considered links are dependent (Y_j's are order statistics). In order to find $F_{eq(r)}(\gamma)$ we should integrate the joint PDF over all of the contributing channels in the space specified by

$$
\Gamma_{r,1,D} \leq \gamma, \Gamma_{r,2,D} \leq \gamma, ..., \Gamma_{r,N-r+1,D} \leq \gamma.
$$

In the first step we calculate the probability of $\Gamma_{r,1,D} \leq \gamma$ by using the approximation in Theorem 2. For brevity, let $L = N - r + 1$. Then,

$$\begin{aligned}
&\Pr\{\Gamma_{r,1,D} \leq \gamma\} \\
&= P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2, \ldots, y_L\right) \Pr\{\Gamma_{r,1} \geq \gamma\} + P_{1,2,\ldots,L:N}\left(y_1 \leq y_2, y_2, \ldots, y_L\right) \Pr\{\Gamma_{r,1} \leq \gamma\} \\
&= P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2, \ldots, y_L\right) e^{-k\lambda\gamma} + P_{1,2,\ldots,L:N}\left(y_1 \leq y_2, y_2, \ldots, y_L\right)\left(1 - e^{-k\lambda\gamma}\right),
\end{aligned}$$

where each term is the integral over one of the strips in D_1. For simplicity, let us denote this result by $a_{1,2}(y_2, \ldots, y_L, \gamma)$ and denote $\int_0^\gamma a_{1,2}\left(y_2, \ldots, y_L, \gamma\right) dy_2$ by $A_{1,2}\left(y_2 \leq \gamma, \ldots, y_L, \gamma\right)$. In the second step we calculate the CDF of $\Gamma_{r,2,D} \leq \gamma$ again by using the approximation in Theorem 2. That is,

$$\Pr\{\Gamma_{r,1,D} \leq \gamma, \Gamma_{r,2,D} \leq \gamma\} = A_{1,2}\left(y_2 \leq \gamma, y_3, \ldots, y_L\right) e^{-k\lambda\gamma} + A_{1,2}\left(y_2 \leq y_3, y_3, \ldots, y_L\right)\left(1 - e^{-k\lambda\gamma}\right).$$

By replacing for $A_{1,2}$ we have

$$\begin{aligned}
\Pr\{\Gamma_{r,1,D} \leq \gamma, \Gamma_{r,2,D} \leq \gamma\} = {} & P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2 \leq \gamma, y_3, \ldots, y_L\right) e^{-k\lambda\gamma} + \\
& + P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2 \leq y_3, \ldots, y_L\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma}) \\
& + P_{1,2,\ldots,L:N}\left(y_1 \leq y_3, y_2 \leq y_3, y_3, \ldots, y_L\right)\left(1 - e^{-k\lambda\gamma}\right)^2.
\end{aligned}$$

For simplicity, let us denote this result by $a_{1,2,3}\left(y_3, \ldots, y_L, \gamma\right)$ and denote $\int_0^\gamma a_{1,2,3}\left(y_3, \ldots, y_L, \gamma\right) dy_3$ by $A_{1,2,3}\left(y_3 \leq \gamma, \ldots, y_L, \gamma\right)$. In the third step we calculate the CDF of $\Gamma_{r,3,D} \leq \gamma$ by using the approximation in Theorem 2

$$\begin{aligned}
&\Pr\{\Gamma_{r,1,D} \leq \gamma, \Gamma_{r,2,D} \leq \gamma, \Gamma_{r,3,D} \leq \gamma\} \\
&\qquad = A_{1,2,3}\left(y_3 \leq \gamma, y_4, \ldots, y_L, \gamma\right) e^{-k\lambda\gamma} + A_{1,2,3}\left(y_3 \leq y_4, y_4, \ldots, y_L, \gamma\right)\left(1 - e^{-k\lambda\gamma}\right).
\end{aligned}$$

Substituting for $A_{1,2,3}$, we have

$$\begin{aligned}
&\Pr\{\Gamma_{r,1,D} \leq \gamma, \Gamma_{r,2,D} \leq \gamma, \Gamma_{r,3,D} \leq \gamma\} \\
&\qquad = P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2 \leq \gamma, y_3 \leq \gamma, y_4, \ldots, y_L\right) e^{-k\lambda\gamma} \\
&\qquad\quad + P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2 \leq \gamma, y_3 \leq y_4, y_4, \ldots, y_L\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma}) \\
&\qquad\quad + P_{1,2,\ldots,L:N}\left(y_1 \leq \gamma, y_2 \leq y_4, y_3 \leq y_4, y_4, \ldots, y_L\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma})^2 \\
&\qquad\quad + P_{1,2,\ldots,L:N}\left(y_1 \leq y_4, y_2 \leq y_4, y_3 \leq y_4, y_4, \ldots, y_L\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma})^3.
\end{aligned}$$

We should proceed in the same manner, but the results can be simplified in each step using

$$P_{i,j:N}\left(y_i \le \gamma, y_j \le \gamma\right) = P_{j:N}\left(y_j \le \gamma\right) \tag{4.8}$$

$$P_{i,j:N}\left(y_i \le \gamma, y_j \to \infty\right) = P_{i:N}\left(y_i \le \gamma\right). \tag{4.9}$$

Hence if y_{N-r+2} tends to infinity, for the last step we have

$$\begin{aligned}
F_{eq(r)}(\gamma) = {} & P_{L:N}\left(y_L \le \gamma\right) e^{-k\lambda\gamma} + P_{L-1:N}\left(y_{L-1} \le \gamma\right) e^{-k\lambda\gamma}\left(1 - e^{-k\lambda\gamma}\right) + \\
& + P_{L-2:N}\left(y_{L-2} \le \gamma\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma})^2 + \cdots \\
& + P_{1:N}\left(y_1 \le \gamma\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma})^{L-1} + (1 - e^{-k\lambda\gamma})^L.
\end{aligned}$$

This result can be written in the following shortened form

$$\begin{aligned}
F_{eq(r)}(\gamma) &= \sum_{s=1}^{L} F_{s:N}\left(\gamma\right) e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma})^{L-s} + (1 - e^{-k\lambda\gamma})^L \\
&= \sum_{s=1}^{L}\sum_{t=s}^{N}\binom{N}{t}\left(1 - e^{-\lambda\gamma}\right)^t\left(e^{-\lambda\gamma}\right)^{N-t} e^{-k\lambda\gamma}(1 - e^{-k\lambda\gamma})^{L-s} + (1 - e^{-k\lambda\gamma})^L \\
&= \sum_{s=1}^{L}\sum_{t=s}^{N}\sum_{u=0}^{t}\sum_{v=0}^{L-s}\binom{N}{t}\binom{t}{u}\binom{L-s}{v}(-1)^{u+v} e^{\left(-\lambda\gamma\left(u+N-t+k(1+v)\right)\right)} + (1 - e^{-k\lambda\gamma})^L.
\end{aligned}$$

Based on this result, we can easily calculate the average error probability. For the last term, we can proceed by using integration by parts. Finally we have

$$\begin{aligned}
P_E = {} & \sum_{s=1}^{L}\sum_{t=s}^{N}\sum_{u=0}^{t}\sum_{v=0}^{L-s}\binom{N}{t}\binom{t}{u}\binom{L-s}{v}\frac{(-1)^{u+v}}{2\sqrt{1 + \frac{2}{M}\lambda\left(u + N - t + k\left(1 + v\right)\right)}} \\
& + \sum_{v=0}^{L}\binom{L}{v}\frac{(-1)^v}{2\sqrt{1 + \frac{2}{M}\lambda v}}
\end{aligned}$$

The proof for $k = 1$ follows the same lines.

5

Relaying Based on Max-min Criterion

5.1 Introduction

This chapter deals with relay assignment in cooperative networks based on the max-min criterion. First, we consider this problem in a network comprising multiple source-destination pairs and multiple relays. We need each pair of the nodes to be assigned a relay and each relay serves only one pair. Instead of comparing all possible source-relay combinations, a simple algorithm is proposed in order to find the *optimum* relay assignment permutation based on the max-min criterion. The simplicity of the proposed algorithm stems from the fact that it involves simple matrix manipulations, as opposed to examining all possible permutations, which can be prohibitively complex for large networks. Then it is proved that the optimum permutation based on this criterion achieves maximum diversity. Different adaptations of the proposed algorithm are developed to solve the relay assignment problem in different network configurations, i.e. clustered two-hop networks and clustered multi-hop networks. In carrying out the analysis, we show that the PDF of the E2E SNR after relay assignment can be expressed as a weighted sum of the order statistics of the PDF of the individual E2E links. Since the analytical calculation of the weighting coefficients in the mentioned weighting average becomes difficult, we propose an approximation in order to calculate the mentioned weighting coefficients. The validity of the analytical results is confirmed by computer simulations.

5.1.1 Motivation and Contributions of This Chapter

Among all of the relay-assignment approaches (described in Section 1.4), the max-min criterion is an interesting choice because it can achieve full diversity in many scenarios, but finding the optimal permutation based on this criterion is prohibitively complex. Consider a clustered network where there are N source-destination pairs and M relays in the network where $N \leq M$. The problem is how to assign a relay to each source-destination pair in order to achieve the highest possible diversity order. We assume that each relay can serve only one pair, so that the network achieves fairness among different relays. Finding this optimal permutation through exhaustive search can be very difficult.[1] For example, for a set of 20 source-destination pairs and 20 relay nodes, about 2.4×10^{18} different permutations should be compared in order to find the optimal permutation. Another challenge here is that the statistical analysis of the optimal permutation, which is known to be untractable because of the correlation among different permutations.

In this chapter, we address this problem by proposing a simple approach that achieves the optimal solution without going through all permutations, the traditional way. The proposed algorithm is presented in the context of a network which consists of multiple source-destination pairs and multiple relays. This network configuration has several applications in ad-hoc networks and wireless sensor networks. For instance, in sensor networks, there are various spatially distributed sensors to monitor environmental and physical conditions, such as temperature, sound, vibration, and so on. Each sensor needs to send its information to its corresponding destination. The same concept applies to ad-hoc networks and similar network configurations.

The contributions of the chapter are summarized as follows.

1. We consider a clustered two-hop network in which the number of relays is greater than or equal to the number of transmitting pairs. We propose an algorithm to

1. Later in this chapter, we will prove that the optimal permutation based on max-min criterion brings full diversity.

find the optimal permutation based on the max-min criterion whereby a single relay is assigned to a pair.

2. We show that the proposed algorithm achieves full spatial diversity, i.e., the number of available relays for all pairs.

3. We provide a framework for analyzing the proposed algorithm, which is based on the so-called weighting functions. With this framework, we are able to obtain very accurate expressions for the bit error rate performance.

4. We adapt the proposed algorithm to clustered three-hop and multi-hop networks with favorable results, i.e., we show that the maximum spatial diversity is achieved by all pairs.

5. The proposed algorithm achieves the optimal solution at a lower complexity. It can also be adapted to other network structures.

The rest of this chapter is organized as follows. Section 5.2 describes some preliminaries. The proposed algorithm for different network configurations is studied in Sections 5.3-5.5. Each section includes the diversity analysis of the corresponding proposed algorithm. Section 5.3 also includes the bit-error-rate (BER) analysis of the proposed algorithm.

5.2 The Max-min Criterion

The detailed description of the max-min criterion is as follows.

– For each channel realization, the E2E SNR of all the links is calculated.

– We refer to each set of the one-to-one assignments of relays to the source-destination pairs as a "permutation". The minimum E2E SNR for each permutation is calculated.

– Among all the permutations, the one with the largest minimum SNR is selected. If two permutations satisfy the above condition, the permutation which maximizes the second minimum SNR for the selected links is selected.

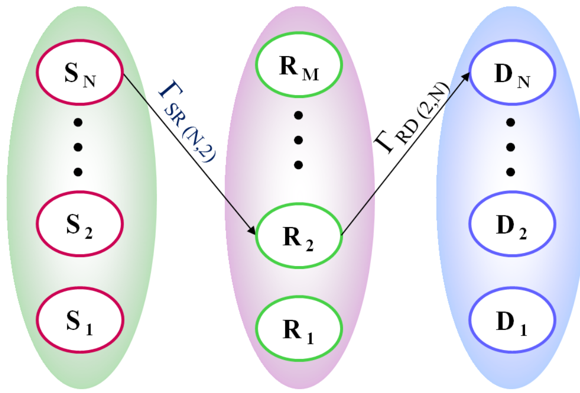

Figure 5.1: System model consisting of N source-destination pairs and M relays

5.3 Clustered Two-hop Network

5.3.1 System Model

Consider the network illustrated in Fig. 5.1, which consists of N source-destination pairs and M relays where $N \leq M$. Each source-destination pair should communicate through one relay node using orthogonal channels and each relay can serve only one pair. We assume that there is no direct path between the sources and destinations. However, the relay assignment algorithm proposed here can also be applied to the case when there is a direct path. Each of the nodes is equipped with a single antenna and operates in a half-duplex mode. We assume that a two-hop relay mode is employed. In the first hop, the source terminals transmit using orthogonal channels and the relays receive. In the second hop, the relay terminals transmit and the destinations receive (again by using orthogonal channels). We assume that a centralized or semi distributed resource allocation is employed and the SNR values of all of the channels are known to the resource allocator. We admit that this assumption requires adding some overhead and the amount of this overhead increases with the size of the network. Therefore difficulty of the implementation of this scheme grows with the number of the nodes, which is a known problem for such networks.

Each relay node retransmits the received signal to the corresponding destination using either amplify-and-forward (AF) or decode-and-forward (DF) relaying. The proposed relay assignment algorithm can be applied to AF or DF cooperation scheme. However, in this chapter, we consider the AF case. Furthermore, the relays transmit their signal using orthogonal channels (either in time or frequency).

There are NM elements in each realization of the channels which can be written in a matrix form as

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} & \dots & \Gamma_{1,M} \\ \Gamma_{2,1} & \Gamma_{2,2} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{N,1} & \dots & \dots & \Gamma_{N,M} \end{bmatrix},$$

where $\Gamma_{i,j}$ represents the equivalent SNR of the link $S_i \to R_j \to D_i$. Assignment of relay j to source-destination pair i corresponds to the selection of $\Gamma_{i,j}$ from the above matrix.

5.3.2 Proposed Algorithm For $N = M$

Since each relay should be assigned to one and only one source-destination pair, we should select only one element from each row and from each column of this matrix. There are $N!$ different permutations. Let us sort the elements of $\mathbf{\Gamma}$ and denote the sorted elements by $\Gamma_{i:N^2}$, where $\Gamma_{1:N^2}$ and $\Gamma_{N^2:N^2}$ denote the smallest element and the largest element of the matrix, respectively. Here is a rough description of the proposed algorithm to select the optimal permutation based on the max-min criterion.

I) Starting from the smallest element $\Gamma_{1:N^2}$, in each step one element is labeled in $\mathbf{\Gamma}$.

II) At any moment, if there is only one remaining element in any row or column of the matrix (which is not labeled), this element is selected for the optimal permutation. In this case we delete the corresponding row and column of the matrix

III) At any moment, if the number of rows or columns with at least one unlabeled element is less than the number of needed elements, we should go back to the state when the last element was labeled. We should select this last element for the optimal permutation and continue from that step.

In the following, we give a pseudocode for the proposed algorithm.

1. $\Omega = \{\}$ *% The set of the elements of the optimal permutation*
2. Set $n_\Omega = 0$ *% Number of the elements in* Ω

3. Set $m_{row} = N$ *% Number of rows in* $\mathbf{\Gamma}$ *that have unlabeled elements*

4. Set $m_{col} = N$ *% Number of columns in* $\mathbf{\Gamma}$ *that have unlabeled elements*

5. Find the smallest element of $\mathbf{\Gamma}$ which is not labeled before ($\Gamma_{r:N^2}$), and label it

6. Save the present state of the variables (Ω, r, and the labeled elements in $\mathbf{\Gamma}$) as state r

7. While {there is only one remaining (unlabeled) element in any row and column of $\mathbf{\Gamma}$}
   (a) $\Omega = \Omega \bigcup$ {the mentioned remaining element}
   (b) Delete the row and column corresponding to the selected element
   (c) Recalculate n_Ω, m_{row}, and m_{col}
   (d) If $m_{row} < N - n_\Omega$ or $m_{col} < N - n_\Omega$
      – Restore the last saved state ($\mathbf{\Gamma}$, Ω, and r from Step 6)
      – Delete the last saved state (the total number of the saved states is decreased by one)
      – $\Omega = \Omega \bigcup$ {the last labeled element}
      – Recalculate n_Ω

8. $r = r + 1$

9. If $r < N^2$ go to step 5

Example 1: Consider the matrix $\mathbf{\Gamma}$ in Fig. 5.2. Different steps resulting from the application of the above algorithm to this matrix are shown in this figure. The first row or column which satisfies the condition in Step 7 is the second row. So $\Gamma_{2,4}$ is selected for the optimal permutation. This means relay 4 is assigned to pair 2. After deleting the second row and the forth column, the 3-by-3 matrix in Fig. 5.2 results. The Second row or column which satisfies the condition in Step 7 is the last row and $\Gamma_{4,3} = 28$ is selected. This means that relay 3 is assigned to pair 4. By deleting the corresponding row and column, a 2-by-2 matrix results, from which, $\Gamma_{1,2} = 19$ and $\Gamma_{3,1} = 29$ are selected. This implies that relays 2 and 1 are assigned to pairs 1 and 3, respectively.

Example 2: Now consider the matrix $\mathbf{\Gamma}$ in Fig. 5.3. The first row or column which satisfies the condition in Step 7 is the fifth column. So $\Gamma_{4,5} = 18$ is selected for the optimal permutation. After deleting the fourth row and the fifth column, a new 4-by-4 matrix results where two elements ($\Gamma_{2,2} = 33$ and $\Gamma_{2,3} = 34$) satisfy the condition in

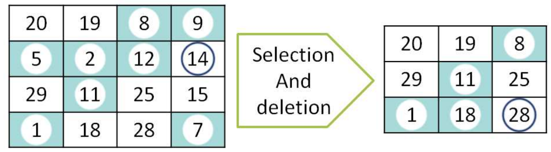

Figure 5.2: Example of relay assignment algorithm for $N = 4$ and $M = 4$. The labeled elements are highlighted and the selected elements are shown by solid line circles. The first row that has only one unlabeled element is the second row, and the first selected element is $\Gamma_{2,4} = 14$

Step 7. By selecting any of them (for example $\Gamma_{2,3} = 34$) and deleting the corresponding row and column, the new 3-by-3 matrix in Fig. 5.3 results. As suggested by this figure, all of the elements in the second column are already labeled, but we need three other elements for the optimal permutation. It means that the remaining elements are not enough for the selection process. In this case, we should go back and restore the last saved state. The mentioned state was saved when we labeled $\Gamma_{5,5} = 17$ in the matrix. By starting from that step, we also select this element ($\Gamma_{5,5} = 17$) for the optimal permutation (the right side matrix in Fig. 5.3). From this point, the remaining steps of applying the algorithm is straightforward.

Optimality analysis of the algorithm: At the end of the algorithm, let us denote the r-th selected element for the optimal permutation by $\Gamma_{s_r:N^2}$ where $r = 1, 2, 3, \cdots, N$. On the other hand, there are some elements that are labeled in the matrix (in Step 5), but are not selected for the optimal permutation. Let us assume that whenever an element $\Gamma_{t:N^2}$ is labeled in the matrix, it is labeled at time instant t. We denote the set of the elements which are selected for the optimal permutation after time instant t by $\mathcal{F}_t$. Then

$$\text{for all } \Gamma_{i,j} \,\epsilon\, \mathcal{F}_t \text{ we have } \Gamma_{i,j} \; > \; \Gamma_{t:N^2}. \tag{5.1}$$

Because in each step of the above algorithm, the elements for the optimal permutation are selected among the remaining (unlabeled) elements of $\mathbf{\Gamma}$.

Theorm 1. *The proposed algorithm leads to picking the optimal permutation based on the max-min criterion, which guarantees the maximum diversity for all pairs.*

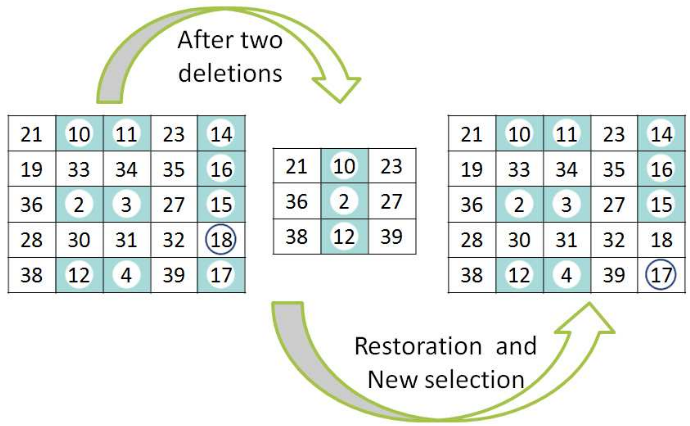

Figure 5.3: Example of relay assignment algorithm for $N = 5$ and $M = 5$. After selecting $\Gamma_{4,5} = 18$ and $\Gamma_{2,3} = 34$ and deleting the corresponding rows and columns of the matrix, we cannot select three elements form remaining unlabeled elements (the middle matrix). In this case, we need to restore the last saved state and select $\Gamma_{5,5} = 17$ instead of $\Gamma_{4,5} = 18$.

Proof. We know that each permutation has one element in each row and each column. The event in Step 7-d checks if there are enough unlabeled elements in the matrix to select relays for all of the remaining pairs. Let us denote this event by H. First we will prove that Γ_{s_1} belongs to the optimal permutation. One of the following events is true.

(A) Γ_{s_1} was the last remaining element in its row or column. In this case, this element is the maximum in its row or column. We denote the last labeled element in this row or column by Γ_{t_1}. From (5.1), we know that

$$\text{For all } \Gamma_{i,j} \,\epsilon\, \Omega \text{ we have } \Gamma_{i,j} \;>\; \Gamma_{t_1}.$$

This clearly means that by selecting Γ_{t_1} or any other element in this row, the selected permutation has a smaller minimum and it is not optimal.

(B) Γ_{s_1} was not the last remaining element in its row or column, but it was the largest element in its row or column that makes event H possible. According to the argument in (A), the smaller elements of this row cannot be selected for the optimal permutation. On the other hand, according to the validation of the event H (Step 7-d in the proposed algorithm), by selecting any larger element from the mentioned row or column, we cannot select Γ_{s_r}, $r = 2, 3, \cdots, N$ from the remaining elements of the matrix, i.e. at least one element will be selected from $\Gamma_{i:N^2}$, $i < s_1$. Hence, in this row, only Γ_{s_1} satisfies the necessary conditions for the optimal permutation.

Similarly, we can prove that other Γ_{s_i}, $i = 2, 3, \cdots, N$ belong to the optimal permutation. □

5.3.3 Proposed Algorithm For $N < M$

When the number of relays is larger than that of source-destination pairs, some of the relays do not contribute to the optimal permutation, which means that no elements will be selected from $M - N$ columns of the matrix. Based on this observation, we extend the algorithm in Section 5.3.1 as follows.

1. $\Omega = \{\}$ *% The set of the elements of the optimal permutation*
2. Set $n_\Omega = 0$ *% Number of the elements in* Ω
3. Set $m_{row} = N$ *% Number of rows in* $\mathbf{\Gamma}$ *that have unlabeled elements*
4. Set $m_{col} = M$ *% Number of columns in* $\mathbf{\Gamma}$ *that have unlabeled elements*
5. Find the smallest element of $\mathbf{\Gamma}$ which is not labeled before ($\Gamma_{r:N^2}$), and label it
6. Save the present state of the variables (Ω, r, and the labeled elements in $\mathbf{\Gamma}$) as state r
7. Check if
   - {A: There is any row with only one remaining element}
   - {B: There is any column with only one remaining element}
8. While A or B
   (a) If A
      i. $\Omega = \Omega \bigcup$ {the mentioned remaining element}

ii. Delete the row and column corresponding to the selected element

iii. Recalculate n_Ω, m_{row}, and m_{col}

iv. If $m_{row} < N - n_\Omega$ or $m_{col} < N - n_\Omega$

- Restore the last saved state ($\mathbf{\Gamma}$, Ω, and r from Step 7)
- Delete the last saved state
- $\Omega = \Omega \bigcup$ {the last labeled element}
- Recalculate n_Ω

v. Recalculate A and B

(b) If B

- If {there are already $N - M$ columns with all elements labeled}

i. $\Omega = \Omega \bigcup$ {the mentioned remaining element}

ii. Delete the row and column corresponding to the selected element

iii. Delete the mentioned $N - M$ columns of $\mathbf{\Gamma}$, this deletion happens only once.

iv. Recalculate n_Ω, m_{row}, and m_{col}

v. If $m_{row} < N - n_\Omega$ or $m_{col} < N - n_\Omega$

- Restore the last saved state similar to the steps in 9a-iv

vi. Recalculate A and B

9. $r = r + 1$

10. If $r < N^2$ go to Step 5

In the above algorithm, whenever there is one remaining element in any row of the matrix, it is selected for the optimal permutation, but we do not select any element from the first $M - N$ completely labeled columns of the matrix (those columns having all their elements labeled). Because the last labeled element from any other column of the matrix is larger than all of the elements of the first $M - N$ labeled columns.

Example 3: Fig. 5.4 shows an example for this case. In this figure, the first labeled column is the first column and we do not select any element from this column. After the deletion of this column, the first row or column that is going to be labeled, is the first row and $\Gamma_{1,2} = 21$ is selected.

8	21	9	11
4	2	14	16
1	13	19	18

Figure 5.4: Example of relay assignment for $N = 3$ and $M = 4$. The first labeled column is the first column and the first selected element is $\Gamma_{1,2} = 21$ which is $\Gamma_{12:12}$

5.3.4 Weighting Coefficients

The proposed relay assignment algorithm in the last section suggests a new way to calculate the PDF and average probability of error when the max-min criterion is used. Let us start with an example. Consider a network consisting of 2 source destination pairs and 2 relays. There are totally 4 E2E SNR value which form the 2-by-2 matrix $\mathbf{\Gamma}$. We know that there are totally $4! = 24$ possible orderings to put $\Gamma_{i:4}$, $i = 1, \cdots, 4$ in $\mathbf{\Gamma}$ (see Fig. 5.5). In each ordering, two elements out of four are selected based on max-min criterion. $\Gamma_{1:4}$ and the element which is in the same diagonal with $\Gamma_{1:4}$ are never selected. By averaging over all orderings, the possibility of the presence of each $\Gamma_{i:4}$, $i = 2, 3, 4$ in the optimal max-min permutation is $2/3$ where $\Gamma_{1:4}$ is never selected for the optimal permutation. Therefore, we have $f_{optimal}(\gamma) = \frac{1}{2}\sum\limits_{r=2}^{4}\frac{2}{3}f_{r:4}(\gamma)$. Because of the symmetry property of the network, the PDF of the SNR of the source-destination pair $S_i \to R_x \to D_i$ will be independent of i. Generally speaking, we have the following lemma which is the basis for our analysis.

Lemma 1. *After ordering the E2E SNRs, the optimal permutation is comprised of some of those ordered SNRs, which are picked according to the proposed algorithm. The indices of those selected elements are only a function of the ordering of the elements in* $\mathbf{\Gamma}$.

Proof. Assume that we change the value of the variables $\Gamma_{i,j}$, but we keep their ordering, i.e. if $\Gamma_{i,j}$ was greater than $\Gamma_{k,l}$, it would still be greater. Obviously, nothing changes in the algorithm of Section 5.3.3. Hence, if $\Gamma_{i,j}$ was selected for the optimal permutation, it would still be selected. In other words, as far as the ordering of the elements in the

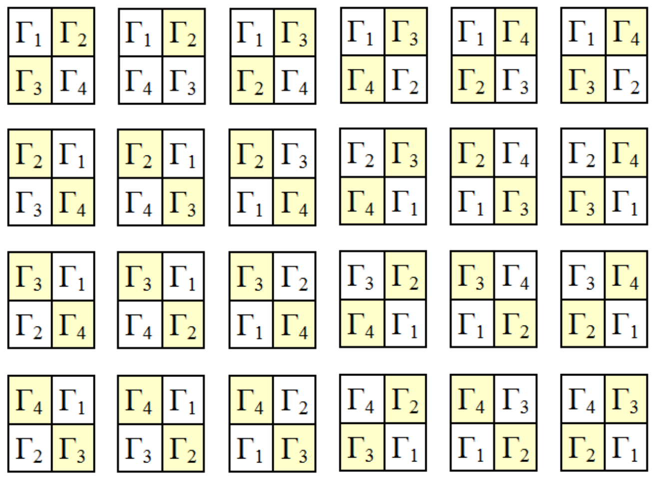

Figure 5.5: Different orderings of 4 SNR values in a 2-by-2 matrix

matrix is preserved, the same elements will be in the optimal permutation. □

Lemma 1 means that for an infinitely large number of channel realizations, the SNR of $S_i \to R_x \to D_i$ will be $\Gamma_{r:NM}$ for a fixed percentage of channel realizations. This means that the PDF of the SNR for the E2E channel assigned to S_i can be expressed in terms of the order-statistical PDFs, i.e. $f_{optimal}(\gamma)$ is a weighted sum of the PDFs of $\Gamma_{r:NM}$, $1 \leq r \leq NM$. Let us denote these weighting coefficients by $w_{N,M}(r)$. Then we have

$$f_{optimal}(\gamma) = \sum_{r=1}^{N^2} w_{N,M}(r) f_{r:NM}(\gamma), \tag{5.2}$$

where $f_{optimal}(\gamma)$ is the PDF of the E2E SNR of each pair which is a result of relay-assignment based on max-min criterion. The same conditions hold true for the average probability of error, that is

$$P_{E(optimal)} = \sum_{r=1}^{N^2} w_{N,M}(r) P_{E(r:NM)}, \tag{5.3}$$

where $P_{E(r:NM)}$ is given by (3.15). Here, we assume $k = 1$ for simplicity (that is the channels in both hops have the same average SNR values).

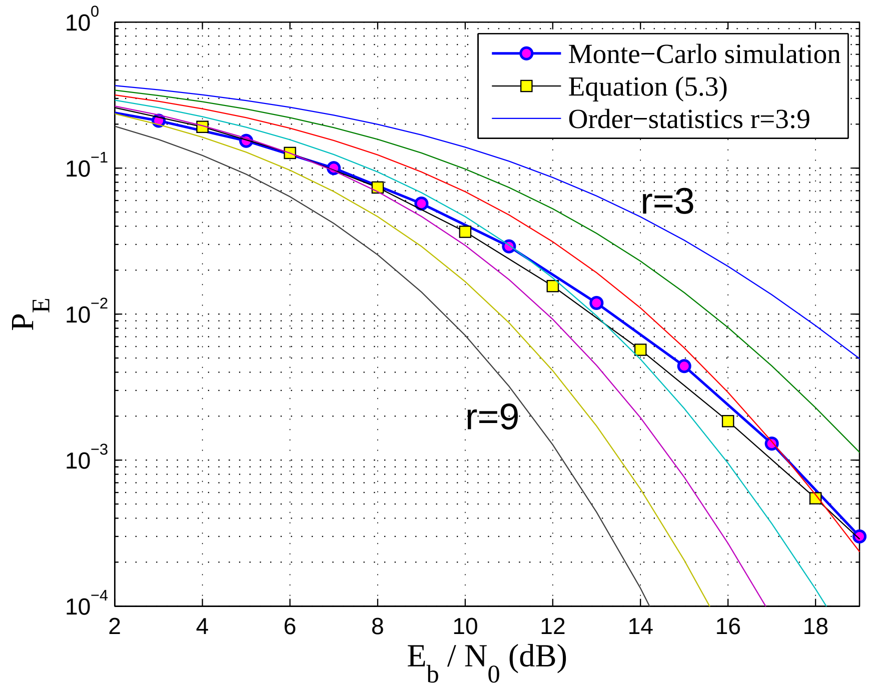

Figure 5.6: Monte-Carlo simulation of the average error probability compared with the Monte-Carlo simulation of equation (5.3)

Example 3: Consider a network with $N = M = 3$. For this network $w_{3,3}(r)$ is calculated by counting all possible combinations of 9 elements in a 3×3 matrix: $\mathbf{w}_{3,3} = [0\ \ 0\ \ 0.0238\ \ 0.0952\ \ 0.1540\ \ 0.1762\ \ 0.1836\ \ 0.1836\ \ 0.1836]$. Using AF cooperation scheme, the results of Monte-Carlo simulation of the average error probability for this network are shown in Fig. 5.6. The results are compared with (5.3). This figure shows an example of the validity of this analysis. This figure also illustrates $P_{E(r:9)}$ for $r = 3, 4, ..., 9$. Obviously there is a very good match between the Monte-Carlo simulation and $P_{E(optimal)}$ in (5.3). The small difference between the two curves rises from the approximation error in (3.15).

5.3.5 Performance Analysis: Diversity Order and BER

Lemma 2. *By using the max-min criterion for relay assignment, the diversity order for all users equals M.*

Proof. The algorithm proposed in Section 5.3.3 shows that the first $M-1$ smallest elements are never selected for the optimal permutation. On the other hand, the diversity order offered by $\Gamma_{r:NM}$ is r [16]. Since in the summation formula (5.3), the lowest diversity order is dominant, it is concluded that the overall diversity order of the system will be equal to M. □

Equation (5.3) and Lemma (2) constitute a new simple way to estimate the BER of the network. It is enough to calculate the NM weights for any given N and M. This calculation is independent of the average SNR values and can be done only once. Then, by using (5.3), the overall BER rate is calculated.

Fig. 5.7 shows the average probability of error for the proposed method and that of max-sum-SNR relaying and sequential relaying. In sequential relaying, the priority of relay selection is given to the source that has the weakest direct channel to the destination. However, this figure only shows the performance achieved through cooperation. In this figure, $N = M = 3$. From the figure it can be inferred that, for sequential relaying, the user that gets assigned its relay first enjoys the maximum diversity, whereas the diversity degrades for other users. As for the max-sum-SNR criterion, the diversity achieved is one, as expected. Contrary to those schemes, the proposed scheme achieves maximum diversity for all pairs. On the other hand, compared to using the max-min criterion with suboptimal relay selection [9], an improvement of about 1.5 dB is observed in this example.

5.3.6 Calculation of $w_{N,N}(r)$

In order to complete the analytical study of the PDF and BER for the optimal permutation, we need to find $w_{N,M}(r)$. Fig. 5.8 shows $w_{N,N}$ for different values of N. These values are achieved by Monte-Carlo simulation. Since the analytical calculation of $w_{N,M}(r)$ is very difficult for general values of N and M, we propose a general fit in order to approximate $w_{N,M}(r)$ for $N = M$. According to the algorithm in Section 5.3, the $M-1$ smallest elements are never selected, i.e. $w_{N,N}(r) = 0$ for $r \leq N-1$. For $N \leq r \leq 2N-3$ we have

$$w_{N,N}(r) = \binom{r-1}{N-1} P_N^N, \tag{5.4}$$

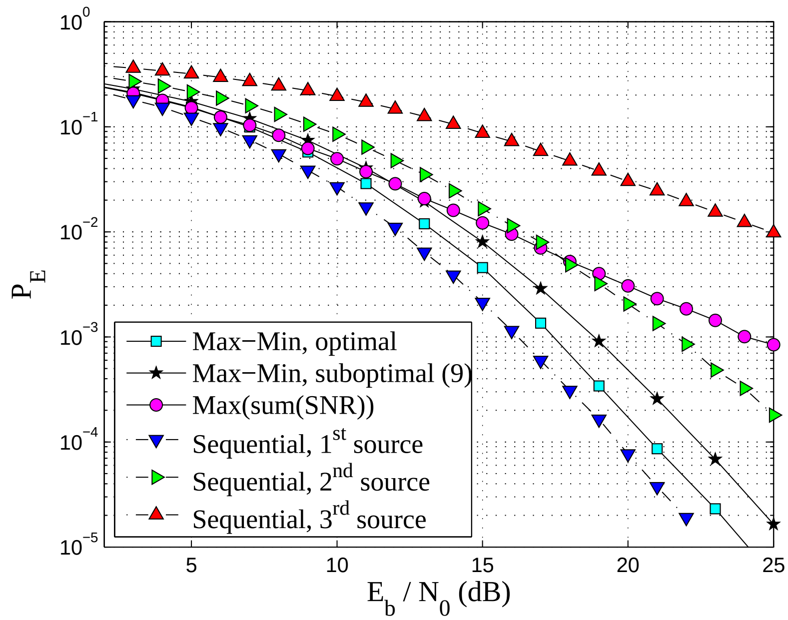

Figure 5.7: Comparison of BER for different relay assignment scenarios

where

$$P_N^N = N^2(2N-2)(N-2)!/\left(\frac{N^2!}{(N^2-N)!}\right). \tag{5.5}$$

To elaborate on (5.4), note that $w_{N,N}(r)$ represents the possibility of having $\Gamma_{r:N^2}$ in the optimal permutation. The only possibility of selecting $\Gamma_{r:N^2}$, $N \leq r \leq (2N-3)$ for the optimal permutation is when it is in the same row or column with $N-1$ elements ($\Gamma_{i:N^2}$, $i \leq (2N-4)$). There are $\binom{r-1}{N-1}$ combinations of the elements for this purpose. Let P_N^N represent the probability of N specific elements being in the same row or column in an $N \times N$ matrix. As for (5.5), suppose that N elements ($\Gamma_{r:N^2}$ plus $(N-1)$ elements amongst $N \leq r \leq (2N-4)$) are specified and we are looking for the number of ways to put them in the same row or column. The first element in the set can be put anywhere in the matrix (a total of N^2 places). Suppose that this element is placed in row i and column j. Since the second element must be in the same row or column with the first one, there are only $2N-2$ remaining places for the second element ($N-1$ places in row i and $N-1$ places in column j). Putting the second element in the matrix, the row or column is specified; hence there are only $N-2$ remaining places for the remaining $N-2$ elements. Therefore the total number of ways is basically the numerator of (5.5).

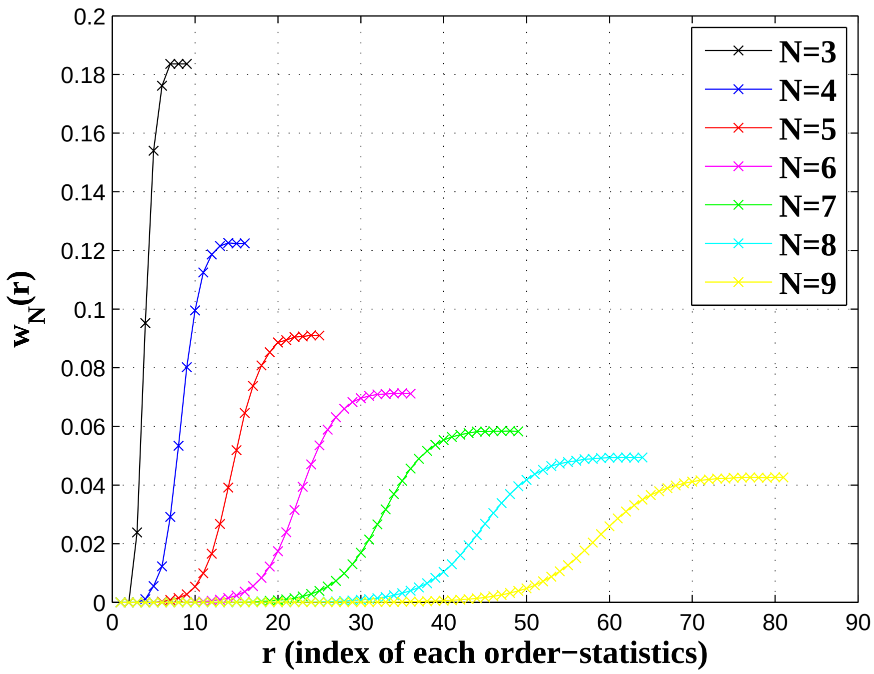

Figure 5.8: Monte-Carlo simulation of $w_{N,M}$ for different values of N

As for the denominator in (5.5), it shows the number of different possibilities for N elements to be placed in the matrix.

For larger values of r, the analytical calculation of $w_{N,N}(r)$ becomes more difficult. In this part we propose an approximation in order to find $w_{N,N}(r)$ for general values of N. For $N \leq r \leq 2N-3$, we have $w_{N,N}(r+1) - w_{N,N}(r) = \binom{r}{N-2}$, i.e., the difference between two successive weighting coefficients is a binomial coefficient. It means that the behavior of $w_{N,N}(r)$ in this interval is like the integral of a Gaussian function (binomial coefficients can be well approximated by a Gaussian function). This property does not hold true for other values of r ($2N-2 \leq r \leq N^2$), however Monte-Carlo simulations show that the integral of the Gaussian function is a very good approximation for $w_{N,N}(r)$ for all r, i.e.,

$$w_{N,N}(r) = \alpha_N \sum_{i=N}^{N^2} \exp(-(r-\mu(N))^2/\sigma(N)^2). \tag{5.6}$$

Fig. 5.9 shows the Monte-Carlo simulations of the backward difference of $w_{N,N}$ for different values of N. Based on this approximation (Gaussianity of the backward difference of $w_{N,N}(r)$), a general approximation for $w_N(r)$ for any r and $N = 3,4,5,6$ is found. This approximation is as follows.

$$\begin{aligned}
\mu(N) &= 0.9297N^2 - 2.148N + 2.488, \\
\sigma(N) &= (-818.2N^2 + 18640N - 37940)/(N + 8618), \\
\alpha(N) &= 1/\sum_{r=N}^{N^2} \exp(-(r-\mu(N))^2/\sigma(N)^2).
\end{aligned}$$

The Monte-Carlo simulations in Fig. 5.10 confirm the validity of the proposed approximation for other values of $N = 7,8,9$.

5.4 Extension to Clustered Three-hop Networks

In this section we extend the proposed algorithm to three-hop clustered networks. We capitalize on the results presented in the previous section, but with less detail.

5.4.1 System Model

Consider the network shown in Fig. 5.11, which consists of N source-destination pairs, M_1 relays in the first hop and M_2 relays in the second hop, where $N \leq M_1$, M_2.

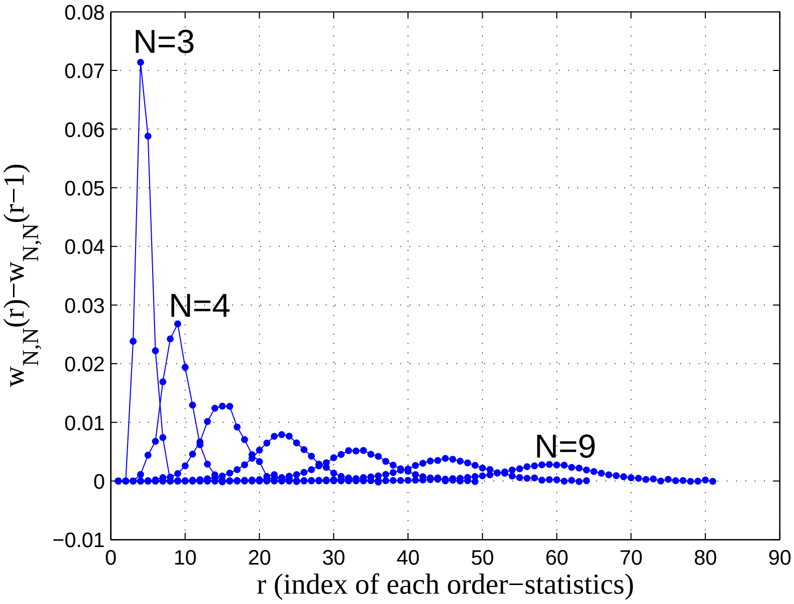

Figure 5.9: Monte-Carlo simulation of $w_{N,N}(r) - w_{N,N}(r-1)$ for different values of N

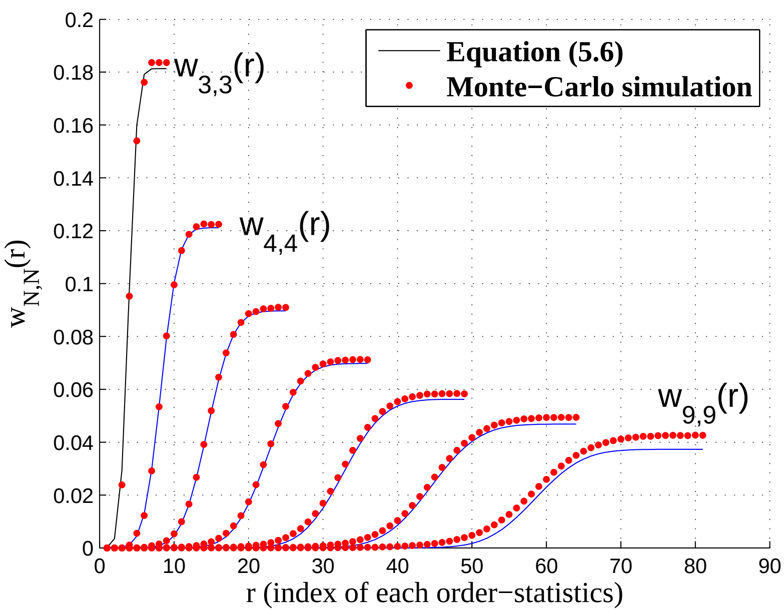

Figure 5.10: Monte-Carlo simulation of $w_{N,N}$ compared to the proposed approximation

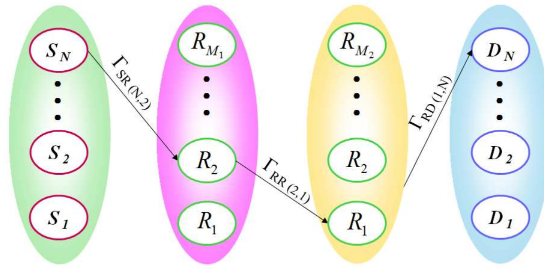

Figure 5.11: System model consisting of N source-destination pairs and M_1 relays in the first cluster and M_2 relays in the second cluster

There is no constraint on the values of M_1 and M_2. This model is a generalization of the model in Section 5.3.3. Again each source-destination pair communicates through one relay node in each hop using orthogonal sub-channels. Since there are M_1 relays in the first hop and M_2 relays in the second hop, there are totally $N \times M_1 \times M_2$ different E2E channels in the network and there are $M_1!M_2!/|M_1 - M_2|!$ different relay assignment permutations.

5.4.2 Proposed Algorithm

Since there are $N \times M_1 \times M_2$ distinct E2E channels in total, their SNRs can be written in a three dimensional matrix $\mathbf{\Gamma}$, where $\Gamma_{i,j,k}$ represents the equivalent SNR of the link $S_i \rightarrow R_{1,j} \rightarrow R_{2,k} \rightarrow D_i$. Assignment of the relays $R_{1,j}$ and $R_{2,k}$ to the source-destination pair i corresponds to the selection of $\Gamma_{i,j,k}$ from the above matrix. The relay assignment algorithm is as follows.

For $\mathbf{N = M_1 = M_2}$: We propose a modified adaptation of the algorithm in Section 5.3.2. Again we sort the elements of $\mathbf{\Gamma}$ and find $\Gamma_{r:NM_1M_2}$. By starting from the smallest element $\Gamma_{1:NM_1M_2}$, each element is labeled in $\mathbf{\Gamma}$. Since every relay is present in every permutation, the optimal permutation has one element in any plane of the matrix. Therefore, at any moment, if there is only one remaining element in any plane of $\mathbf{\Gamma}$, this last element belongs to the optimal permutation. In this case, we delete the corresponding planes of this element from the matrix and start from the beginning. Hence, the size of each dimension of the matrix $\mathbf{\Gamma}$ is reduced by one (See Fig. 5.12).

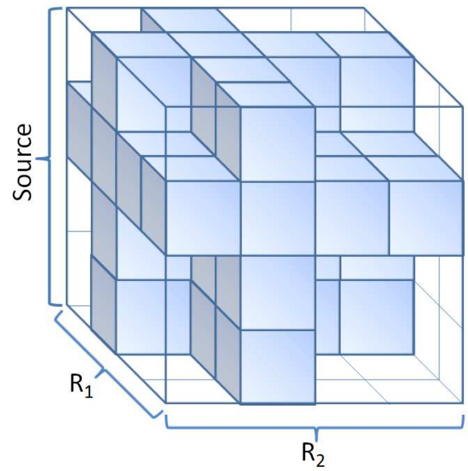

Figure 5.12: Example of relay assignment algorithm for $N = M_1 = M_2 = 4$. Each element shows the SNR of one end-to-end link $S_i \to R_{1,j} \to R_{2,k} \to D_i$. Here the selected element is $\Gamma_{2,2,2}$ and the corresponding planes are shown by solid cubes.

For $\mathbf{N \leq M_1, M_2}$: When the number of relays is larger than that of source-destination pairs, some of the relays do not contribute to the optimal permutation. The number of those relays is $M_1 - N$ in the first hop and $M_2 - N$ in the second hop. This means that no elements should be selected from the first K columns of $\mathbf{\Gamma}$ that have all elements labeled, where $K = M_1M_2 - N^2$. In other words, when all of the elements of a column are labeled and this column is among the first K columns that have all elements labeled, we delete this column without selecting any element, because there are still better choices.

5.4.3 Performance Analysis

Each channel in the second hop $R_{1,j} \to R_{2,k}$ contributes to N E2E channels. So the elements of the matrix $\mathbf{\Gamma}$ are not independent. This means that we cannot use (5.2) and (5.3) in order to express the statistical behavior of the optimal answer. However, we can still calculate the diversity order offered by the optimal permutation.

Lemma 3. *The diversity order offered by the optimal permutation is* $\min\{M_1, M_2\}$.

Proof. The first selected element of the optimal permutation is the largest element in its plane. If this plane is a horizontal plane (denoted by $\mathbf{\Gamma}_{i,:,:}$), the selected element will be larger than M_1M_2 independent elements in this plane. If this plane is a vertical one shown by $\mathbf{\Gamma}_{:,j,:}$ (or $\mathbf{\Gamma}_{:,:,k}$), its elements can be partitioned into M_1 (or M_2) non-overlapping sets where each set contains N dependent subchannels, which are independent from the subchannels in other sets. In this case, the selected SNR is larger than M_1 (or M_2) independent SNRs. We can conclude that the selected element is at least larger than $\min\{M_1, M_2\}$ independent elements. Hence its diversity order is $\min\{M_1, M_2\}$. □

Fig. 5.13 shows some Monte-Carlo simulation results for the average error probability referred to this scenario. The simulations are performed by using BPSK modulation and contain the results for four different values of M_1 and M_2. By assuming $N = M_1 = M_2 = 2$, diversity two is achieved. By increasing the number of relays in the second hop to $M_2 = 3$, still the diversity order is two, but there is a significant improvement in the coding gain. This is because the second relay-cluster provides a diversity order of three. However, this diversity order is dominated by that of the first relay-cluster, which plays the role of a bottleneck, but still provides some coding gains. Another increase in the number of the relays in the second relay-cluster ($M_2 = 4$) will not bring a significant improvement in the performance of the network. Besides that, in high SNR region, when $M_2 = M_1 + 2$, the performance of the relaying link converges to that of $M_2 = M_1 + 1$. This result shows that, when there is a bottleneck in one of the hops (M_1 is fixed), it is almost enough to have $M_2 = M_1 + 1$ in order to achieve the best possible performance. An increase in the diversity order occurs when we increase the number of relays in both relay-clusters.

5.5 Extension to Clustered Multi-hop Networks

The proposed algorithm of Section 5.4 can be easily generalized in order to find the optimal permutation for multi-hop clustered network. Consider an $L+1$-hop clustered network consisting of N source-destination pairs and M_i relays in the relay-cluster i, $1 \leq$

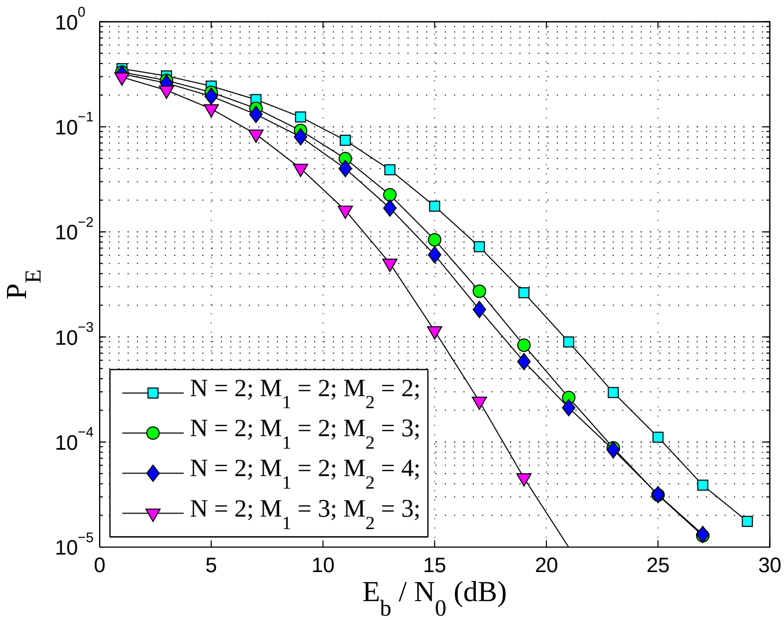

Figure 5.13: BER for a three-hop network with different number of relays in each relay-cluster

$i \leq L$. Again each source-destination pair communicates through one relay node in each relay-cluster using orthogonal sub-channels. There are totally $N \times M_1 \times M_2 \times \ldots \times M_L$ different E2E subchannels in the network. In this case we have an $(L+1)$-dimensional matrix $\mathbf{\Gamma}$.

5.5.1 Proposed Algorithm

Again we sort the elements of $\mathbf{\Gamma}$ and form $\Gamma_{r:NM_1\ldots M_L}$. By starting from the smallest element $\Gamma_{1:NM_1\ldots M_L}$, each element is labeled in the matrix $\mathbf{\Gamma}$.

For $\mathbf{N = M_1 = M_2 = ... = M_L}$: At any moment, if there is only one remaining element in any L-dimensional sub-matrix of $\mathbf{\Gamma}$, this last element belongs to the optimal permutation. In this case we delete the corresponding sub-matrices of this element from $\mathbf{\Gamma}$ and start from the beginning. Hence the size of each dimension of the matrix $\mathbf{\Gamma}$ is reduced by one.

For $\mathbf{N \leq M_1, ..., M_L}$: When the number of relays is larger than the number of source-destination pairs, some of the relays do not contribute to the optimal permutation. The number of those relays is $M_i - N$ relays in relay-cluster i. This means that no elements should be selected from the first K $(L-1)$-dimensional sub-matrices $\mathbf{\Gamma}_{i,:,:,\ldots,:}$ that have all elements labeled where $K = M_1 \ldots M_L - N^L$. In other words, when all of the elements of a sub-matrix $\mathbf{\Gamma}_{i,:,:,\ldots,:}$ are labeled and this sub-matrix is among the first K sub-matrices that have all elements labeled, we delete this sub-matrices without selecting any element, because there are still better choices.

5.5.2 Performance Analysis

Similar to Section 5.4.3, the dependency among different permutations makes the statistical analysis very difficult. However, generalizing the proof for Lemma 3, we can conclude that the proposed algorithm achieves the diversity $\min\{M_1, M_2, \ldots, M_L\}$.

6

Relaying Based on Max-Sum Criterion

6.1 Introduction

In this chapter, we consider relay the assignment problem in cooperative networks based on maximizing the sum of rate values (sum-rate) and maximizing the sum of SNR values (sum-SNR). We aim to analyze the performance of these schemes. These criteria are interesting in the sense that they aim the maximal use of the resources. Our analysis in this chapter is also motivated by the fact that there are many formulations in the literature to find the optimum answer based on the above criteria. We also aim to propose a new method to find the optimal permutation based on the above mentioned criteria.

6.1.1 Contributions of This Chapter

The contributions of this chapter are summarized as follows.

1. We show that the scheme based on sum-rate achieves full diversity, assuming that all of the E2E channels are independent, but the scheme based on sum-SNR does not achieve diversity, however, it achieves a good coding gain at low average SNR values.

2. In order to perform this analysis, first we propose a new method to calculate the diversity order of fading channels at high SNR values. In the previous method, the diversity order was expressed in terms of the Taylor expansion of the random SNR$'$ s PDF only at the origin [41] (see Section 3.1.4). However that method fails to calculate the diversity when the PDF is not *well behaved* at the origin. Our proposed method does not suffer from the same problem.

3. We propose a new method to find the optimal permutation based on sum-SNR or sum-rate criteria. We introduce a Vehicle Routing Problem (VRP) formulation for the problem at hand which can be efficiently solved with Binary Integer Programming (BIP). The proposed formulation can solve many clustering and relay assignment problems in a unified framework. Two different scenarios are described to show this flexibility. In the first scenario, only one of the nodes in each cooperating set benefits from the cooperation whereas in the other scenario both nodes benefit.

The rest of this chapter is organized as follows. Section 6.2 proposes a new method to calculate diversity order in fading channels. Section 6.3 analyzes the performance of relay assignment when sum-rate is the optimization criterion. Section 6.4 performs the same analysis when max-sum-SNR is used as optimization criterion. Some simulation results are provided in these two sections to show the validity of the analysis. Section 6.5.1 reviews the vehicle routing problem. Section 6.5.2 describes how to formulate the clustering and relay assignment as a VRP problem. Some simulation results are provided in Section 6.5.3 where a complementary discussion about this method is presented.

6.2 A New Method To Calculate Diversity Order

In this section, we propose a new method to calculate the diversity order of fading channels at high SNR values. The average error probability of a fading channel is defined as

$$P_E = \int_0^\infty Q(\sqrt{k\varphi}) f_\Phi(\varphi) d\varphi. \tag{6.1}$$

where $f_\Phi(\varphi)$ is the PDF of SNR and the Q-function represents the instantaneous error probability. Here, k is a fixed value which is determined by the modulation format.

There are some cases that the average error probability cannot be found in closed form. In these cases, one solution is to evaluate the integral numerically. Although in this way we have the numerical evaluation of the system performance, in general it does not offer clear insights to the behavior of the system. Zhengdao and Giannakis [41] tried to fill this gap between analytical results and intuition with approximate (yet accurate) parameterizations. Their method quantifies average error probability and outage, both in terms of diversity gain (diversity order) and coding gain. This analysis is motivated by the fact that the outage probability pattern behaves similarly to that of the average error rate and they have the same slope. Their analysis allows us to gain insights to the factors determining the performance in the presence of fading. In [41], the Taylor series of the PDF of the SNR around origin is used to determine the diversity order, however, when the PDF is not *well behaved* at the origin, this method fails to analyze the performance.

To solve this problem, we propose a simple method to calculate diversity order over fading channels. The proposed method works anywhere the method of [41] works (coded or uncoded, coherent or noncoherent; and over different types of fading channels such as Rayleigh, Nakagami-m, Nakagami-n, and Nakagami-q types). It also works in some cases where (or when) the PDF of SNR is not well behaved or its Taylor expansion does not exist.

Throughout this section, the instantaneous and the average SNR at the receiver are denoted by φ and $1/\lambda$, respectively. Here, $\frac{1}{\lambda}$ is the average SNR which is a deterministic positive quantity, and φ is a channel-dependent nonnegative random variable ($E[\varphi] = \frac{1}{\lambda}$). We are interested in large SNR performance, which is equivalent to $\lambda \to 0^+$ (λ tends to 0 from above). The instantaneous error probability is given by $P_E(\varphi) = Q\left(\sqrt{k\varphi}\right)$, where k is a positive fixed constant and specifies the type of modulation.

Lemma 1. *Consider single-user uncoded communication over a random fading channel. We assume that the PDF of SNR for $\lambda \to 0^+$ can be approximated by a single "polynomial" term as*

$$f_\Phi(\varphi, \lambda) = a(\varphi)\lambda^t + \underbrace{b(\varphi, \lambda)}_{O(\lambda^{t+\epsilon})}$$

where $\epsilon > 0$. The parameter t quantifies the order of smoothness of $f_\Phi(\varphi, \lambda)$ in terms of λ at the origin. In this case, the achieved diversity order is t.

Proof. We have

$$\begin{aligned} P_E(\lambda) =& \int_0^\infty Q(\sqrt{k\varphi}) f_\Phi(\varphi) d\varphi \\ =& \int_0^\infty Q(\sqrt{k\varphi}) \left(a(\varphi)\lambda^t + b(\varphi, \lambda)\right) d\varphi \\ =& A\lambda^t + \underbrace{B(\lambda)}_{O(\lambda^{t+\epsilon})} \end{aligned}$$

This result obviously shows that the systems achieves diversity t. □

The problem is that, both proposition 1 in [41] and also the above Lemma sometimes fail to determine the diversity order of the system under consideration. This is because $f_\Phi(\varphi, \lambda)$ is not well behaved at $\lambda \to 0^+$. Moreover, let us assume that the first nonzero term in the Taylor expansion of $f_\Phi(\varphi)$ in terms of λ is infinite. In this case, the following proposition can be useful.

Proposition 1. *Consider a single-user uncoded communication over a random fading channel. The system achieves diversity t, if and only if*

$$\lim_{\lambda \to 0^+} \frac{f_\Phi(\varphi, \lambda)}{f_\Phi(\varphi, k\lambda)} = k^t \tag{6.2}$$

Proof. Diversity order is the slope of BER curve in the logarithmic scale.

$$G_d = \lim_{\lambda \to 0^+} \frac{\log\left(\frac{P_E(\lambda)}{P_E(k\lambda)}\right)}{\log(k)}. \tag{6.3}$$

We can multiply both numerator and the denominator of (6.2) by $Q(\sqrt{k\varphi})$ and integrate them in the same interval without changing the result. This is because both operations are linear. Then we have

$$\lim_{\lambda \to 0^+} \frac{\int_0^\infty Q(\sqrt{2\varphi}) f_\Phi\left(\varphi, \lambda\right) d\varphi}{\int_0^\infty Q(\sqrt{2\varphi}) f_\Phi\left(\varphi, k\lambda\right) d\varphi} = k^t$$

or equivaently

$$\lim_{\lambda \to 0^+} \frac{P_E(\lambda)}{P_E(k\lambda)} = k^t$$

Plugging this result into (6.3) yields $G_d = t$. On the other hand, the inverse is obvious, because it is the definition of the diversity order. $\square$

6.3 Diversity Analysis of the Sum-rate Criterion

Wireless communication is facing the scarcity of radio resources, such as time slots and subcarriers. Due to this reason, optimal use of the resources becomes mandatory. In the context of relay-assignment, this issue leads us to one widely accepted and commonly used criterion which is sum-rate (see Section 1.4.2). According to this criterion, the resources should be assigned in a way that that the overall rate of information exchanged in the network is maximized. Nowadays, there are many formulations in the literature to implement this criterion. In this section, we aim to statistically analyze the diversity order achieved by this criterion. In our analysis, the PDF turns out not to be well behaved at the origin. Consequently, the proposition in [41] cannot be used to calculate the diversity order, but our proposed method in Section 6.2 shows that full diversity is achieved.

6.3.1 Problem Formulation

Consider the network shown in Fig. 6.1-a. There are N source-destination pairs and N relays where each source-destination pair can use only one relay to transmit its data to the destination. Suppose that general assumptions are similar to the general assumptions in previous sections (i.i.d. Rayleigh fading channels in each cluster and two-hop AF relaying mode). We know that at high average SNR values, each E2E channel can be well approximated by another Rayleigh fading channel (see Eq. (2.7)). For the E2E channels in Fig. 6.1-b, the same statement can be true if the channels in the second hop do not experience fading. In the network in Fig. 6.1-b, there are N source nodes, N relay nodes and a single destination. This scenario corresponds to fixed relays that have very good direct channels to the destination. The problem is to

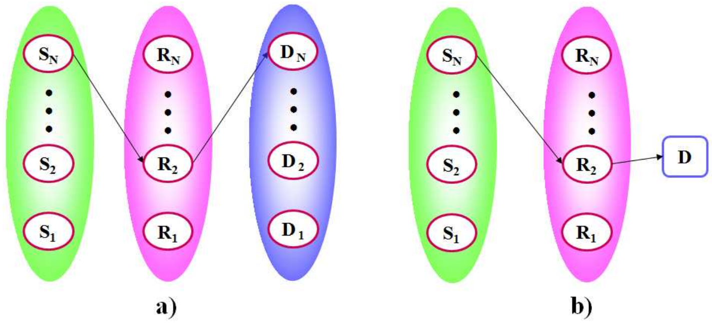

Figure 6.1: Network model

calculate the diversity achieved by each source through the use of sum-rate criterion. Our analysis holds true for both networks in Fig. 6.1, but to be more specific, we consider the network of Fig. 6.1-b.

The source node set and relay node set are respectively denoted by $S = \{S_1, ..., S_N\}$ and $R = \{R_1, ..., R_N\}$. We denote the SNR of the channel $S_i \to R_j$ by $\Gamma_{i,j}$. In this way, all SNR values of the channels in the first hop form the matrix $\Gamma = [\Gamma_{i,j}]$. Since each relay should be assigned to one and only one source, we should select one $\Gamma_{i,j}$ in each row and column of matrix Γ.

6.3.2 Diversity Order Analysis

Theorem 1. *The relay-selection based on the sum-rate criterion achieves full diversity, assuming that all entries of the matrix* $\mathbf{\Gamma}$ *are independent:*

Proof. The proof is based on induction. First we show that the statement is true for $N = 2$. For this case, consider the matrix of SNR values $\mathbf{\Gamma}$

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} \\ \Gamma_{2,1} & \Gamma_{2,2} \end{bmatrix}$$

The maximum information exchange rate for this network is

$$\mathbf{I} = \begin{bmatrix} \log(1+\Gamma_{1,1}) & \log(1+\Gamma_{1,2}) \\ \log(1+\Gamma_{2,1}) & \log(1+\Gamma_{2,2}) \end{bmatrix}$$

Using sum-rate criterion, we should compare

$$\log(1+\Gamma_{1,1}) + \log(1+\Gamma_{2,2}) \gtrless \log(1+\Gamma_{1,2}) + \log(1+\Gamma_{2,1}) \tag{6.4}$$

Simplifying (6.4) yields

$$(1+\Gamma_{1,1})(1+\Gamma_{2,2}) \gtrless (1+\Gamma_{1,2})(1+\Gamma_{2,1})$$

Let us denote the SNR of the channel which is assigned to S_1 by Γ_1, i.e. the goal of this analysis is to find the diversity order achieved by Γ_1. According to the sum-rate criterion, $\Gamma_{1,1}$ is assigned to S_1 if $(1+\Gamma_{1,1})(1+\Gamma_{2,2}) > (1+\Gamma_{1,2})(1+\Gamma_{2,1})$ and otherwise, $\Gamma_{1,2}$ is assigned to S_1. For the sake of abbreviation, let us denote $1+\Gamma_1$ by Φ_1. We also assume that $1+\mathbf{\Gamma}$ equals to the following matrix.

$$1+\mathbf{\Gamma} = \begin{bmatrix} X & W \\ Z & Y \end{bmatrix}$$

Hence, the CDF of SNR for Φ_1 can be written as follows:

$$\Pr\{\Phi_1 < \varphi\} = \Pr\{X < \varphi, XY > WZ\} + \Pr\{Y < \varphi, XY < WZ\}$$

The symmetric nature of the problem implies that

$$\Pr\{\Phi_1 < \varphi\} = 2\Pr\{X < \varphi, XY > WZ\} \tag{6.5}$$

In order to calculate (6.5), we should integrate the joint PDF over a four-dimensional space specified by $X < \varphi$ and $XY > WZ$. This space is shown in Fig. 6.2 assuming that X is fixed. The upper limits of integration are specified by:

$$XY > WZ \Rightarrow w < xy/z$$

$$XY > Z \Rightarrow z < xy$$

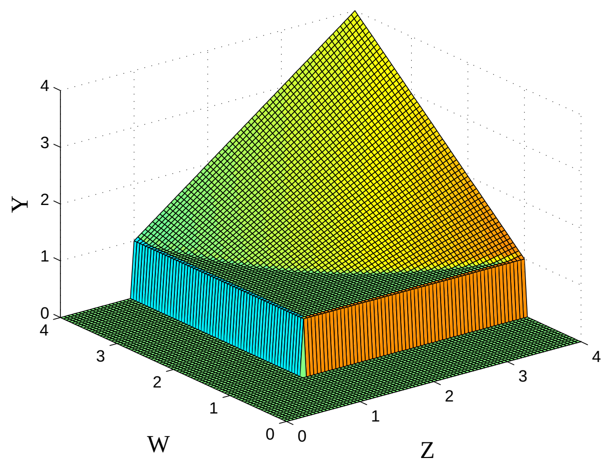

Figure 6.2: Integration space for Eq. 6.6

Now, we can write $\Pr\{X < \varphi, XY > WZ\}$ as follows

$$\Pr(X < \varphi, XY > WZ) = \int_1^{\varphi} \int_1^{\infty} \int_1^{xy} \int_1^{xy/z} f_W(w) f_Z(z) f_Y(y) f_X(x) dw dz dy dx \tag{6.6}$$

Differentiating (6.6) with respect to φ yields the PDF:

$$f_\Phi(\varphi) = \frac{d\Pr(X < \varphi, XY > WZ)}{d\varphi} = \varphi e^{-\lambda(\varphi-1)} \underbrace{\int_1^{\infty} \overbrace{\int_1^{\varphi y} \underbrace{\int_1^{\varphi y/z} f_W(w) f_Z(z) f_Y(y) dw}_{1^{st}\text{ integral}}\, dz}^{2^{nd}\text{ integral}}\, dy}_{3^{rd}\text{ integral}} \tag{6.7}$$

The integration space is shown in Fig. 6.2. We do the integration step-by-step. For the first integral, we have:

$$\int_1^{\varphi y/z} f_w(w) dw = \int_1^{\varphi y/z} \lambda e^{-\lambda(w-1)} dw = -\exp\left(-\lambda\left(\frac{\varphi y}{z} - 1\right)\right) + 1$$

and for the the second integral we have

$$
\begin{aligned}
&\int_1^{\varphi y}\int_1^{\varphi y/z} f_z(z)f_w(w)dwdz\\
&=\int_1^{\varphi y}\left(-\exp\left(-\lambda\left(\frac{\varphi y}{z}-1\right)\right)+1\right)\lambda\exp\left(-\lambda\left(z-1\right)\right)dz\\
&=\underbrace{-\int_1^{\varphi y}\lambda\exp\left(-\lambda\left(z+\frac{\varphi y}{z}-2\right)\right)dz}_{A(\varphi,y)}+\int_1^{\varphi y}\lambda\exp\left(-\lambda\left(z-1\right)\right)dz\\
&=A\left(\varphi,y\right)-\exp\left(-\lambda\left(\varphi y-1\right)\right)+1
\end{aligned}
$$

To the best of our knowledge, a closed form solution for the third step of integration is not known. Therefore, in order to proceed, we divide the integration interval of the third integral into two intervals

$$
\begin{aligned}
&\int_1^{\infty}\int_1^{\varphi y}\int_1^{\varphi y/z} f_y(y)f_z(z)f_w(w)dwdzdy\\
&=-\frac{1}{\varphi+1}e^{-\lambda(\varphi-1)}+1+\int_{1/\varphi}^{\infty}A\lambda e^{-\lambda(y-1)}dy-\int_{1/\varphi}^{1}A\lambda e^{-\lambda(y-1)}dy\\
&=-\frac{1}{\varphi+1}e^{-\lambda(\varphi-1)}+1\\
&\quad-\int_{1/\varphi}^{\infty}\int_1^{\varphi y}\lambda\exp\left(-\lambda\left(z+\frac{\varphi y}{z}-2\right)\right)\lambda e^{-\lambda(y-1)}dzdy\\
&\quad+\int_{1/\varphi}^{1}\int_1^{\varphi y}\lambda\exp\left(-\lambda\left(z+\frac{\varphi y}{z}-2\right)\right)\lambda e^{-\lambda(y-1)}dzdy
\end{aligned}
$$

Let us denote the last two integrals by $B(\varphi)$. Then by changing the order of integrations, we have:

$$
\begin{aligned}
B\left(\varphi\right)&=-\int_1^{\infty}\int_{z/\varphi}^{\infty}\lambda\exp\left(-\lambda\left(z+\frac{\varphi y}{z}-2\right)\right)\lambda e^{-\lambda(y-1)}dydz\\
&\quad+\int_1^{x}\int_{z/\varphi}^{1}\lambda\exp\left(-\lambda\left(z+\frac{\varphi y}{z}-2\right)\right)\lambda e^{-\lambda(y-1)}dydz\\
&=-\int_1^{\infty}\lambda^2e^{-\lambda(z-3)}\int_{z/\varphi}^{\infty}\exp\left(-\lambda\left(\frac{\varphi y}{z}+y\right)\right)dydz\\
&\quad+\int_1^{\varphi}\lambda^2e^{-\lambda(z-3)}\int_{z/\varphi}^{1}\exp\left(-\lambda\left(\frac{\varphi y}{z}+y\right)\right)dydz
\end{aligned}
$$

$$\begin{aligned}
=& -\int_1^{\infty} \lambda e^{-\lambda(z-3)} \left[-\frac{\exp\left(-\lambda y \left(\frac{\varphi}{z}+1\right)\right)}{\frac{\varphi}{z}+1} \right]_{z/\varphi}^{\infty} dz \\
&+ \int_1^{\varphi} \lambda e^{-\lambda(z-3)} \left[-\frac{\exp\left(-\lambda y \left(\frac{\varphi}{z}+1\right)\right)}{\frac{\varphi}{z}+1} \right]_{z/\varphi}^{1} dz \\
=& -\int_1^{\infty} \lambda e^{-\lambda(z-3)} \frac{\exp\left(-\lambda\left(1+\frac{z}{\varphi}\right)\right)}{\frac{\varphi}{z}+1} dz \\
&+ \int_1^{\varphi} \lambda e^{-\lambda(z-3)} \frac{-\exp\left(-\lambda\left(\frac{\varphi}{z}+1\right)\right)+\exp\left(-\lambda\left(1+\frac{z}{\varphi}\right)\right)}{\frac{\varphi}{z}+1} dz \\
=& \underbrace{-\int_{\varphi}^{\infty} \lambda \left(\frac{z}{\varphi+z}\right) \exp\left(-\lambda\left(z+\frac{z}{\varphi}-2\right)\right) dz}_{C(\varphi)} \\
&- \int_1^{\varphi} \lambda \left(\frac{z}{\varphi+z}\right) \exp\left(-\lambda\left(z+\frac{\varphi}{z}-2\right)\right) dz
\end{aligned}$$

where

$$\begin{aligned}
C(\varphi) &= -\int_{\varphi}^{\infty} \lambda \left(1-\frac{\varphi}{\varphi+z}\right) \exp\left(-\lambda z\left(1+\frac{1}{\varphi}\right)\right) e^{2\lambda} dz \\
&= -\left[-\frac{\exp\left(-\lambda z\left(1+\frac{1}{\varphi}\right)\right)}{1+\frac{1}{\varphi}} \right]_x^{\infty} e^{2\lambda} + \int_x^{\infty} \lambda \left(\frac{\varphi}{\varphi+z}\right) \exp\left(-\lambda z\left(1+\frac{1}{\varphi}\right)\right) e^{2\lambda} dz \\
&= -\frac{\varphi}{\varphi+1} \exp\left(-\lambda\left(\varphi+1\right)\right) e^{2\lambda} + \int_x^{\infty} \lambda \left(\frac{\varphi}{\varphi+z}\right) \exp\left(-\lambda z\left(1+\frac{1}{\varphi}\right)\right) e^{2\lambda} dz
\end{aligned}$$

Now, substituting $B(\varphi)$ and $C(\varphi)$ in $f_\Phi(\varphi)$ yields

$$\begin{aligned}
f_\Phi(\varphi) =& 2\varphi e^{-\lambda(\varphi-1)} \left(-\frac{1}{\varphi+1} e^{-\lambda(\varphi-1)} + 1 - \frac{\varphi}{\varphi+1} e^{-\lambda(\varphi+1)} e^{2\lambda} \right) + 2\varphi e^{-\lambda(\varphi-1)} \\
& \left(\int_x^{\infty} \lambda \left(\frac{\varphi}{\varphi+z}\right) \exp\left(-\lambda z\left(1+\frac{1}{\varphi}\right)\right) e^{2\lambda} dz - \int_1^{\varphi} \lambda \left(\frac{z}{\varphi+z}\right) \exp\left(-\lambda\left(z+\frac{\varphi}{z}-2\right)\right) dz \right)
\end{aligned}$$

$$
\begin{aligned}
&= \underbrace{2\varphi e^{-\lambda(\varphi-1)}\left(1-e^{-\lambda(\varphi-1)}\right)}_{D_1(\varphi)} \\
&\quad + \underbrace{2\varphi e^{-\lambda(\varphi-1)}\left(\int_{\varphi}^{\infty}\lambda\left(\frac{\varphi}{\varphi+z}\right)\exp\left(-\lambda z\left(1+\frac{1}{\varphi}\right)\right)e^{2\lambda}dz\right)}_{D_2(\varphi)} \\
&\quad + \underbrace{2\varphi e^{-\lambda(\varphi-1)}\left(-\int_{1}^{\varphi}\lambda\left(\frac{z}{\varphi+z}\right)\exp\left(-\lambda\left(z+\frac{\varphi}{z}-2\right)\right)dz\right)}_{D_3(\varphi)}
\end{aligned} \tag{6.8}
$$

In this result, there is no Taylor series in terms of λ for $D_2(\varphi)$ and $D_3(\varphi)$, but Proposition 1 applies and for each of them, we have

$$
\lim_{\lambda\to 0^+}\frac{D_i(\varphi,\lambda)}{D_i(\varphi,k\lambda)} = k^2, \;\; i=1,2,3 \tag{6.9}
$$

Now, let us assume that the statement is true for a network of $(N-1)$ source-destination pairs and $(N-1)$ relays. We want to prove the statement for a system consisting of N source-destination pairs and N relays. In this case, $\mathbf{\Gamma}$ is a N-by-N matrix and there are $N!$ different permutations (denoted by Y_i). We define $\mathbf{X} = 1 + \mathbf{\Gamma}$. Again, let us denote the SNR of the channel which is assigned to S_1 as Γ_1, i.e. the goal of this analysis is to find the diversity order achieved by Γ_1. We denote $1+\Gamma_1$ by Φ_1. Hence, similarly as (6.5), we have

$$
\Pr\{\Phi_1 < \varphi\} = N!\Pr\{X_{1,1} < \varphi, Y_1 > Y_i, (i = 2, \cdots, N!)\} \tag{6.10}
$$

where Y_1 is one of the permutations involving $X_{1,1}$. For instance, we assume that (See Fig. 6.3)

$$
\begin{aligned}
Y_1 &\triangleq \prod_{i=1}^{N} X_{i,i} \\
Y_2 &\triangleq X_{1,2}X_{2,1}\prod_{i=3}^{N} X_{i,i}
\end{aligned}
$$

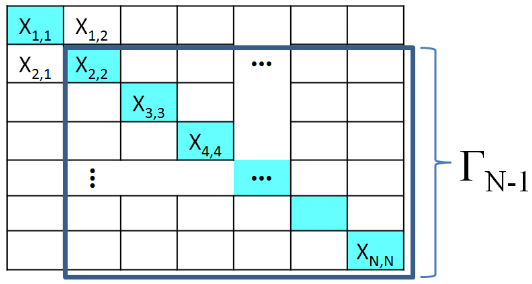

Figure 6.3: Permutations Y_1

This time, as a generalization of (6.7), we have a N^2-dimensional integral. Let us denote the space specified by the inequality $Y_1 \geq Y_i$ as D_{1i}. Hence, (6.10) becomes.

$$\begin{aligned} F_{\Phi_1}(\varphi) &= N!\Pr\{X_{1,1} \leq \varphi, Y_1 > Y_i, (i = 2, \cdots, N!)\} \\ &= N! \underset{\bigcap_{i=2}^{N!} D_{1i}}{\int \cdots \int} f_{X_{11}, X_{12}, \cdots, X_{NN}}(X_{1,1}, X_{1,2}, \cdots, X_{N,N})\, dX_{1,1}\, dX_{1,2} \cdots dX_{N,N} \end{aligned}$$

Now, let us consider $\mathbf{\Gamma_{N-1}}$. In this matrix, $Y = \prod_{i=2}^{N} X_{i,i}$ is larger than the product of elements in any other permutation. We denote by D the space specified by this event. Then we have

$$\begin{aligned} &\underset{\bigcap_{i=2}^{N!} D_{1i}}{\int \cdots \int} f_{X_{1,1}, X_{1,2}, \cdots, X_{NN}}(X_{1,1}, X_{1,2}, \cdots, X_{N,N})\, dX_{1,1}\, dX_{1,2} \cdots dX_{N,N} \\ &\leq \underset{D_{1,2} \bigcap D}{\int \cdots \int} f_{X_{1,1}, X_{1,2}, \cdots, X_{N,N}}(X_{1,1}, X_{1,2}, \cdots, X_{N,N})\, dX_{1,1}\, dX_{1,2} \cdots dX_{N,N} \end{aligned}$$

The above inequality means that by increasing the integration space, the result of integration becomes larger. It is enough to prove that the expression in the right-hand side achieves diversity N.

Expanding D_{12} and D, it is found that the integration space does not put any limits on the variables $X_{1,i}, i = 2, \cdots, N$ and $X_{i,1}, i = 2, \cdots, N$, which means that the integration interval over these variables is $[1, \infty)$. In other words, the result of integration does not depend on these variables.

$$F_{\Phi_1}(\varphi) = \int_0^{\varphi} \lambda \exp\left(-\lambda X_{1,1}\right) \underset{D_{1,2}}{\int \cdots \int} \underset{D}{\int \cdots \int} f_{X_{1,1},X_{1,2},\cdots,X_{3,3}}\left(X_{1,1}, X_{1,2}, \cdots, X_{N,N}\right)$$

$$dX_{1,1}\, dX_{1,2} \cdots dX_{N,N}$$

$$= \int_0^{\varphi} \lambda \exp\left(-\lambda X_{11}\right) \underbrace{\int \cdots \int}_{D_{1,2} \equiv X_{1,1}\, X_{2,2} \geq X_{1,2}\, X_{2,1}} \underbrace{\int \cdots \int}_{D} \underbrace{\int_1^{\infty} \cdots \int_1^{\infty}}_{X_{1,i}, i=2,\cdots,N} \underbrace{\int_1^{\infty} \cdots \int_1^{\infty}}_{X_{i,1}, i=2,\cdots,N}$$

$$f_{X_{1,1},X_{1,2},\cdots,X_{N,N}}\left(X_{1,1}, X_{1,2}, \cdots, X_{N,N}\right) dX_{1,1}\, dX_{1,2} \cdots dX_{N,N}$$

Using (6.8), for the integration over $D_{1,2}$, we have

$$\begin{aligned}
&\underbrace{\int \cdots \int}_{D_{1,2} \equiv X_{1,1}\, X_{2,2} \geq X_{1,2}\, X_{2,1}} f_{X_{1,2},X_{2,1},X_{2,2}}\left(X_{1,2}, X_{2,1}, X_{2,2}\right) dX_{1,2}\, dX_{2,1}\, dX_{2,2} \\
=&1 - e^{-\lambda(X_{1,1}-1)} + \int_x^{\infty} \lambda \left(\frac{X_{1,1}}{X_{1,1}+z}\right) \exp\left(-\lambda z \left(1 + \frac{1}{X_{1,1}}\right)\right) e^{2\lambda} dz \\
&- \int_1^{X_{1,1}} \lambda \left(\frac{z}{X_{1,1}+z}\right) \exp\left(-\lambda \left(z + \frac{X_{1,1}}{z} - 2\right)\right) dz.
\end{aligned} \tag{6.11}$$

Let us denote this result as $f_{D_{1,2}}$. For the integration over D, we have

$$\lim_{\lambda \to 0^+} \frac{f_{N-1}(X_{1,1}, \lambda)}{f_{N-1}(X_{1,1}, k\lambda)} = k^{N-2} \tag{6.12}$$

which is the assumption of the induction. Then using (6.11) and (6.12), after some manipulations, we obtain

$$\lim_{\lambda \to 0^+} \frac{f_{\Phi_1}(\varphi, \lambda)}{f_{\Phi_1}(\varphi, k\lambda)} = \lim_{\lambda \to 0^+} \frac{f_{N-1}(\varphi, \lambda) f_{D_{1,2}}(\varphi, \lambda)}{f_{N-1}(\varphi, k\lambda) f_{D_{1,2}}(\varphi, k\lambda)} = k^N \tag{6.13}$$

Since $\Phi_1 = 1 + \Gamma_1$, it can be inferred that S_1 achieves full diversity. The symmetricity of the problem implies that all source nodes achieve full diversity. □

Figures 6.4 and 6.5 show the Monte-Carlo simulation results of BER for the different scenarios shown in Fig. 6.1. From these figures, it can be inferred that network achieves full diversity.

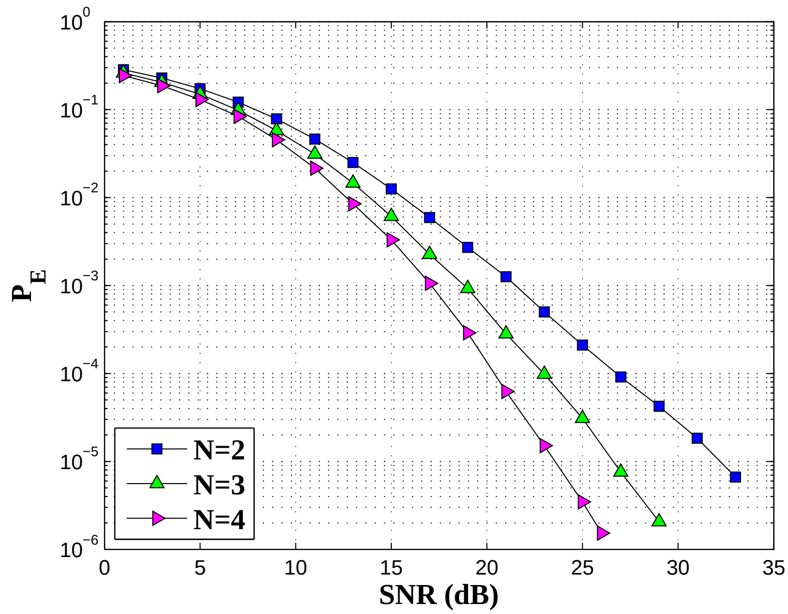

Figure 6.4: BER curves for relay-assignment based on sum-rate criterion in the network of Fig. 6.1-a

6.4 Diversity Analysis of Sum-SNR Criterion

Among different criteria for relay-assignment in cooperative networks, the maximization of the overall network SNR values is very effective in a wide range of SNR values. This section deals with the statistical analysis of relay-assignment based on this criterion, i.e. we analyze the distribution of some random variables when their summation is maximized. Each random variable follows exponential distribution, because channels are assumed to be Rayleigh flat fading channels. It is assumed that the system model is similar to the system model in Section 6.3. We will show that the improvement offered by this approach lies in the coding gain and the diversity order is one. Then through some simulations we will show that this approach offers the best performance among other approaches for a wide range of SNR values (less than about 15 dB).

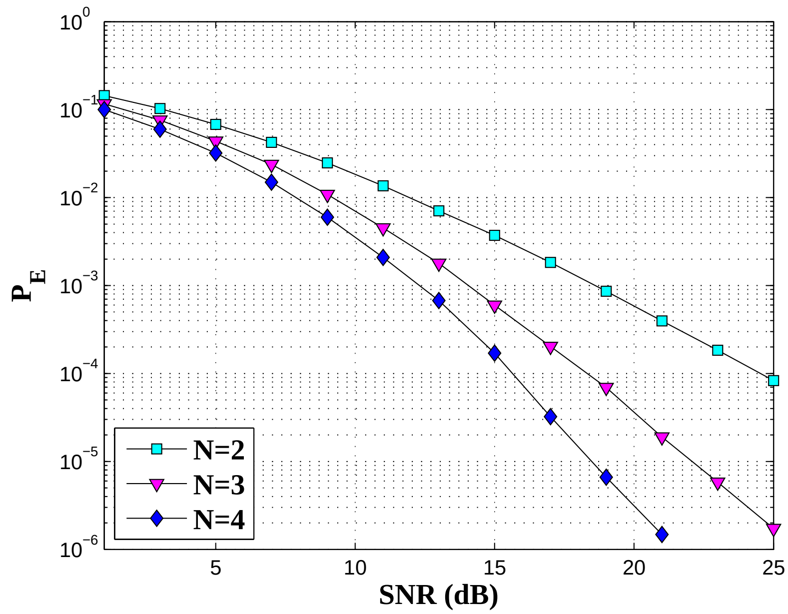

Figure 6.5: BER curves for relay-assignment based on sum-rate criterion in the network of Fig. 6.1-b

6.4.1 Problem Formulation

Again consider the networks shown in Fig. 6.1. This time we consider the diversity analysis of the system when max-sum-SNR criterion is used for relay selection. We denote the SNR of the channel $S_i \to R_j$ as $\Gamma_{i,j}$. In order to solve this problem, first we consider the simplest case where $N = 2$ and then we generalize the analysis to other values of N.

6.4.2 Diversity Order Analysis For $N = 2$

We start with the matrix of SNR values

$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} \\ \Gamma_{2,1} & \Gamma_{2,2} \end{bmatrix}$$

In this case there are two possibilities for relay-assignment. For each possibility, we denote the sum of SNR values by Y_i:

$$\begin{cases} Y_1 = \Gamma_{1,1} + \Gamma_{2,2} \\ Y_2 = \Gamma_{1,2} + \Gamma_{2,1} \end{cases}$$

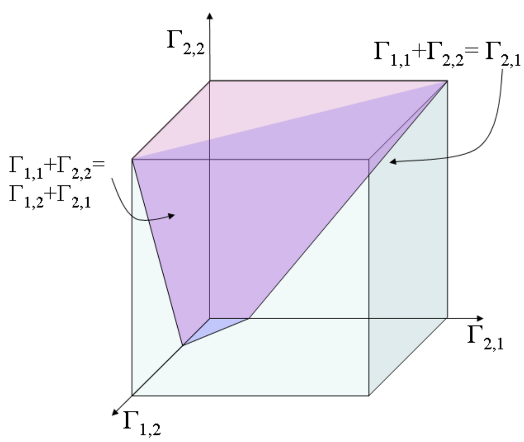

Figure 6.6: Integration space for $\Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_2)$

We choose $Z = max\{Y_1, Y_2\}$. We denote the channel assigned to the node S_i by Γ_i. Our final goal in this analysis is to find the diversity order offered to S_i. Because of the symmetrical nature of the problem, the distribution of all Γ_is are identical. The CDF of the random variable Γ_1 can be expressed as

$$\Pr(\Gamma_1 < \gamma) = \Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_2) + \Pr(\Gamma_{1,2} < \gamma, Y_1 < Y_2) \tag{6.14}$$

In order to calculate the $\Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_2)$, we should integrate the joint PDF over a four-dimensional space specified by $\Gamma_{1,1} < \gamma$ and $Y_1 > Y_2$. This space is shown in Fig. 6.6 assuming that $\Gamma_{1,1}$ is fixed. The upper limits of integration are specified by:

$$\Gamma_{1,1} + \Gamma_{2,2} > \Gamma_{1,2} + \Gamma_{2,1} \Rightarrow \Gamma_{1,2} < \gamma_{1,1} + \gamma_{2,2} - \gamma_{2,1}$$
$$\Gamma_{1,1} + \Gamma_{2,2} > \Gamma_{2,1} \Rightarrow \Gamma_{2,1} < \gamma_{1,1} + \gamma_{2,2}$$

Now, we can write $\Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_2)$ as follows

$$\Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_2)$$
$$= \int_0^{\gamma}\int_0^{\infty}\int_0^{\gamma_{1,1}+\gamma_{2,2}}\int_0^{\gamma_{1,1}+\gamma_{2,2}-\gamma_{2,1}} f_{\Gamma_{1,1}}(\gamma_{1,1}) f_{\Gamma_{2,2}}(\gamma_{2,2}) f_{\Gamma_{2,1}}(\gamma_{2,1}) f_{\Gamma_{1,2}}(\gamma_{1,2}) d\gamma_{1,2} d\gamma_{2,1} d\gamma_{2,2} d\gamma_{1,1} \tag{6.15}$$

Here $1/\lambda$ denotes the average SNR value and i.i.d Rayleigh fading channels are assumed. Then, we obtain

$$\Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_2) = \frac{1}{4}((\lambda\gamma + 2)e^{-2\lambda\gamma} - 4e^{-\lambda\gamma} + 2)$$

Same calculation holds true for the second term in (6.14), i.e. $\Pr(\Gamma_{1,2} < \gamma | Y_1 < Y_2) = \frac{1}{4}((\lambda\gamma + 2)e^{-2\lambda\gamma} - 4e^{-\lambda\gamma} + 2)$. Plugging the result into (6.14) yields

$$\Pr(\Gamma_1 < \gamma) = \frac{1}{2}(\lambda\gamma e^{-2\lambda\gamma} + 2e^{-2\lambda\gamma} - 4e^{-\lambda\gamma} + 2)$$

The first two terms in the Taylor expansion of the above expression are

$$\Pr(\Gamma_1 < \gamma) \cong \frac{1}{2}\lambda\gamma - \frac{1}{12}\lambda^4\gamma^4 \tag{6.16}$$

According to proposition 1 in [41], when the PDF of SNR can be approximated by a single polynomial term for $\gamma \to 0^+$ ($p_\Gamma(\gamma) = a\gamma^t + O(\gamma^{t+\epsilon})$), the system has a diversity order of $(t + 1)$. Here, $\epsilon > 0$ and a is a positive constant. Using this proposition, the result in (6.16) implies that the diversity order offered by Γ_1 is one.

The integral in (6.15) is the core expression to calculate the distribution of Γ_i. As we saw for $N = 2$, the Taylor expansion of the result of this integral involves the first order term γ. The Taylor expansion of the integrand in terms of γ starts with the fixed term (γ^0). On the other hand, the maximum value for the upper limit of the first integral in (6.15) is γ. For the remaining three integrals in (6.15) the maximum value of the upper limit is infinity. This property holds true for any other value of N. In the next section we will see this property in detail.

6.4.3 Diversity Order Analysis for General Values of N

For the general case of N sources and N relays, there are $N!$ possibilities for relay assignment. After the process of relay-assignment based on max-sum-SNR criterion, we denote the SNR of the channel assigned to S_1 by Γ_1. Similar to (6.14), the CDF of Γ_1 can be written as $F_{\Gamma_1}(\gamma) = \sum_i \Pr(\Gamma_{1,i} < \gamma, Y_i > Y_j)$ where $j = 1, ..., N!, j \neq i$. Because of the symmetricity of the problem in terms of each $\Gamma_{1,i}$, all of the terms participating in the above expression have the same form. Hence, the result can be simplified as

$$\Pr(\Gamma_1 < \gamma) = N \Pr(\Gamma_{1,1} < \gamma, Y_1 > Y_j, j = 2, 3, ..., N!)$$

In order to evaluate the above expression we need to solve an N^2-dimensional integral. The integrand is the product of all probability density functions $f_{i,j}(\gamma_{i,j})$ and the integration space is an N^2 dimensional space specified by $Y_1 > Y_j, j = 2, 3, ..., N!$. Hence the upper limit of each integral is a linear combination of different $\Gamma_{i,j}$ (similar to (6.15)) and we have:

1) The Taylor expansion of the integrand in terms of γ starts with the fixed term (γ^0), because the Taylor expansion of each terms $f_{i,j}(\gamma_{i,j}) = \lambda \exp(\lambda \gamma_{i,j})$ starts with the fixed term γ^0.

2) The maximum value for the upper limit of the integral over $\Gamma_{1,1}$ is γ. In this step of integration, the lowest degree in Taylor expansion of the integrand is increased by one.

3) For the remaining steps of integration, the upper limit is a linear combination of the other $\Gamma_{i,j}$s and the maximum value of these upper limits is infinity, because when $\Gamma_{1,1}$ is upper bounded with γ, other $\Gamma_{i,j}$s can be increased up to infinity and still the inequality $Y_1 > Y_j, j = 2, 3, ..., N!$ holds true. This means that the lowest degree in Taylor expansion of the integrand is not increased in this step.

Hence, the smallest power in Taylor expansion of $\Pr(\Gamma_1 < \gamma)$ is one, because the only increase in the power of integrand occurs in the integration over $\Gamma_{1,1}$. Again, using the proposition 1 in [41], this result implies that the diversity order offered by Γ_1 is one.

6.4.4 Simulations and Discussion

Fig. 6.7 shows the Monte-Carlo simulation result of BER for max-sum-SNR criterion compared with other relay-assignment criteria when $N = 4$. This figure shows that for low average SNR values, using max-sum-SNR criterion brings the best performance among all other relay-assignment techniques through cooperation. However in sequential relaying, those sources who have the right to select their relays first, achieve higher performance.

In Fig. 6.8 shows an estimate of have calculated the average number of users who experience an outage through cooperation. This figure shows that the mentioned relay-assignment approach offers the best performance for SNR values less than 15 db and a very good performance for all SNR values.

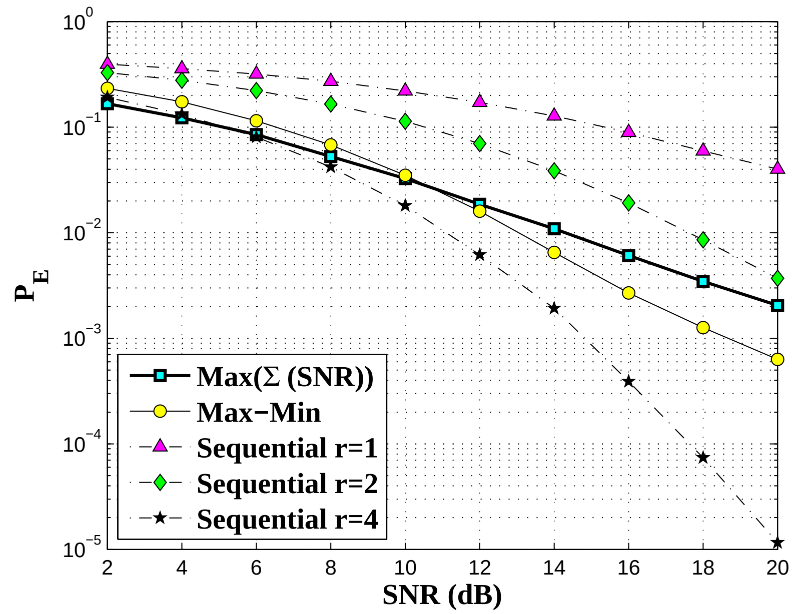

Figure 6.7: Comparison of P_E for different relay-assignment methods

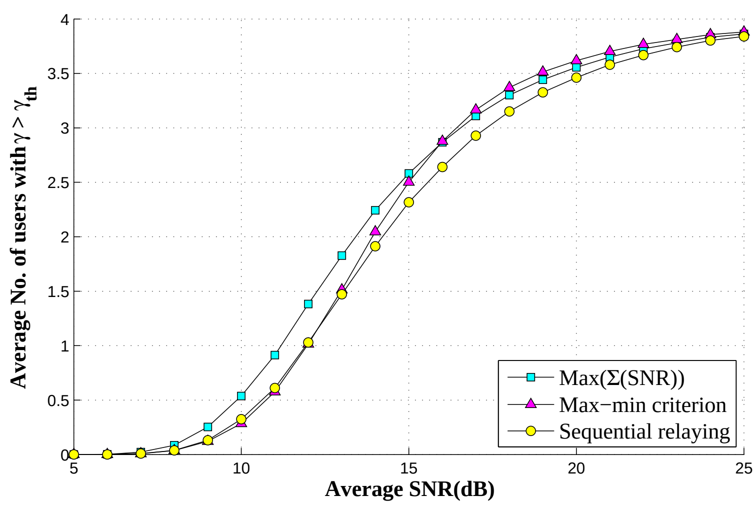

Figure 6.8: Average No. of users with $\gamma > \gamma_{threshold}$

The algorithm of relay assignment based on max-min criterion maximizes the minimum of the selected SNR values. In low average SNR values, it is almost impossible to have an acceptable SNR for all of the selected channels. It means that we should forget some nodes in order to assign an acceptable channel for some other nodes. Due to the low average SNR value, we cannot assign good channels to all of the nodes at the same time. This is what happens in relay assignment based on max-sum-SNR criterion. In this method, some low SNR channels may be assigned to some nodes in order to maximize the overall network SNR which consequently involves some good conditioned nodes.

6.5 A New Formulation to Find Max-sum-SNR

Exhaustive search is the last solution for many optimization problems in wireless communications such as resource allocation, channel assignment (or subcarrier assignment for OFDMA) and relay assignment. However, for large networks, finding the optimum answer through exhaustive search becomes impractical. For example for $N = 20$, there are more than 10^{18} different permutations in Fig. 6.1. Howeve if the problem could be described in canonical form, the solution could be easily found by using different LP methods. We will give a simple formulation to find this permutation in this section.

In mathematics, linear programming (LP) is a technique for optimization of a linear objective function, subject to linear equality and linear inequality constraints. Binary integer programming is the special case of linear programming where variables are required to be 0 or 1. In order to use LP, the problem of relay assignment should be expressed in terms of maximization of one objective function. One widely accepted and commonly used objective function is the sum of SNR/rate values for all of the links. Recently many interesting formulations have been proposed for this purpose [13, 53, 54, 55]. Also authors in [15] have provided a heuristic to find a close-to-optimal relay assignment. Authors in [56] have provided bounds for multiple-sources single-relay scenario. More recently, [57] has investigated the fairness issues in an orthogonal frequency division multiple-access (OFDMA) uplink scenario with multiple sources, multiple relays and a single destination.

The main contributions of this section are as follows.

1. We present a flexible Vehicle Routing Problem (VRP) model for the problem of relay assignment in cooperative networks. The proposed model incorporates the

problems of clustering and relay assignment into a unified problem and can be solved efficiently by using binary integer programming (BIP).

2. The flexibility of the proposed approach allows us to solve many relay assignment problems by using the same algorithm. Two different scenarios are described to show this flexibility. In the first scenario, only one of the nodes in each cooperating set benefits from the cooperation, whereas in the other scenario both nodes benefit.
3. The proposed approach achieves fairness by providing the same average performance to all the source nodes. It also distributes the load equally among the relay nodes.
4. The proposed algorithm is also applicable to other network configurations. As an example we can consider a clustered network with N source-destination pairs and M relays.

For compliance, we assume that a centralized resource allocation is employed and that the SNR information of all the nodes is known to the resource allocator. The channels are assumed to be slow fading and remain constant during resource allocation process. All the wireless nodes work in half-duplex mode, i.e. cannot transmit and receive at the same time.

6.5.1 Vehicle Routing Problem

A rough description of the VRP is as follows. Suppose that a number of goods need to be moved from a specific pickup location to some drop-off locations. The goal is to find optimal (shortest) routes for a fleet of vehicles to visit the pickup and delivery locations. This problem is called the Vehicle Routing Problem.

The VRP is a combinatorial problem whose ground set is the edges of a graph $G(V, A)$ where $V = \{v_0, v_1, ..., v_N\}$ is a vertex set and v_is form a set of n nodes (cities or clients). $A = \{(v_i, v_j)|v_i, v_j \in V, i \neq j\}$ is an arc set and a depot is located at v_0. Also it's assumed that

- C denotes a matrix of non-negative costs (or gains) $c_{i,j}$ between customers v_i and v_j
- $\mathbf{d}$ is a vector of the customer demands
- R_i is the route for vehicle i

– m is the number of vehicles (all identical) where one route is assigned to each vehicle.

When $c_{i,j} = c_{j,i}$ for all i and j, the problem is said to be symmetric. With each client v_i is associated a quantity q_i of some goods to be delivered by a vehicle. The VRP thus consists of determining a set of m vehicle routes of minimal total cost (or maximal total gain), starting and/or ending at a depot, such that every vertex in V is visited exactly once by one vehicle. It is required that the total cost of any vehicle route may not surpass a given bound D. A feasible solution is composed of:

– A partition $\{R_0, R_1, ..., R_N\}$ of V.

– A permutation σ_i of $R_i \cup 0$ specifying the order of the customers on route i.

There is a class of VRP problems called capacitated vehicle routing problem (CVRP). In the CVRP, the number of the nodes in each route is limited to Q.

6.5.2 The Proposed Formulation

In this section, we introduce our formulation for clustering and relay assignment based on VRP. Assuming the same formulation given above, let $V = \{v_0, v_1, ..., v_{N+1}\}$ be the set of wireless nodes in the network where node v_0 and v_{N+1} corresponds to the destination and nodes $\{v_1, ..., v_N\}$ correspond to communication nodes. The destination has been split into two nodes to make modeling easier: node v_0 corresponds to the start of the routes and v_{N+1} corresponds to the end of the routes. We assume that the SNRs or the rates are organized as a matrix $\{c_{ij} | 1 \leq i, j \leq N+1\}$. This assumption implies that the objective function should be maximized (whereas in the first definition, the objective function was to be minimized).

A legal route $\bar{r}$ must be a simple path (that is, no node is visited twice) from node v_0 to node v_{N+1}. We can write such a path as $\bar{r} = \{v_0, v_g, ..., v_h, v_{N+1}\}$ where $1 \leq g \leq h$. The nodes v_i, $g \leq i \leq h$ are the nodes visited on the route. The number of the nodes visited on the route is $h - g + 1$. We can also assume a legal route $\bar{r}$ as a simple path from node v_0 to node v_h, i.e. no return to the depot.

The route should satisfy the capacity requirement, i.e. the number of the nodes in each set of cooperating nodes should be less than Q. The cost $c_{\bar{r}}$ of a route $\bar{r}$ is

$$c_{\bar{r}} = \sum_{i \epsilon \bar{r}} c_{v_i, v_{i+1}}. \tag{6.17}$$

Let R be the set of all feasible routes and let $\mathbf{A}_{\bar{r}} = [a_{i\bar{r}}]_{N\times|R|}$ be a Boolean matrix. Let $a_{i\bar{r}} = 1$ if and only if route $\bar{r}$ serves customer i. As an example consider a network which consists of a destination and 4 nodes. Suppose that our feasible routes are the routes consisting of only one or two nodes. In this case, $\mathbf{A}_{\bar{r}}$ takes the following form: (the last row specifies the destination)

$$\mathbf{A}_{\bar{r}} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Then the CVRP can be formulated as

$$\min \sum_{\bar{r}\in R} c_{\bar{r}} x_{\bar{r}} \tag{6.18}$$

subject to

$$\sum_{\bar{r}\in R} a_{i\bar{r}} x_{\bar{r}} = 1, \forall i \in \{1, 2, ..., N\} \tag{6.19}$$

$$\sum x_{\bar{r}} = m \tag{6.20}$$

where m shows the maximum number of sets of cooperating nodes in our formulation and

$$x_{\bar{r}} \in \{0, 1\}, \bar{r} \in R \tag{6.21}$$

specifies if the route $\bar{r}$ is selected through solving the VRP. The objective function (6.18) selects a set of feasible routes that minimizes the sum of the route costs while equation (6.19) ensures that all customers are served exactly once and (6.20) ensures that exactly m vehicles are used. In some variants of the CVRP, equation (6.20) is relaxed such that at most m vehicles are used or such that there are no restrictions on the number of vehicles used.

A variant of the CVRP that is often studied in the heuristic literature is the distance constrained CVRP, where a distance measure $\{d_{i,j}\}$ (possibly different from $\{c_{i,j}\}$) is assigned to each arc. An upper bound on distance D is also given and no routes must be longer than D. This constraint is easily added to our model: we simply require that the nodes $\{v_g, ..., v_h, v_{N+1}\}$ in our feasible path $\bar{r}$ should satisfy the equation

$$\sum_{i\epsilon\bar{r}} d_{v_i, v_{i+1}} \leq D. \tag{6.22}$$

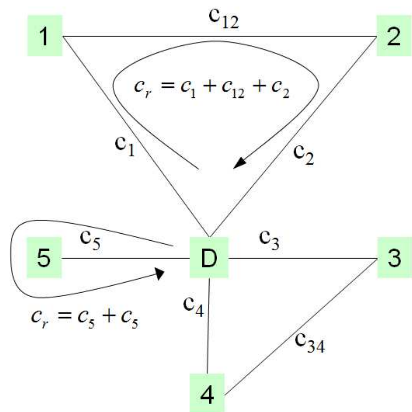

Figure 6.9: An example for a solution of VRP problem

In the context of our wireless network, this constraint can be very useful and it can be assumed as a condition on minimum SNR achieved through each set of cooperating nodes. In this way, we can avoid some bad conditioned nodes to waste their energy.

Now consider the framework of cooperation between pairs of users as shown in Fig. 6.9. In order to comply with CVRP formulation, first of all we need to have a cost-function or benefit-function: we can use SNR, bit-rate, BER and SER. For instance we choose SNR, but to completely comply with the above formulations, we need to make a little bit modification. We denote the SNR between node i and the destination by $\gamma_i = |h_i|^2 / N_i$ and the SNR between node i and node j by $\gamma_{i,j} = |h_{i,j}|^2 / N_{i,j}$. We need to specify the elements of the benefit matrix such that the route gain for each set of the cooperating nodes equals the true amount of SNR for the corresponding nodes. For this purpose, we assume the elements of the benefit-matrix as follows:

$$
\begin{aligned}
c_i &= \frac{1}{2}\gamma_i \\
c_{i,j} &= \frac{\gamma_{i,j}\gamma_j}{\gamma_{i,j} + \gamma_j + 1} + \frac{\gamma_{j,i}\gamma_i}{\gamma_{j,i} + \gamma_i + 1} + \frac{1}{2}\gamma_i + \frac{1}{2}\gamma_j
\end{aligned}
$$

To illustrate the usefulness of this formulation, we need to distinguish the following two cases:

Case 1: User i cooperates with user j and both of them benefit from the cooperation

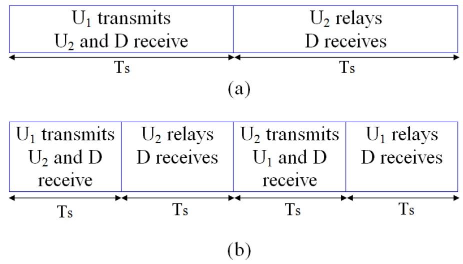

Figure 6.10: Time division channel allocations for (a) Only one user enjoys cooperation, (b) Both users cooperate with each other

(Fig. 6.10-b). If we write the route gain (SNR) for the route $\{v_0, v_i, v_j, v_{N+1}\}$, we have:

$$
\begin{aligned}
SNR_{v_0,v_i,v_j,v_{N+1}} &= c_1 + c_{1,2} + c_2 \\
&= \gamma_i + \frac{\gamma_{i,j}\gamma_j}{\gamma_{i,j} + \gamma_j + 1} + \frac{\gamma_{j,i}\gamma_i}{\gamma_{j,i} + \gamma_i + 1} + \gamma_j
\end{aligned}
\tag{6.23}
$$

On the other hand it is well known that in a two-user cooperation scenario the total SNR for the first user U_i is:

$$
SNR_i = \gamma_i + \frac{\gamma_{i,j}\gamma_j}{\gamma_{i,j} + \gamma_j + 1} \tag{6.24}
$$

and for the second user U_j

$$
SNR_j = \gamma_j + \frac{\gamma_{j,i}\gamma_i}{\gamma_{j,i} + \gamma_i + 1} \tag{6.25}
$$

Therefore, the total SNR of the both users i and j is the same as the route gain in (6.23).

Case 2: User i does not cooperate with any other user, in this case, U_i is only the member of route $\{v_0, v_i, v_{n+1}\}$ which corresponds to the following SNR

$$
c_i + c_i = \frac{1}{2}\gamma_i + \frac{1}{2}\gamma_i = \gamma_i \tag{6.26}
$$

This result is also consistent with the assumption of absence of cooperation.

Generally speaking, vehicle routing problems belong to a class of problems that is proved to be difficult to solve and only moderately sized problems can be solved consistently. The above problem formulation enables us to use BIP to solve the problem.

The proposed algorithm can be also used to solve the relay assignment problem in clustered networks. In a clustered network, the nodes are divided into two clusters (a source cluster and a relay cluster) by using a long term routing process. In order to use the proposed relay assignment method, it is enough to change $\mathbf{A}_{\bar{r}}$ to reflect the different relay assignment permutations in the clustered networks, i.e. each column of $\mathbf{A}_{\bar{r}}$ specifies a set of cooperating nodes with one node in each cluster.

6.5.3 Simulations and Discussion

In this section, first some simulations are presented in order to graphically show how the clustering and relay assignment are achieved by the proposed algorithm. We considered 40 terminals located randomly in the X-Y plane where each coordinate is a uniformly distributed random variable. In order to have a convenient demonstration, it is assumed that the SNR of the link between each pair of nodes is proportional to the inverse square of their distance. The type of cooperation is type (a) in Fig. 6.10, i.e. only one user in each set of cooperating nodes benefits from the cooperation. The terminals are not forced to contribute to cooperation. This condition is applied through writing the routes with 1 and 2 terminals in each columns of matrix $\mathbf{A}$. The result is shown in Fig. 6.11. As it is shown in this picture, two of the terminals do not contribute to the cooperation. In the second simulation, again we considered 40 terminals, but the type of cooperation is type (b) in Fig. 6.10, i.e. both users of each set of cooperating nodes benefit from the cooperation. The result is shown in Fig. 6.12.

In the next simulation, we apply the proposed algorithm to a clustered network which consists of 4 source-nodes, 4 relay-nodes and one destination. It is assumed that only one user in each set of cooperating nodes benefits from the cooperation (Fig. 6.10-a) and all terminals are forced to contribute to cooperation. In this case, each set of cooperating nodes has one node in the source cluster and one node in the relay cluster. In addition it is assumed that each source node uses its direct channel to the destination, however the average SNR of this channel is assumed to be one fourth of the source-relay channels. The BER performance of each source node through the usage of the proposed algorithm is compared with other relay assignment algorithms. The type of modulation is BPSK and the channels in each hop are i.i.d. Rayleigh fading channels. We assume complex AWGN with PSD $N_0/2$ per dimension. Fig. 6.13 shows

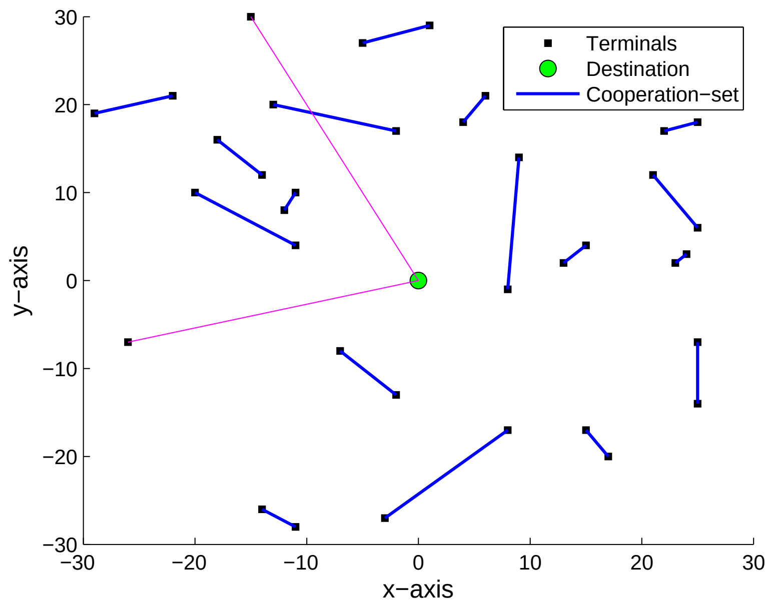

Figure 6.11: Result of the proposed method when only one user benefits from the cooperation in each set of cooperating nodes

some BER results. One interesting result of this simulation is the superior performance of the proposed algorithm for low SNR values. This is because at low average SNR, normally, we cannot have acceptable SNR for all of the selected channels in the same time. This means that selecting some channels with acceptable SNR comes at the price of neglecting some other nodes. This is exactly what is happening in the max-sum-SNR criterion. However, in max-min criterion, the objective of the algorithm is to maximize the minimum SNR for the selected permutation. At low average SNR, normally, this minimum SNR is not large enough, but its maximization may result in relatively low SNR values for all of the selected links.

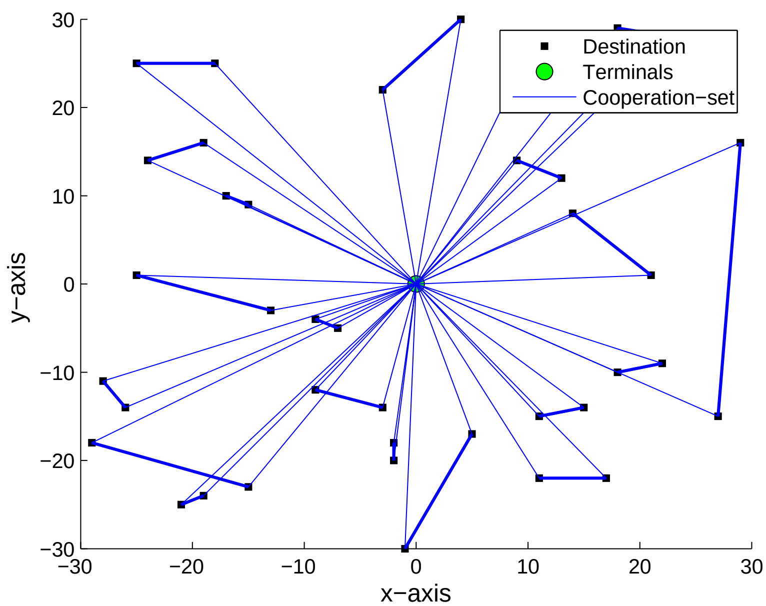

Figure 6.12: Result of the proposed method when both users benefit from the cooperation and all nodes contribute to the cooperation

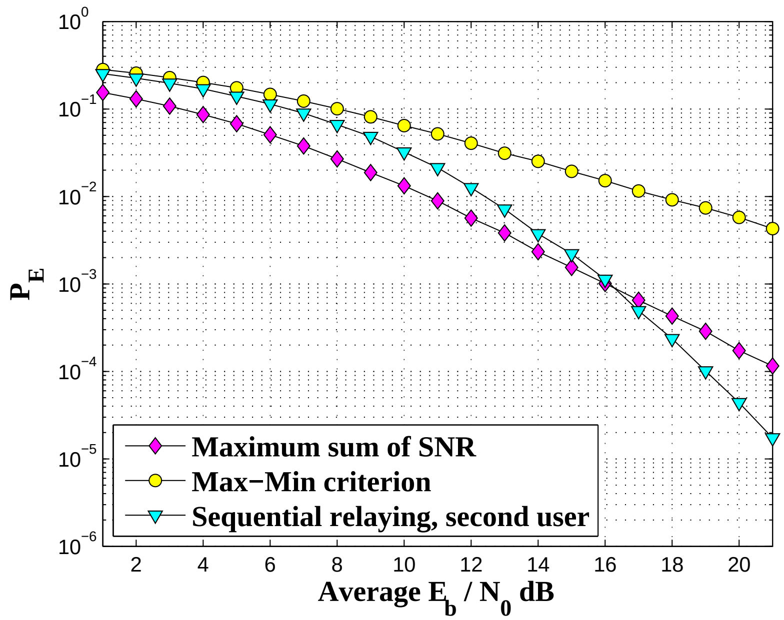

Figure 6.13: BER comparison of the proposed method (maximum sum of SNR values) with other methods for relay assignment

7

Cooperative Relaying Based on Distributed Implementation of Linear Channel Codes

7.1 Introduction

In this chapter, a novel scheme is proposed in order to achieve diversity in a network which consists of multiple sources, multiple relays, and a single destination. The proposed scheme is based on a distributed implementation of linear block codes or convolutional codes. In this scheme, each relay node implements one column of the generator matrix of the code, i.e. different symbols of the codeword are sent to the destination by different relays. Each relay receives the symbols from one or more source nodes and performs a modulo-q addition on the decoded symbols and retransmits the result to the destination. In order to achieve the maximum diversity order, an appropriate source relay pairing has to be employed. For this relay assignment process, we assume that only the channel state information (CSI) of the source-relay channels is available. The main advantage of this scheme is that it achieves diversity without using the CSI of the relay-destination channels for relay assignment. We propose an algorithm based on the max-min criterion for the resulting relay assignment problem. The result of this algorithm is not the optimal solution based on the max-min criterion, but still achieves full diversity in the relay nodes. Finally, soft decoding is employed at the destination to retrieve the transmitted information. We prove that the proposed scheme achieves diversity d_{min} or d_{free} for the E2E performance, where, d_{min} and d_{free} are the minimum distance and the free distance of the corresponding implemented codes, respectively. The proposed scheme achieves complete fairness among all of the source nodes.

7.1.1 Aims of the Proposed Scheme

A basic challenge in the design of cooperative networks is how to assign the relay nodes to the source nodes (relay assignment problem). This is a part of a bigger problem which concerns the architecture of the network. The answer to this problem depends on the amount of available CSI. Many authors have assumed that perfect CSI is available at the resource allocator, although this perfect CSI is difficult to achieve in practice. This is because unavoidable errors happen in the estimation of the channel coefficients at the receiver [58] and also errors happen when feeding back the estimated CSI to the transmitter [59]. In some applications such as resource-constrained ad-hoc and sensor networks, those errors are more likely to happen. On the other hand, monitoring the connectivity among all nodes consumes a considerable amount of network resources. Motivated by the mentioned reasons, some authors have analyzed the effects of such CSI imperfections on the overall performance of the cooperative network, where they have shown that diversity drops to one for many network schemes [60, 61, 62]. In this chapter, we are seeking a model to deploy network coding to achieve diversity at the destination, but we intend the scheme to be less dependent on the CSI. More specifically, we intend not to use the CSI of the second hop (relay destination channels).

7.1.2 Contributions of the Proposed Scheme

1. We will introduce a new scheme which is built in the intersection of two active areas of research; i.e. cooperative networks and network coding. We consider a network which consists of multiple sources (say k), multiple relays (say n_t), and a single destination, where the number of the source nodes is less than the number of the relays ($k < n_t$). Each source node has some information to be sent to the common destination. We assume that only the CSI of the source-relay

channels are available for the design of the cooperation strategy. The goal is to achieve diversity at the destination (See Fig. 7.1). The proposed scheme uses a combination of network coding and relay assignment in order to transmit the information of each source node on several channels which results in diversity. We use a distributed implementation of linear block codes or convolutional codes as a pattern for this purpose.

2. For any utilization of network coding in our network, we need to avoid the weaker channels in the first hop (those channels that are assumed to be known). This issue is very crucial for the proposed scheme in this chapter, because it can strongly affect the diversity achieved at the destination. This relay assignment should be optimum in the sense that it maximizes the diversity achieved in the relay nodes. As it was described in Section 1.4, max-min is a very promising criterion because it can achieve the maximin possible diversity in some network configurations such as a network consisting of k source-destination pairs and n_t relays where $n_t \geq k$ [9, 63]. However it fails to achieve diversity in a network consisting of k sources, n_t relays, and one destination. This result stems from the fact that the information from each source passes through only one channel in the second hop. Hence, if this channel is in deep fading, the corresponding signal is faded at the destination. This fact dominates the performance of this scheme, because in the long term, each of the source nodes experiences this fading. We use this criterion for relay assignment in our scheme. To avoid an exhaustive search, we propose an algorithm for the mentioned relay assignment which finds a sub-optimal permutation based on the max-min criterion and achieves the maximum possible diversity for all of the received signals at the relays.

3. The proposed scheme in this section achieves fairness among source nodes by providing the same diversity order for all of them. It also achieves fairness among relay nodes by distributing the load equally among them (each relay node transmits only signal). This means that we do not need to put extra conditions on the service-time or power consumption of the relays to achieve fairness.

4. In the classical literature, to obtain diversity using linear block codes or the convolutional codes, different coded symbols should undergo different channel fades. Therefore, the use of an interleaver, whose role is to scramble the coded symbols

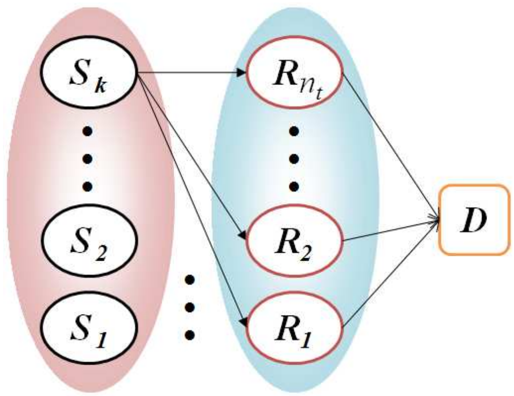

Figure 7.1: System model consisting of k source nodes and n_t relays

before transmission, is necessary. the proposed scheme is free of this limitation, because different coded symbols are transmitted over completely separate channels. Hence, the system will not suffer the corresponding delay.

7.1.3 Chapter Outline

The rest of this chapter is organized as follows: Section 7.2 describes the system model. Section 7.3 reviews some preliminaries concerning the diversity analysis of linear block codes and convolutional codes. The proposed scheme is introduced in Section 7.4 and the corresponding algorithms for relay assignment are described in Section 7.5. Section 7.6 presents the performance analysis of the proposed algorithm.

7.2 System Model

Consider the network shown in Fig. 7.1 consisting of k source nodes, n_t relay nodes, and a single destination. The set of the source nodes, the set of the relay nodes, and the destination are respectively denoted by $S = \{S_1, S_2, ..., S_k\}$, $R = \{R_1, R_2, ..., R_{n_t}\}$, and D. All relays operate in the half-duplex mode. The fading in all source-relay and relay-destination channels is assumed to be independent but not identically distributed (i.n.d.) according to the Rayleigh distribution. The complex channel gain between the source i and relay j is denoted by $g_{i,j}$ and the complex channel gain between the relay j and the common destination is denoted by h_j. We assume there is no direct link between the sources and the destination. A two-phase relay mode is employed. In the first time slot, the source nodes broadcast their messages using k orthogonal channels and the relays receive. The symbol transmitted by the i-th source node is denoted by a_i, $1 \leq i \leq k$ and the result of decoding a_i at j-th relay is denoted by $b_{i,j}$. Some relay nodes decode the message from one or more source nodes and perform an exclusive-OR operation on the decoded bits before retransmitting them to the destination. The result of exclusive-OR operation at the j-th relay node is denoted by c_j, $1 \leq j \leq n$. In the second time slot, the relay terminals transmit using orthogonal channels and the destination receives. We assume that a centralized resource allocation is employed and the signal-to-noise ratio (SNR) information of the channels in the first hop is known to the resource allocator. As a practical example, currently, the 802.16j Mobile Multihop Relay (MMR) working group is focused on integrating relay schemes into 802.16-based networks with centralized or semi-distributed resource allocation [64].

Throughout this section, the vectors and matrices are respectively denoted by lowercase bold and uppercase bold letters. $[\cdot]^T$ denotes transposition of a matrix. The PDF and the CDF of a random variable Γ are denoted by $f_\Gamma(\gamma)$ and $F_\Gamma(\gamma)$, respectively. We recall that if the random variables $\Gamma_1, \Gamma_2, ..., \Gamma_N$ are arranged in increasing order and written as $\Gamma_{1:N} \leq \Gamma_{2:N} \leq ... \leq \Gamma_{N:N}$, then $\Gamma_{r:N}$ is called the r-th order statistic. Although random variables Γ_i are assumed to be independent, the $\Gamma_{r:N}$ are dependent because of the ordering.

7.3 Preliminary: Diversity Analysis of Channel Codes

In this part, first we review the diversity analysis of a linear block code, employed in a fully interleaved Rayleigh fading channel. We assume that soft decision decoding is employed at the receiver. The proof can be found in [65].

7.3.1 Diversity Analysis of Linear Block Codes

Since the code is linear, without any loss of generality, we can assume that the all-zero codeword ($\mathbf{0}$) is transmitted and then we have $P_E = \Pr(\text{Error}|\mathbf{0} \text{ is transmitted})$, where P_E is the probability of decoding a wrong codeword at the destination. Using the union bound, we can upper bound this expression by a simpler expression $P_E \leq$

$\sum_{\mathbf{c}\neq\mathbf{0}} \Pr(\mathbf{0} \to \mathbf{c})$ [65, 66], while $\Pr(\mathbf{0} \to \mathbf{c})$ is the pairwise error probability (PEP) of receiving a codeword "closer" to the incorrect codeword $\mathbf{c}$ when the all-zero codeword is transmitted. Let us denote the received signal at the destination by $\mathbf{y} = [y_1, y_2, \ldots, y_n]$. We have $y_i = h_i u_i + n_i$, where h_i, u_i, and n_i denote the channel fading coefficient, the transmitted symbol, and additive white Gaussian noise, respectively. Assuming that the channel coefficients are perfectly known at the receiver, the optimal maximum likelihood (ML) decoder decodes the codeword which is closest to the received signal in the Euclidean distance sense, i.e., the ML decoder minimizes the Euclidean distance between $\mathbf{y}$ and the vector $[h_1u_1, h_2u_2, \ldots, h_nu_n]$ over all the codewords $\mathbf{u}$. Hence, the PEP is a function of the squared Euclidean distance between the all-zero codeword and the codeword $\mathbf{c}$ which is given by $4\left(|h_{j_1}|^2 + |h_{j_2}|^2 + \ldots + |h_{j_d}|^2\right)$ where d is the weight of $\mathbf{c}$ and j_is are the positions with the component of $\mathbf{c}$ being 1. Then

$$\Pr(\mathbf{0} \to \mathbf{c}) = E\left[Q\left(\sqrt{2\rho_c\left(|h_{j_1}|^2 + |h_{j_2}|^2 + \ldots + |h_{j_d}|^2\right)}\right)\right]$$

where ρ_c is the average SNR per coded bit and the expectation is over the fading coefficients. Let us denote by ρ_c the average SNR per coded bit. By plugging the PDF of the instantaneous SNR ($\gamma_i = \rho_c\,|h_i|^2$), this can be written explicitly as:

$$\begin{aligned}\Pr(\mathbf{0} \to \mathbf{c}) = \iiint Q\left(\sqrt{2\left(\gamma_{j_1}^2 + \gamma_{j_2}^2 + \ldots + \gamma_{j_d}^2\right)}\right) \\ \times \prod_{i=1}^{d} \frac{1}{\rho_c} \exp(-\gamma_i) d\gamma_1 d\gamma_2 \cdots d\gamma_d\end{aligned}$$

By upper bounding the Q-function using $Q(x) \leq \frac{1}{2}\exp(-\frac{x^2}{2})$, we obtain

$$\Pr(\mathbf{0} \to \mathbf{c}) = \prod_{i=1}^{d}\left(\frac{1}{2\rho_c}\int \exp(-\gamma_i)\exp(-\frac{\gamma_i}{\rho_c})d\gamma_i\right) = \frac{1}{(1+\rho_c)^d}.$$

Substituting this upper bound on the PEP expression in $P_E \leq \sum_{\mathbf{c}\neq\mathbf{0}} \Pr(\mathbf{0} \to \mathbf{c})$, an upper bound on the codeword error rate is achieved:

$$P_E \leq \sum_{\mathbf{c}\neq\mathbf{0}} \frac{1}{(1+\rho_c)^d} \leq \left(2^k - 1\right)\frac{1}{(1+\rho_c)^{d_{min}}}.$$

where k is the number of the message bits. This result clearly shows that a diversity order of d_{min} is achieved.

7.3.2 Diversity Analysis of Convolutional Codes

Next, we briefly review the diversity analysis of the convolutional codes over fully interleaved Rayleigh flat fading channels. As a simple and intuitive approach, if the convolutional code is terminated by adding some bits at the end to make sure that the final state and the initial state of the code are the same, we obtain a linear block code. For this linear block code, the minimum distance is the free distance of the convolutional code d_{free}. Thus, for large block lengths, we can conclude that the diversity order with soft decision decoding is d_{free}. We neglect the detailed proof which is normally based on the transfer function of the code [65].

7.4 Proposed Scheme

In this section, we introduce the proposed relaying scheme which is based on a distributed implementation of the various linear codes. We start with the binary systematic linear block code. Then the proposed scheme is extended to the distributed Reed-Solomon code relaying (non-binary codes). Finally, we introduce the distributed convolutional code relaying where the codes are generally non-systematic.

7.4.1 Distributed Linear Block Code Relaying

Consider a network which consists of k source nodes, n_t relay nodes and a single destination. In this network, we want to implement a (n, k) systematic linear block code [67] in a distributed manner. We assume $n \leq n_t$ and also we assume that the mentioned code is a systematic binary code. Let us denote the generator matrix of the mentioned code by $\mathbf{G} = [\mathbf{P}, \mathbf{I}_k]_{k\times n}$ where $\mathbf{P}$ is a k-by-$(n-k)$ matrix and $\mathbf{I}_k$ is the identity matrix. Fig. 7.2 shows how the relays form a linear block code in a distributed manner. In this figure, there are seven relays selected for the cooperation $(n = 7)$.

In general, since we have k source nodes and n_t relay nodes, there are totally $k \times n_t$ different source-relay channels. We can collect the SNR values of these channels in a k-by-n_t matrix $\mathbf{\Gamma}$ where $\Gamma_{i,j}$ represents the equivalent SNR of the link $S_i \to R_j$. As an

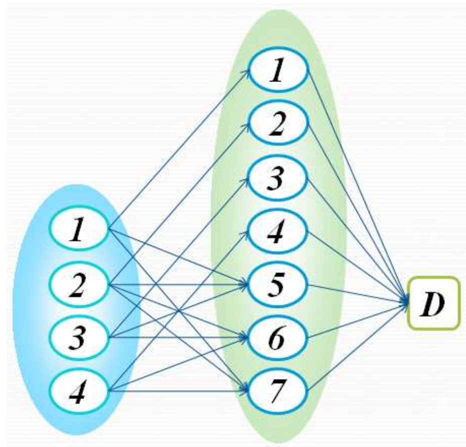

Figure 7.2: An example of the proposed distributed linear block code relaying. Four relays only decode and forward the received signal while the other three relays transmit the XOR of the received signal from the three sources

example consider the Hamming code $(7,4)$ with generator matrix

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7.1}$$

Assignment of relay j to the source i corresponds to the selection of $\Gamma_{i,j}$ from $\mathbf{\Gamma}$. In other words, each relay corresponds to one column of the generator matrix $\mathbf{G_{k \times n}}$. When $G_{i,j}$ is 1, relay j has to decode the message of source i. Since there are multiple 1s in $n-k$ columns of $\mathbf{G}$, the corresponding relays are responsible for multiple sources. These relays should perform an exclusive-OR operation on the decoded bits from the corresponding sources and then retransmit the result to the destination. Each relay corresponding to one of the k remaining columns of $\mathbf{G}$, only decodes and forwards the information of one source. The proper selection of the relays for each set is the subject of Section 7.5.

7.4.2 Distributed Reed-Solomon Code Relaying

Reed-Solomon codes are non-binary cyclic codes where the input symbols and the generator matrix elements are selected from $GF(q)$ [67]. The Reed-Solomon code is optimal in the sense that its minimum distance has the maximum possible value for a linear code of the same size; this is known as the Singleton bound. On the other hand, Koetter et al. presented a polynomial-time soft decision algebraic list-decoding algorithm for RS codes [68]. These two properties makes the RS code a promising candidate for the distributed code relaying. As an example, let us consider the RS code with the following generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 3 & 6 & 3 & 1 & 0 & 0 \\ 4 & 6 & 6 & 4 & 0 & 1 & 0 \\ 3 & 6 & 3 & 1 & 0 & 0 & 1 \end{bmatrix}. \tag{7.2}$$

The structure of the scheme will be similar to Fig. 7.2 where the number of the source nodes is 3. The only difference is that instead of exclusive-OR operation in the relays, we need to perform summation in the Galois field $GF(q)$. We will show that using these codes, we can achieve a more bandwidth efficient coded scheme compared to the distributed Hamming code relaying.

7.4.3 Distributed Convolutional Code Relaying

In this section, we introduce distributed convolutional code relaying scheme. Convolutional codes are one subset of linear codes where their encoding operation can be viewed as convolution operation. Fig. 7.3 shows how the relays form a convolutional code in a distributed manner. The structure of this scheme is similar to that of the distributed linear block coding relaying, except the fact that each element of the generator matrix is a polynomial and hence, its implementation requires the delayed (buffered) copies of the input signal. Again, each column of the generator matrix is assigned a relay. The corresponding relay receives the signal from some or all the source nodes and buffers them in order to calculate their convolution. Fig. 7.3 shows a nonsystematic convolutional code with the Generator matrix

$$\mathbf{G} = \begin{bmatrix} 1+x^2 & x & x+x^2 \\ 1 & x^2 & 1+x+x^2 \end{bmatrix}. \tag{7.3}$$

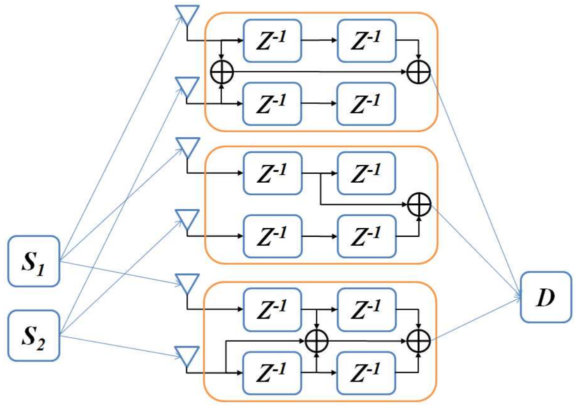

Figure 7.3: System model for distributed convolutional code relaying

7.5 Proposed Relay-Assignment Algorithm

The proposed scheme (Fig. 7.2 and Fig. 7.3) requires a proper assignment of the relays to the source nodes in order to achieve the highest performance at the relays. In this section we propose a sub-optimal algorithm for this relay assignment problem based on the max-min criterion. The algorithm is described in Section 7.5.1. Section 7.5.2 involves the performance analysis of the received signal at the relays. Although the proposed algorithm does not necessarily finds the optimal permutation (based on the max-min criterion), it guarantees a minimum diversity of $n_t - n + k$ for all of the selected channels, which equals the diversity achieved by the optimal max-min permutation. In the analysis of Section 7.6, we will show that the E2E diversity achieved by this scheme is strictly dependent on the diversity achieved at the relay nodes. Selection of the max-min criterion for this application is based on the fact that it achieves the maximum possible diversity at the relays.

7.5.1 Proposed Algorithm

First, let us consider the proposed scheme for the systematic codes. By accepting the matrix representation of the SNR values (introduced in Section 7.4.1), the problem of relay assignment turns into the selection of some elements from $\mathbf{\Gamma}$.

As it was mentioned before, each relay implements one column of the generator matrix. On the other hand, each relay corresponds to one column of $\mathbf{\Gamma}$. Hence, the relay assignment problem translates to finding the correspondence between the columns of $\mathbf{G}$ and the columns of $\mathbf{\Gamma}$. The proposed relay assignment algorithm consists of two stages. In the first stage, we select some columns of $\mathbf{\Gamma}$ corresponding to the nonsystematic part of $\mathbf{G}$. In the second stage, we have the remaining part of $\mathbf{\Gamma}$ which is a $(n_t - n + k)$-by-k matrix and we need to select k elements from this matrix based on the max-min criterion, in a way that:

- No couple of elements are selected from the same row
- No couple of elements are selected from the same column
- The selected elements are the optimal selection based on the max-min criterion

Chapter 5 describes an algorithm to select this optimal permutation based on the max-min criterion.

First stage: A rough description of the proposed algorithm to select the elements corresponding to the nonsystematic part of $\mathbf{G}$ is given below:

1. In the source-relay SNR matrix $\mathbf{\Gamma}$, label the smallest element
2. Label the next smallest element of $\mathbf{\Gamma}$
3. If {there are only $n - k$ columns without any labeled elements}
   (a) Denote the union of the mentioned $n - k$ columns by $\mathbf{\Delta}$
   (b) $\mathbf{\Delta}$ has the same dimensions as $\mathbf{P_k}$. Select all elements of $\mathbf{\Delta}$ corresponding to 1s in $\mathbf{P_k}$ for the suboptimal permutation

   Else: Go to Step 2
4. Denote the other $n_t - n + k$ columns of $\mathbf{\Gamma}$ by $\mathbf{\Phi}$. Use the next algorithm in order to select k elements from $\mathbf{\Phi}$ based on the max-min criterion

Second stage: $\mathbf{\Phi}$ is a k-by-$(n_t - n + k)$ matrix. Let us denote $n_t - n + k$ by N. A rough description of the proposed algorithm to select k elements from $\mathbf{\Phi}$ based on the max-min criterion is as follows:

1. By starting from the smallest element $\Phi_{1:kN}$, each element is labeled in the matrix.

2. At any moment, if there is only one remaining element in any row or column of the matrix, this element is selected for the optimal permutation. In this case we delete the corresponding row and column of the matrix
3. In each step, we denote the number of the elements selected for the optimal permutation by n_M. At any moment, if the number of the remaining rows or columns with at least one empty space is less than n_M, it means that we cannot select the required elements from the remaining rows or columns. In this case, we should go back to the state when the last element was labeled in the matrix. We should select this last element for the optimal permutation and continue from that point.
4. When the number of relays is more than the number of source nodes, some of the relays do not contribute to the optimal permutation, which means that no elements will be selected from the first $n_t - n$ completely labeled columns of the matrix.

In this algorithm, whenever there is one remaining element in any row of the matrix, it is selected for the optimal permutation, but we do not select any element from the first $n_t - n$ completely labeled columns of the matrix. Because the last labeled element from any other column of the matrix is bigger than all of the elements of the first $n_t - n$ labeled columns. The detailed algorithm is as follows:

---

1. $M = \{\}$ *% The set of the elements of the optimal permutation*
2. Set $n_M = 0$ *%Number of the elements in M*
3. Set $m_{row} = k$ *% Number of rows that have unlabeled elements in* $\mathbf{\Phi}$
4. Set $m_{col} = N$ *% Number of columns that have unlabeled elements in* $\mathbf{\Phi}$
5. Find the smallest element ($\Phi_{i,j} = \Phi_{1:kN}$) and label it
6. Find the next smallest element and label it
7. Save the present state of the variables ($\mathbf{\Phi}$, M, and r) as state r
8. Check if
   - A: There is any row with only one remaining unlabeled element in $\mathbf{\Phi}$
   - B: There is any column with only one remaining unlabeled element in $\mathbf{\Phi}$

9. While A or B
   (a) If A
      i. Select this remaining element for the optimal permutation, i.e. $M = M \bigcup \{\text{the mentioned remaining element}\}$
      ii. Delete the row and column of $\mathbf{\Phi}$ corresponding to the selected element
      iii. Recalculate n_M, m_{row}, and m_{col}
      iv. If $m_{row} < n_t - n - n_M$ or $m_{col} < n_t - n - n_M$
         – Restore the last saved state ($\mathbf{\Phi}$, M, and r from Step 7)
         – Delete the last saved state (the total number of the saved states is decreased by one)
         – $M = M \bigcup \{\text{the last labeled element}\}$
         – Recalculate n_M
      v. Recalculate A and B
   (b) If B
      – If $\{$There are already $n_t - n$ completely labeled columns$\}$
         i. $M = M \bigcup \{\text{the mentioned remaining element}\}$
         ii. Delete the row and column of $\mathbf{\Phi}$ corresponding to the selected element
         iii. Delete the mentioned $n_t - n$ columns of $\mathbf{\Phi}$, this deletion happens only once.
         iv. Recalculate n_M, m_{row}, and m_{col}
         v. If $m_{row} < n_t - n - n_M$ or $m_{col} < n_t - n - n_M$
            – Restore the last saved state similar to the steps in 9a-iv
         vi. Recalculate A and B
10. $r = r + 1$
11. If $r < kN$ go to step 6

Example 1: Consider $\mathbf{\Gamma}$ in Fig. 7.4-a. The labeled elements are highlighted and the selected elements for the optimal permutation are specified by solid circles. Different steps of applying the above algorithm are shown in this figure. By starting from the smallest element, we label the elements one-by-one in the matrix until there are only three (which is $n - k$) columns without any labeled element (the first stage). In this

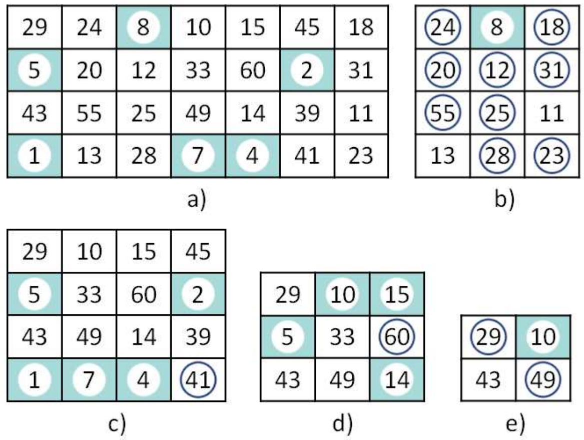

Figure 7.4: Example of relay assignment algorithm for $n = 7$ and $k = 4$

case, we have $\mathbf{\Delta} = \Gamma(:, [2, 3, 7])$ which is illustrated in Fig. 7.4-b. We select all of the elements of $\mathbf{\Delta}$ corresponding to 1s in $\mathbf{P_k}$ for the optimal permutation. The selected elements are shown by solid circles in Fig. 7.4-b. Matrix $\mathbf{\Phi}$ is illustrated in Fig. 7.4-c and the remaining steps are obvious from Fig. 7.4-d,e.

For the relaying based on the distributed non-systematic codes, the relay assignment algorithm is exactly the first algorithm in this section.

7.5.2 Performance Analysis of the Received Signal at the Relays

We analyze the performance of the proposed algorithm in two steps. To avoid the difficulty of dealing with i.n.d. channels, we assume that the channels in each hop are independent and identically distributed (i.i.d.) fading channels. Obviously, the results of diversity order analysis holds true for i.n.d. channels (assuming that they are still Rayleigh fading channels with different average SNR values). This analysis is mainly based on the fact that when the distribution of SNR for a Rayleigh fading channel follows the r^{th} order-statistics, it achieves diversity r [16].

First, let us consider the elements selected from $\mathbf{\Delta}$. Obviously, these elements are not among the $n_t - n + k$ smallest elements of the matrix. This means that the diversity order corresponding to the contribution of these elements to the selected permutation is at least $n_t - n + k + 1$. This is because when the distribution of the SNR for a Rayleigh fading channel follows the r^{th} order-statistics, it achieves diversity r [16].

Now, let us consider the elements selected from $\mathbf{\Phi}$. These elements constitute the optimal permutation from $\mathbf{\Phi}$ [63]. Without loss of generality, let us sort the elements of $\mathbf{\Phi}$ in increasing order and denote the result by $\Phi_{1:kN}, \Phi_{2:kN}, ..., \Phi_{kN:kN}$ where $\Phi_{r:kN}$ is the r-th order statistic. In [63], the average error probability of relay assignment based on max-min criterion is calculated and it is shown that the PDF of the SNR of each selected source-relay pair $S_i \to R_j$ is a "weighted" sum of the PDFs of $\Phi_{r:kN}$, $r = k, \cdots, kN$, i.e.

$$f_{optimal}(\gamma) = \sum_{r=k}^{kN} w_{k,N}(r) f_{r:kN}(\gamma) \tag{7.4}$$

where $w_{k,N}(r)$ are the mentioned weighting coefficients. Each weighting is the probability that the corresponding entry belongs to the selected optimal permutation. The same conditions hold true for the average error probability.

$$P_{E(optimal)} = \sum_{r=k}^{kN} w_{k,N}(r) P_{E(r:kN)} \tag{7.5}$$

We know that $P_{E(r:kN)}$ achieves diversity r [16]. Since in the right-hand side of (7.5), the minimum diversity belongs to $r = k$ and this minimum diversity is dominant, we can conclude that $P_{E(optimal)}$ achieves diversity k,

7.6 Diversity Order Analysis

In this section, we analyze the E2E diversity of the proposed scheme. For this purpose, let us define the following events.

A_0 :No detection error in the 1^{st} hop

A_1 :One detection error in the 1^{st} hop

A_2 :Two or more detection errors in the 1^{st} hop

B :Decoding error at the destination

The E2E average error probability can be expressed in terms of the conditional probabilities $\Pr\{B|A_0\}$, $\Pr\{B|A_1\}$, and $\Pr\{B|A_2\}$, that is

$$P_E = \Pr\{A_0\}\Pr\{B|A_0\} + \Pr\{A_1\}\Pr\{B|A_1\} + \Pr\{A_2\}\Pr\{B|A_2\}. \tag{7.6}$$

where $\Pr\{B|A_i\}$ is the conditional probability of B given A_i. In (7.6), we have distinguished between two different error scenarios at the destination. In the first scenario, all of the information symbols are correctly decoded at the relays, but a wrong codeword is decoded at the destination ($\Pr\{B|A_0\}$). The analysis of this scenario is a part of the present literature about channel coding schemes which is shortly described in the preliminaries in Sections 7.3. In the second scenario, detection error happens at one or more relays. This scenario is analyzed in the sequel (Section 7.6.1). The conclusion about the overall E2E diversity is expressed in Section 7.6.2.

7.6.1 Effect of Detection Errors at the Relays

Theorem 1. *Detection error at one of the relays, creates a fixed irreducible error term at the destination for both distributed linear block code relaying and distributed convolutional code relaying, i.e. the diversity order achieved by the second hop in this scenario is zero.*

Proof. First let us consider the distributed linear block code relaying. We assume that there is one bit error among n bits at the relays. This error may be the result of exclusive-OR operation at relays. However, we know that two errors cancel each other in the exclusive-OR operation. We assume BPSK modulation where coded bits are transmitted equivalently as -1 (for 0) and as 1 (for 1). This assumption normalizes the energy per coded bit to unity. Without loss of generality, we assume that the codeword without error was

$$\mathbf{c}_0 = [-1, -1, \cdots, -1, c_{d_{min}+1}, \cdots, c_n],$$

and the decoded signal at the relays is

$$\hat{\mathbf{c}}_0 = [+1, -1, -1, \cdots, -1, c_{d_{min}+1}, \cdots, c_n]$$

where the first bit is in error. We want to show that even in a noiseless system, detection errors at the relays results in a fixed irreducible decoding error probability at the destination. The received codeword at the destination in the absence of noise is

$$\mathbf{r} = [+h_1, -h_2, -h_3, \cdots, -h_{d_{min}}, c_{d_{min}+1}h_{d_{min}+1}, \cdots, c_n h_n] .$$

Here, without loss of generality, we have denoted the index of the selected relays by j where $1 \leq j \leq n$ and hence, $\mathbf{h} = [h_1, h_2, \cdots, h_n]$ denotes the complex gains of the selected relay-destination channels. The ML-decoder at the destination minimizes the Euclidean distance between the received signal $\mathbf{r}$ and the vector $\mathbf{c}_i = [h_1 u_1, h_2 u_2, \cdots, h_n u_n]$ over all the codewords $\mathbf{u}$. Let us assume that the codeword $\mathbf{c}_1 = [+1, +1, \cdots, +1, c_{d_{min}+1}, \cdots, c_n]$ is a valid codeword. The following condition is a case for error

$$Distance\left(\mathbf{r}, \mathbf{c}_0 \circ \mathbf{h}\right) \geq Distance\left(\mathbf{r}, \mathbf{c}_1 \circ \mathbf{h}\right) \tag{7.7}$$

where $\circ$ denotes the elementwise multiplication. This means that the Euclidean distance between the received signal $\mathbf{r}$ and $\mathbf{c}_0$ is larger than the distance between $\mathbf{r}$ and $\mathbf{c}_1$. (7.7) implies that

$$2\left|h_1\right|^2 \geq 2\left|h_2\right|^2 + \cdots + 2\left|h_{d_{min}}\right|^2 \tag{7.8}$$

This probability of decoding error is not reduced by increasing the average SNR value. This result shows that one bit error at the relays produces a fixed error probability at the destination which makes an error surface for soft decision decoding in the mentioned scenario and the diversity equals zero.

A similar analysis holds true for the distributed convolutional code relaying. For a rate $R = k/n$ code with $k > 1$, there are k transmitted bit sequences (coming from k

source nodes). Let us denote the set of the transmitted bits from k source nodes at time t by $\mathbf{m}_t = [m_t(1), m_t(2), \cdots, m_t(k)]$ where $m_t(i)$ denotes the transmitted bit by source i. The block of L sets of input bits is denoted by $\mathbf{m} = [\mathbf{m}_0, \mathbf{m}_1, \cdots, \mathbf{m}_{L-1}]$. At time t, the corresponding decoded bits at the relays are denoted by $\mathbf{c}_t = [c_t(1), c_t(2), \cdots, c_t(n)]$. The entire decoded output block is $\mathbf{c} = [\mathbf{c}_0, \mathbf{c}_1, \cdots, \mathbf{c}_{L-1}]$. The set of bits in $\mathbf{c}_t$ pass through the relay-destination channels, resulting in the received set $\mathbf{r}_t$. In the decoding process, we deal with negative log likelihood functions $\|\mathbf{r}_t - \mathbf{c}_t\|^2$.

In the trellis of the encoder, the state at time t is denoted as Ψ_t. States are represented with integer values in the range $0 \leq \Psi_t < 2^\nu$, where ν is the constraint length for the encoder. (We use 2^ν since we are assuming binary encoders for convenience. For a q-ary code, the number of states is q^ν.) It is always assumed that the initial state is $\Psi_t = 0$. Quantities associated with the transition from state p to state q are denoted with (p, q) . The dependency among inputs means that optimal decisions are based upon an entire received block of symbols. The likelihood function to be maximized is thus $f(\mathbf{r}|\mathbf{c})$, where by assuming memoryless channels we have

$$f(\mathbf{r}|\mathbf{c}) = \prod_{t=0}^{L-1} f(\mathbf{r}_t|\mathbf{c}_t) \tag{7.9}$$

It is convenient to deal with the log likelihood function,

$$\log f(\mathbf{r}|\mathbf{c}) = \sum_{t=0}^{L-1} \log f(\mathbf{r}_t|\mathbf{c}_t) \tag{7.10}$$

Let us assume that at time t there is one bit error among n decoded bits at the relays. We know that two errors cancel each other in the exclusive-OR operation in each relay. Again, we assume BPSK modulation where coded bits are transmitted (equivalently) as -1 (for 0) and as 1 (for 1).

Similar to distributed linear block codes, we can assume that the codeword without error, the decoded signal at the relays, and the received codeword at the destination are

$$\mathbf{c}_0 = [-1, -1, \cdots, -1, c_0(d_{free}+1), \cdots, c_0(n)]$$
$$\hat{\mathbf{c}}_0 = [+1, -1, -1, \cdots, -1, c_0(d_{free}+1), \cdots, c_0(n)]$$
$$\mathbf{r} = [+h(1), -h(2), -h(3), \cdots, -h(d_{free}), c_0(d_{free}+1)h(d_{free}+1), \cdots, c_0(n)h(n)]$$

, respectively. Here, the system is assumed to be noise free and the first bit in $\hat{\mathbf{c}}_0$ is in error. From this point forward, the proof is similar to that of linear block codes, i.e. the following condition is a case for error in the branch metric

$$Distance\,(\mathbf{r}, \mathbf{c}_0 \circ \mathbf{h}) \geq Distance\,(\mathbf{r}, \mathbf{c}_1 \circ \mathbf{h})$$

where $\mathbf{c}_1 = [+1, +1, \cdots, +1, c_0(d_{free}+1), \cdots, c_0(n)]$ is a valid codeword. Even for a noiseless system, there is a fixed possibility for this event, which means a deviation in the survival path in the Viterbi algorithm which does not reduce by increasing SNR value. □

7.6.2 End-to-end Diversity Analysis

Using Theorem 1, we are ready to analyze the diversity order achieved by the proposed scheme at the destination. We perform this analysis by assessing the diversity order offered by each term in (7.6). Since in the analysis of the diversity order only the powers of the average SNR are important, we can neglect the fixed coefficients in this analysis. Here, we assume binary codes are used. The analysis for RS codes is straightforwardly similar. In the sequel, the probability of one error in one of the selected source-relay channels is denoted by p_1. We have

$$\Pr\{A_0\} = (1-p_1)^m + \sum_i \alpha_i {p_1}^{2i} (1-p_1)^{m-2i} \tag{7.11}$$

where m denotes the total number of 1s in $\mathbf{G}$ and α_is are fixed values to be determined. In here, $(1-p_1)^m$ specifies the probability of only one error in the selected source-relay channels and the second term specifies the probability of $2i$ errors where those errors cancel each other. Similarly, we have:

$$\Pr\{A_1\} = mp_1(1-p_1)^{m-1} + \sum_i \beta_i {p_1}^{2i+1}(1-p_1)^{m-2i} \tag{7.12}$$

$$\Pr\{A_2\} = \sum_{i=2}^{m} \gamma_i {p_1}^{i}(1-p_1)^{m-i}, \tag{7.13}$$

where β_is and γ_is are fixed values to be determined. According to (7.5), the diversity order corresponding to p_1 is k, equivalently, from (7.11), (7.12) and (7.13) we can conclude that

$$\Pr\{A_0\} \propto \frac{1}{\rho_c^0}, \quad \Pr\{A_1\} \propto \frac{1}{\rho_c^k}, \quad \Pr\{A_2\} \propto \frac{1}{\rho_c^{2k}}$$

where ρ_c is the energy per coded bit. In other words, the diversity corresponding to $\Pr\{A_i\}$ is i. Now we can rewrite the expression for P_E in the following form. The contribution of each term to the diversity order is already calculated in this section. We write the corresponding results below each term.

$$P_E \approx \underbrace{\Pr\{A_0\}}_{0} \underbrace{\Pr\{B|A_0\}}_{d_{free} \text{ or } d_{free}} + \underbrace{\Pr\{A_1\}}_{k}\underbrace{\Pr\{B|A_1\}}_{0} + \underbrace{\Pr\{A_2\}}_{2k}\underbrace{\Pr\{B|A_2\}}_{0} \tag{7.14}$$

The minimum diversity in the above terms is dominant. This minimum diversity is d_{min} and belongs to the first term, because $d_{min} < k$. On the other hand, we can see that if the first hop does not achieve diversity ($k = 1$), the diversity order corresponding to the second term will be one and the overall achieved diversity becomes one.

In order to efficiently utilize the resources in the proposed scheme, we need to achieve the same diversity at the relay nodes and at the destination. Otherwise, according to (7.14), the extra diversity is dumped. It means that the total number of relays should be selected according to

$$n_t = \begin{cases} d_{min} + n - k & \text{for systematic codes} \\ d_{min} + n - k - 1 & \text{for non-systematic codes} \end{cases}$$

where, for convolutional codes we should replace d_{min} with d_{free}.

7.7 Simulations and Discussion

In this section, we illustrate the Monte-Carlo simulation results for the proposed scheme and compare them with the results of some other schemes. First, consider the network configuration of Fig. 7.2 and the $(7, 4)$ Hamming code corresponding to G in (7.1). We assume that the channels in each hop (source-relay and the relay-destination) are i.i.d. Rayleigh fading channels. We compare the performance of the proposed scheme with the sequential relaying when the relay assignment is based on the "outdated" CSI for the second hop [62]. The so called "outdated" CSI means that the decision regarding the best relay does not correspond to the current time instance because of, e.g., feedback delay. This is motivated by the fact that our proposed scheme does not need the CSI of the relay-destination channels. Therefore, in order to have a relatively fair comparison, we assume that in the rival scenario (sequential relaying) relay assignment is based on the outdated CSI in the second hop. We denote by $h_{R_i,D}$ the circularly symmetric complex Gaussian channel gain between the relay R_i and D and we denote by $\hat{h}_{R_i,D}$ the partially known channel corresponding to $h_{R_i,D}$ at the time of relay assignment. The outdated CSI for relay assignment is modeled as [69, 70]

$$\hat{h}_{R_i,D} = \rho h_{R_i,D} + \sqrt{1-\rho^2} w_{R_i,D}$$

where $w_{R_i,D}$ is a circularly symmetric complex Gaussian RV having the same variance as $h_{R_i,D}$ and ρ is a fixed parameter specifying the correlation coefficient between $h_{R_i,D}$ and $w_{R_i,D}$.

Fig. 7.5 shows some simulated BER results for the proposed scheme versus E_b/N_0 (the energy per bit to noise power spectral density ratio) compared to that of sequential relaying for different correlation coefficients ρ. As evidenced by this figure, our proposed scheme achieves diversity $d_{min} = 3$ for the Hamming codes. This figure obviously shows that the outdated CSI significantly affects the performance of sequential relaying. This degradation in the performance is both in the diversity order and the coding gain, i.e., by selecting any value for ρ other than one, the achieved diversity order drops to one. For low average SNR values, our proposed scheme has a lower performance compared to sequential relaying, which is a result of employing the channel code in our scheme.

In the next simulation, we check the sensitivity of the proposed scheme (same network as the last simulation) against different average SNR values at the source-relay

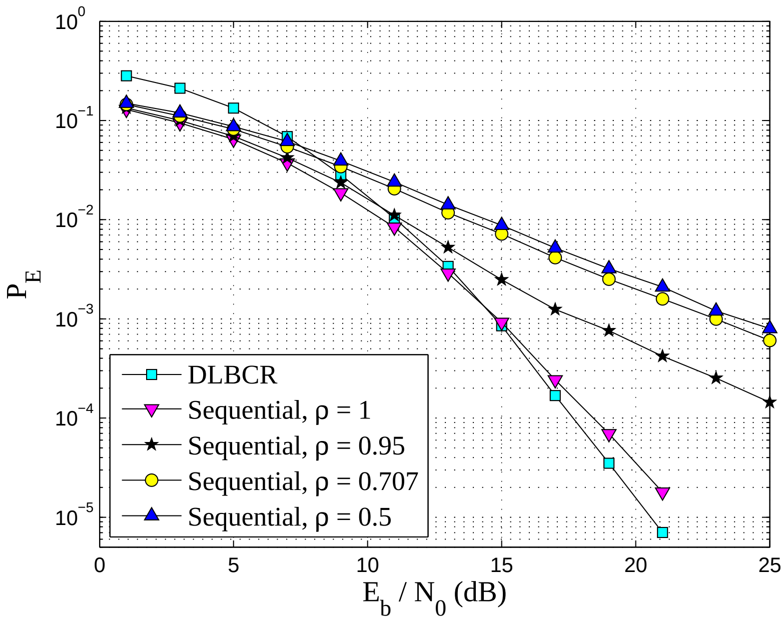

Figure 7.5: Monte-Carlo simulation of the average error probability of the proposed distributed linear block code relaying (DLBCR) versus the normalized average SNR of the $S-R$ and $R-D$ links

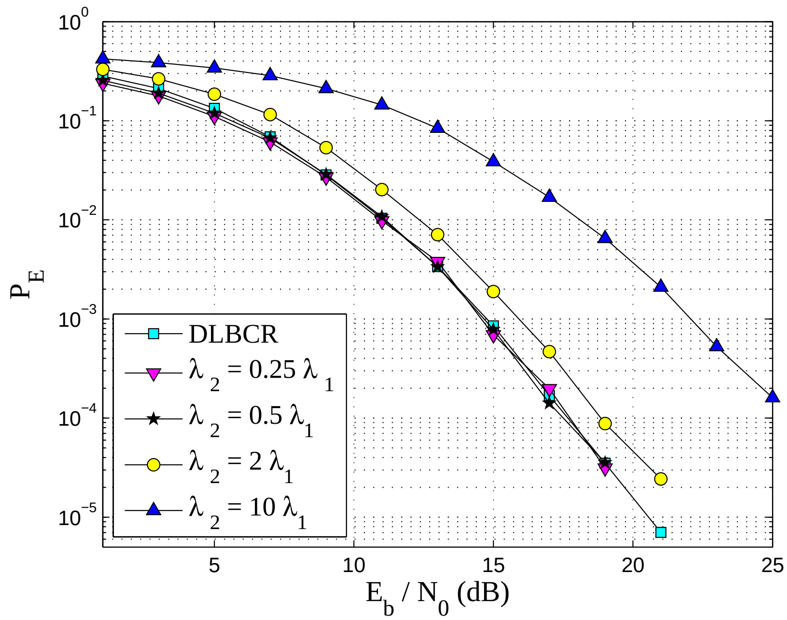

Figure 7.6: Monte-Carlo simulation of the average error probability when the source-relay and the relay-destination channels have different average SNR values

and the relay-destination links. Fig. 7.6 shows the results. Since Rayleigh flat fading is assumed for all of the underlying links, the SNR follows exponential distribution, i.e., $f_\Gamma(\gamma) = \lambda \exp(-\lambda\gamma)$ where $1/\lambda$ is the average SNR value. Let us denote the average SNR value for the first and the second hop by $1/\lambda_1$ and $1/\lambda_2$, respectively. This simulation shows that the different average SNR values in the first and the second hop results in a degradation of the performance. However, the proposed scheme retains the same diversity order.

Next, we consider the performance of $(7, 3)$ distributed Reed-Solomon code relaying corresponding to G in (7.2). We compare the performance of the proposed scheme with the rival sequential relaying when the relay assignment is based on the “outdated” CSI for the second hop. For both scenarios, the transmitted symbols are selected from $GF(8)$. Fig. 7.7 shows some simulation results for the average symbol-error-rate (SER) of the proposed scheme versus E_s/N_0 (the energy per symbol to noise power spectral density ratio) compared to that of sequential relaying for different correlation coefficients ρ. As it is evidenced by this figure, our proposed scheme nearly achieves diversity 5,

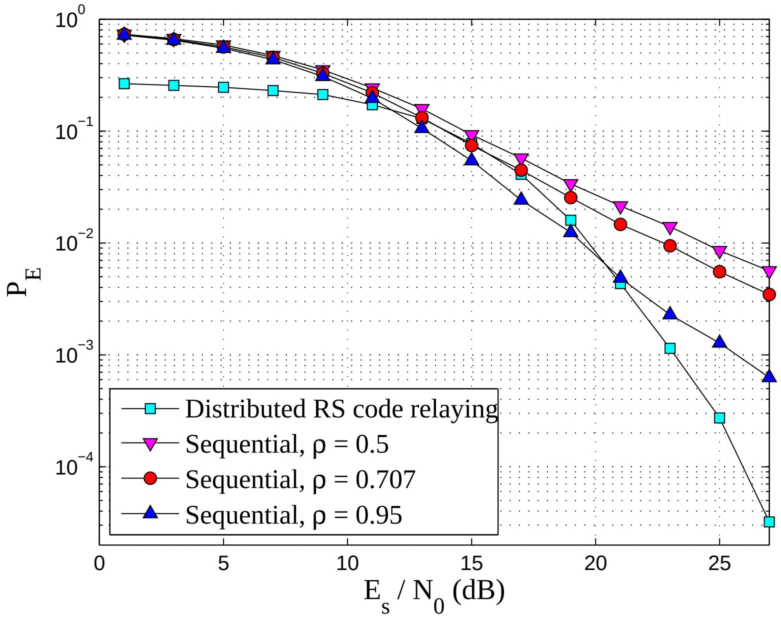

Figure 7.7: Monte-Carlo simulation of the average error probability of the distributed Reed-Solomon codes compared to sequential relaying with outdated CSI of the second hop

where all of the other curves fail to achieve diversity. Another interesting observation from this figure is the superiority of the proposed scheme, even for low average SNR values. This superiority goes back to the inherent performance of the RS codes.

In the last figure, we simulate the performance of the proposed distributed convolutional code relaying corresponding to G in (7.3) and also we check its sensitivity against different average SNR values at the source-relay and the relay-destination links. Fig. 7.8 shows the results. Again we can see that the different average SNR values in the first and the second hop results in a degradation of the performance. However, the proposed scheme retains the same diversity order. In addition, it is learnt that the coding gain for $\lambda_2 = 0.5\lambda_1$ is better than the coding gain for $\lambda_2 = 0.5\lambda_1$. This fact shows that the sensitivity of this scheme to the SNR values in the first hop is more than the that of the second hop. This result can be explained based on the discussion in Section 7.6.1.

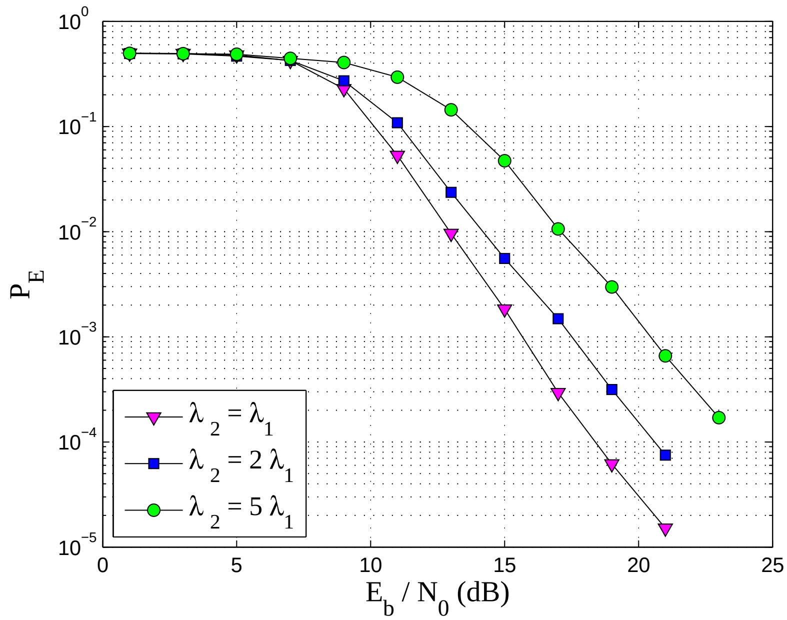

Figure 7.8: Monte-Carlo simulation of the average error probability for distributed convolutional code relaying when the source-relay and the relay-destination channels have different average SNR values

8

Conclusions and Perspectives

In this thesis, we have explored the problem of relay assignment in cooperative networks. The performance of different relay assignment schemes is statistically analyzed, new schemes are proposed to achieve diversity and some algorithms are proposed to find the optimum permutation based on some existing criteria.

As a means for our analysis, first we considered different scenarios where the PDF of the relaying channels' SNR involves order statistics. These scenarios are generalizations of the best-relay selection scheme and correspond to the case where the best relay is unavailable due to some reasons. The results have several applications in the performance analysis of various relay assignment schemes and are used in the remaining chapters of this thesis. In order to perform our analysis, we proposed a new approximation for the first order modified Bessel function of the second kind. This function widely appears in the Rayleigh fading and Nakagami-m fading AF relay channels. Our proposed approximation which is more accurate than the classical approximation, is still easy to handle and enables us to find much more accurate expressions for the performance of the r^{th} weakest E2E relay channel in a set of N two-hop channels.

In the following three sections, we have considered the problem of relay assignment in cooperative networks based on different criteria. Each criterion has its own pros and cons, and some are suitable for some network configurations but not for other configurations. We considered each criterion in a suitable network configuration and analyzed its performance. As the performance metric, we calculated the PDF of the end-to-end SNR, the diversity order and the BER. The mentioned criteria are sequential relay assignment (Chapter 4), relay assignment based on max-min criterion (Chapter 5), and relay assignment based on max-sum criterion (Chapter 6). It is assumed that only one relay node is assigned to a single transmitting node, which has been shown to have the capability to maximize the network throughput.

Chapter 4 proposes a simple sequential relay assignment scheme for the networks consisting of multiple sources, multiple relays, and a single destination. In this scheme, for each set of channel realizations, the sources sequentially choose their relays among the remaining relays. In the relay assignment process, the priority of the source nodes for relay-selection is based on the quality of the source-destination links, i.e. the source nodes that have weaker source-destination channels, have higher priority in getting assigned relays with stronger relay-destination links. As such, the number of relays could be more than N. Since each source benefits from both its direct channel to the destination and that through the assigned relay, the proposed scheme achieves balance among different sources, and therefore all sources achieve the same diversity. We calculated the PDF of the E2E SNR and BER for the proposed scheme. The proposed scheme offers the highest performance among our known relay assignment schemes in the literature.

Chapter 5 deals with relay assignment based on max-min criterion. According to this criterion, the minimum SNR of all possible permutations are compared, and the permutation whose minimum SNR is the maximum, is selected. There were two basic questions about this criterion: First, there was no algorithm to find the optimum permutation based on this criterion (the only solution was exhaustive search). Second, statistical analysis of the optimum permutation was unknown and it was assumed to be very difficult (because of the dependency among different permutations). In this chapter, we answered both questions, i.e. we offered a simple algorithm to find the optimum permutation based on this criterion and also we statistically analyzed the performance of this optimum answer. The simplicity of the proposed algorithm stems from the fact that it involves simple matrix manipulations. We proved that this scheme achieves full diversity.

In Chapter 6, we considered relay assignment based on maximizing the sum of rate or SNR values. Both schemes are statistically analyzed and their diversity order is calculated. Sum-rate is a widely accepted criterion in the literature, because it maximizes the amount of information exchanged in the network. Our analysis proved that sum rate criterion achieves full diversity; however, using this criterion, the achieved BER for each user is larger than that of using max-min criterion. We also proved that maximizing the sum of SNR values does not achieve diversity; however, it has a relatively good performance for a wide range of low average SNR values. In the remaining of this chapter, we proposed a new flexible formulation to find the optimum permutation based on the mentioned criteria. The proposed scheme is based on the vehicle routing problem.

In the last chapter of this thesis, we have moved forward to a relay assignment scheme which is less dependent on CSI to achieve diversity. This issue is motivated by the fact that we should go through different difficulties to provide CSI. These difficulties involve 1) errors in the estimation of CSI; 2) round-off error due the limited feedback; 3) errors happening in the reporting process; and 4) variations of the channel during the reporting process. To address this problem, we have proposed a new scheme to achieve diversity in relay channels where we only used the CSI of the source-relay channels. The proposed scheme is based on a distributed implementation of linear block codes or convolutional codes. In this scheme, each relay node implements one column of the generator matrix of the code, i.e. different symbols of the codeword are sent to the destination by different relays. Each relay receives the symbols from one or more source nodes and performs a modulo-q addition on the decoded symbols and retransmits the result to the destination. In order to achieve the maximum diversity order, an appropriate source-relay pairing has to be employed. For this relay assignment process, we assumed that only the CSI of the source-relay channels is available. We proposed an algorithm based on the max-min criterion for the resulting relay assignment problem. The result of this algorithm is not the optimal solution based on the max-min criterion, but still achieves full diversity in the relay nodes. Finally, soft decoding is employed at the destination to retrieve the transmitted information. We proved that the proposed scheme achieves diversity d_{min} or d_{free} for the E2E performance, where, d_{min} or d_{free} are the minimum distance and the free distance of the corresponding implemented codes, respectively.

The contents of Chapter 7 reflect a new trend in the design and analysis of cooperative networks where the effects of limited CSI are taken into account. New researches conducted by different scholars show that limited CSI can strongly degrade the performance of different relay assignment schemes. This issue can be analyzed from different points of view. On one hand, the effects of limited CSI should be carefully analyzed in the performance of different relay assignment schemes. On the other hand, new schemes should be proposed to increase the robustness against limited CSI.

One relevant interesting scheme is the distributed space-time-coded cooperative network [5]. This scheme is not given recognition it should have got, because it achieves diversity without using CSI at all, however, it needs careful synchronization among all transmitting nodes. It also needs simultaneous transmitting and receiving by all relay nodes. We think improvements can be achieved by a proper combination of this scheme and max-min or sum-rate criteria.

As the last point, we want to mention the power problem which is still a challenge for the implementation of cooperative networks. The importance of this issue stems from the fact that the relay nodes in many applications have limited energy. From this point of view, it seems that the design of relaying schemes should be much more investigated. Recently, the idea of energy harvesting nodes is proposed to tackle this problem. In fact, solving this issue can promise a big share for the cooperative networks in the future wireless technology.

9

List of Publications

Submitted:

– Amir Minayi-Jalil, Vahid Meghdadi, Ali Ghrayeb, Jean-Pierre Cances "*Relay Assignment in Cooperative Networks Based on Order Statistics*", submitted to **IEEE Trans. on Wireless Communications**.

– Amir Minayi-Jalil, Vahid Meghdadi, Ali Ghrayeb, Jean-Pierre Cances "*A Simple Max-min Based Relay Assignment Approach for Multi-hop Cooperative Networks*", submitted to **IEEE Trans. on Wireless Communications**.

– Amir Minayi-Jalil, Robert Schober, Vahid Meghdadi, Jean-Pierre Cances "*Cooperative Relaying Based on Distributed Implementation of Linear Channel Codes*", To be submitted to **IEEE Trans. on Communications**.

– Amir Minayi-Jalil, Vahid Meghdadi, Jean-Pierre Cances "*Diversity Analysis of Relay Assignment Based on Sum-rate Criterion*", To be submitted as a Correspondence to **IEEE Trans. on Vehicular Technology**.

Accepted:

– Amir Minayi-Jalil, Vahid Meghdadi, Jean-Pierre Cances, "*Order-Statistics-Based Relay Selection for Uplink Cellular Networks*", (**WCNC 2012**) Paris, France.

– Amir Minayi-Jalil, Vahid Meghdadi, Jean-Pierre Cances, "*A Cross-Layer Approach to Clustering and Relay Assignment Based on Vehicle Routing Problem*", (**IWCLD 2011**), Rennes, Brittany, France.

– Amir Minayi-Jalil, Vahid Meghdadi, Jean-Pierre Cances, "*A Simple Optimal Solution for Relay-Assignment in Cooperative Systems Based on the Max-min Cri-*

terion", IEEE Personal, Indoor and Mobile Radio Communications (**PIMRC 2011**), Toronto, Canada.

- Amir Minayi-Jalil, Vahid Meghdadi, Jean-Pierre Cances, "*Relay Assignment in Decode-and-forward Cooperative Networks Based on Order-Statistics*", IEEE Personal, Indoor and Mobile Radio Communications (**PIMRC 2010**) , Istanbul, Turkey.
- Amir Minayi-Jalil, Vahid Meghdadi, Jean-Pierre Cances "*Relay Assignment in Cooperative Networks: Diversity Order Analysis*", 10th International Conference on Information Sciences, Signal Processing and their Applications (**ISSPA 2010**) Kuala Lumpur, Malaysia.
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Affectation de relais dans les réseaux coopératifs sans fil

Résumé : Dans cette thèse, nous explorons le problème de l'affectation des relais dans les réseaux coopératifs. La performance des différents schémas d'affectation de relais est analysée statistiquement. De nouveaux schémas sont proposés pour atteindre la diversité maximale, et certains algorithmes sont proposés pour trouver la permutation optimale basée sur certains critères existants. Dans notre analyse, tout d'abord nous avons considéré différents scénarios où le SNR équivalent source-relais-destination est considéré comme une variable aléatoire. Dans les trois sections suivantes, nous avons considéré le problème de l'affectation de relais dans les réseaux coopératif basés sur différents critères. Nous avons considéré chaque critère dans une configuration de réseau appropriée et analysé sa performance. Comme indicateur de performance, nous avons calculé la PDF du SNR équivalent total, l'ordre de la diversité et le TEB (Taux d'Erreur Binaire). Les critères utilisés sont l'affectation de relais séquentiel (chapitre 4), l'affectation de relais sur la base du critère max-min (chapitre 5), et l'affectation de relais sur la base du critère max-somme (chapitre 6). Dans le dernier chapitre de cette thèse, nous avons proposé un nouveau schéma pour réaliser la diversité dans les canaux de relais où nous avons seulement utilisé la CSI des canaux source-relais. Le schéma proposé est basé sur l'implémentation distribuée de codes linéaires en bloc ou des codes convolutifs.

Mots clés : Affectation de relais, réseaux coopératifs, max-min, sum-rate.

Relay Assignment in Cooperative Networks

Abstract: In this thesis, we have explored the problem of relay assignment in cooperative networks. The performance of different relay assignment schemes is statistically analyzed, new schemes are proposed to achieve diversity and some algorithms are proposed to find the optimum permutation based on some existing criteria. As a means for our analysis, first we considered different scenarios where the PDF of the relaying channels' SNR involves order statistics. In the following three sections, we have considered the problem of relay assignment in cooperative networks based on different criteria. Each criterion has its own pros and cons, and some are suitable for some network configurations but not for other configurations. We considered each criterion in a suitable network configuration and analyzed its performance. As the performance metric, we calculated the PDF of the end-to-end SNR, the diversity order and the BER. The mentioned criteria are sequential relay assignment (Chapter 4), relay assignment based on max-min criterion (Chapter 5), and relay assignment based on max-sum criterion (Chapter 6). In the last chapter of this thesis, we have proposed a new scheme to achieve diversity in relay channels where we only used the CSI of the source-relay channels. The proposed scheme is based on a distributed implementation of linear block codes or convolutional codes.

Keywords: Relay Assignment, Cooperative Networks, Amplify-and-Forward, max-min, sum-rate.

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