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Formation de faisceaux coopératifs pour transmissions multiutilisateurs par relais

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In the loving memory of my brother

Majid Meghdadi
1969-2003

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Hamid Meghdadi

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List of Acronyms

AF amplify-and-forward

AWGN additive white Gaussian noise

BEP bit error probability

BER bit error rate

BS base station

CDMA code division multiple access

CF compress-and-forward

CRC cyclic redundancy codes

CSI channel state information

CVX Matlab Software for Disciplined Convex Programming

DF decode-and-forward

EGC equal gain combining

EM Expectation-Maximization

i.i.d independent identically distributed

LDPC low-density parity-check

MAI multiple access interference

MDF multipath decode-and-forward

MIMO multiple-input multiple-output

MPSK M-ary phase-shift keying

MRC maximum ratio combining

MS mobile station

OFDMA orthogonal frequency division multiple access

pdf probability distribution function

PSK phase-shift keying

PSO Particle Swarm Optimization

QAM Quadrature Amplitude Modulation

QCQP Quadratically Constrained Quadratic Programming

QPSK quadrature phase-shift keying

RS relay station

SC selection combining

SDP semi definite programming

SEP symbol error probability

SER symbol error probability

SINR signal to noise plus interference ratio

SNR signal to noise ratio

ZF zero forcing

Chapter 1

Introduction

During the past few decades the demand for reliable high-speed wireless communication links experienced a tremendous growth. End user clients do not stop asking for more and more bit-rates and better link qualities. In 1984 Jakob Nielsen states that a high-end user's connection speed grows by 50% per year [1]. His prediction has proved to be an accurate estimation over the past few years as shown in Figure 1.1. In practice the average speed increases more slowly due to the conservativeness of telecommunication companies and the fact that users tend to keep their old materials and subscriptions. Another example of the increasing growth of telecommunication speed and reliablity is the mobile phone usage. Studies show that the use of data on the mobile phones has been increasing exponentially over the past few years [2]. Figure 1.2 shows the average amount of cellular phone use per subscriber per month in North America since 2008. It also projects this forward going towards 2013. It can be seen from Figure 1.2 that since 2009, data consumption has surpassed voice calling on mobile phones. While voice usage per subscription remains roughly constant (around 50 MB/month in average), the data use grows exponentially from roughly 26 MB/month in 2008 to 1235 MB/month in 2013. Such trends justify that the wireless communication is by far the fastest growing segment of communications industry and reserach [3].

However, numerous technical challenges remain in designing high-speed reliable links to support emerging applications. These difficulties are mainly due to the fact that, in a wireless environment, unlike many other channels, transmitted signals are received through multiple paths which usually add destructively resulting in serious performance degradations. Furthermore, the medium is normally shared by many different users, thus there is the possibility of significant interference as well. Other challenges for high-speed wireless applications include highly constrained transmit powers, as well as hardware complexity and cost requirements [4].

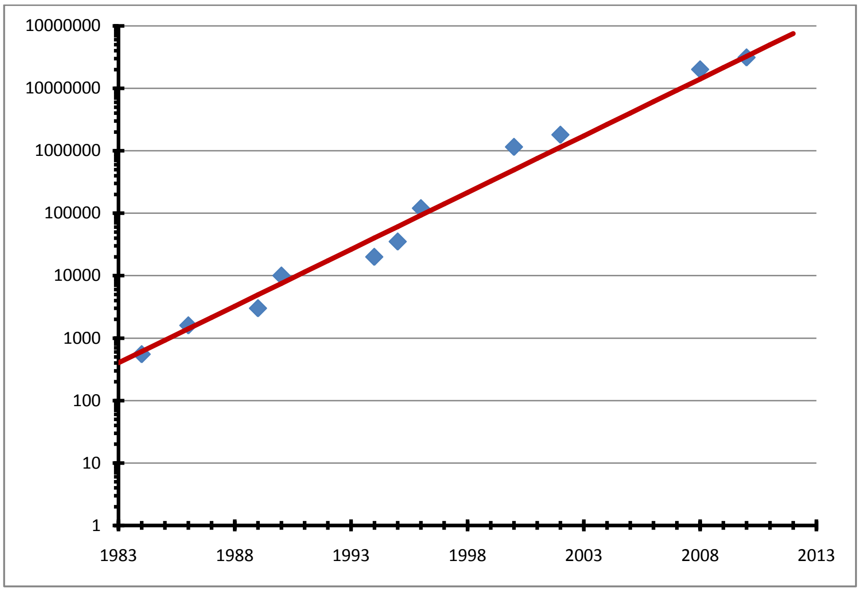


Figure 1.1: Internet speed evolution in bits per seconds over the time. The dots in the diagram show the internet speed measured by Nielsen since 1984.

The most effective technique in order to obtain a reliable link over a fading wireless channel is to use the diversity. Diversity techniques are widely used in practical wireless communication scenarios. The diversity order of a given scheme is a quantitative measure of how useful it is to invest the transmit power into this given scheme instead of simply increasing the transmit power of the original strategy. There are several ways to obtain diversity all of which attempt to obtain independently faded replicas of the transmitted signal at the receiver side. The receiver may then combine the information contained in each independently faded version of the transmitted signal in order to determine the most likely transmitted signal. An informal definition of the diversity is therefore the number of independent replicas of the transmitted signal received at the destination. A more formal definition of diversity gain g_d is given as [5]:

$$g_d = -\lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR} \tag{1.1}$$

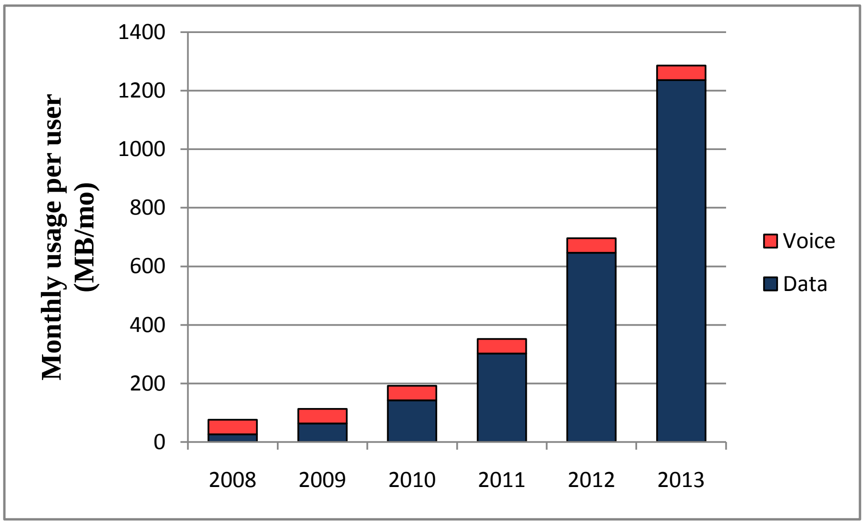


Figure 1.2: Average mobile use per subscriber per month in North America.

where P_e denotes the average error probability and SNR is the signal to noise ratio of the network. As seen in (1.1), the diversity gain is the negative slope of the error probability versus the signal to noise ratio (SNR), traced in log-log scale. Diversity may be realized in different ways, including frequency diversity, time diversity, spatial diversity, etc.

This dissertation focuses on one of the above mentioned diversity techniques: spatial diversity, which is a powerful means of combatting the deleterious effects of fading. In this technique transmitter and/or receiver is equipped with multiple antennas. Systems with multiple antennas are also referred to as multiple-input multiple-output (MIMO) systems. One of the major advantages of MIMO systems is the substantial increase in the channel capacity, which immediately translates to higher data throughputs. Another advantage of MIMO systems is the significant improvement in data transmission reliability. These advantages are achievable without any expansion in the required bandwidth or increase in the transmit power. In the spatial diversity techniques the goal is to obtain the independently faded replicas of the transmitted signal due to the different channel coefficients between the source and the destination. However, due to the size constraints, the antennas on transmitter or receiver may not be placed sufficiently far apart. This limitation induces two major problems that challenge limit the benefits of MIMO systems:

- The channel coefficients from two adjacent antennas may not be statistically independent. As a result the signals received by the destination are not completely independent replicas of the transmitted signal. It is obvious that since the diversity is the number of *independent* replicas of the transmitted signal, the full diversity may not be achieved.

- When an obstacle is placed between the transmitter and the receiver, all transmitter/receiver antennas will simultaneously experience deep attenuation. As a result the telecommunication link will be seriously degraded, no matter how many antennas are used.

One promising solution is to use distributed MIMO systems which consists of distributing antennas among different nodes of the system. This method has also the advantage of increasing the transmission range. In the cases where the source and the destination are not in line-of-sight and/or when the media loss is important, one may use telecommunication relays to limit the transmit power. Distributed MIMO systems combine the benefits of relaying schemes and MIMO systems.

Another very challenging aspect of wireless communication is the number of simultaneous users in the network. Figure 1.3 shows the growth of mobile subscriptions in the world until 2009. Today, there are over 5 billion mobile phones in use worldwide. In 40 countries the number of mobile phones surpasses the population [6]. It is impossible to serve each user with an individual frequency career or time slot. The research for new strategies that can economize the use of frequency careers or dedicated time slots never stops. Such strategies may include MIMO code division multiple access (CDMA) and MIMO orthogonal frequency division multiple access (OFDMA).

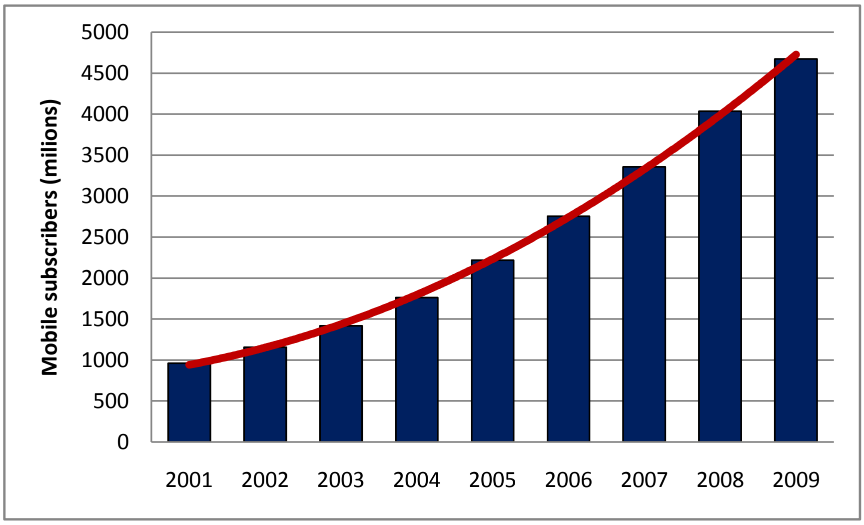


Figure 1.3: The evolution of mobile subscriptions in the world from 2001 to 2009 according to the World Bank.

This dissertation is focused on one promising solution for the multiple access MIMO systems: cooperative communications. This is the case when two or more elements of the system (usually end users), cooperate to deliver a message from the source to the destination.

1.1 Cooperative communications

The classic representation of a communication network is a graph with a set of nodes and edges. The nodes usually represent devices such as a router, a wireless access point, or a mobile telephone. The edges usually represent communication links or channels, for example an optical fiber, a cable, or a wireless link. This work deals mainly with Rayleigh flat fading wireless channels. Both the devices and the channels may have constraints on their operation. For example, a router might have limited processing power, a wireless phone has limited battery resources, the maximum transmission distance of an optical fiber is limited by several types of dispersion, and a wireless link can have rapid time

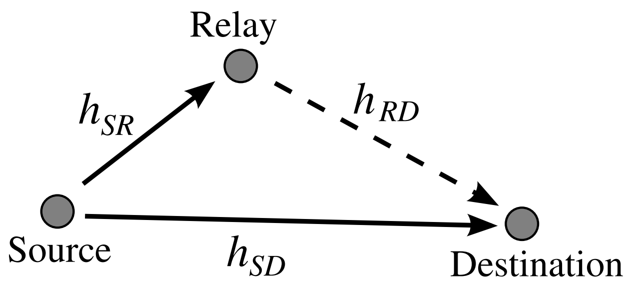


Figure 1.4: System model

variations arising from mobility and multipath propagation of signals. The purpose of a communication network is to enable the exchange of messages between its nodes.

Due to the broadcast nature of wireless links, signal transmissions between two nodes may be received at the neighbor nodes. It has been understood in the information theory for over three decades that wireless communication from a source to destination can benefit from the cooperation of nodes that overhear the transmission, as these intermediate nodes may themselves generate transmissions based on processing of the overheard signals. Let us consider the system depicted in Figure 1.4 where one system node (source) is sending a message to another system node (destination). Due to the broadcast nature of the wireless link, this message is overheard by a third node of the network (relay). During the first phase of transmission (solid lines in Figure 1.4), the source broadcasts the unitary message symbol s to both relay and destination using the power E_s. The second phase (dashed line in Figure 1.4) consists of relay transmitting a transformed version of its received signal to destination while source is silent. Note that two phases indicate two independent transmissions. This may be achieved by using orthogonal codings; e.g. using different time slots or different frequency careers. Let us assume that all three links are independent identically distributed (i.i.d) flat fading Rayleigh channels $h_{ij} \sim \mathcal{CN}(0, 1)$ with $ij \in \{SR, SD, RD\}$. The received signal at relay and destination after the first phase accomplishment may be expressed as:

$$y_R = \sqrt{E_s} h_{SR} s + n_{SR} \tag{1.2}$$

$$y_{D1} = \sqrt{E_s} h_{SD} s + n_{SD} \tag{1.3}$$

with $n_{SR} \sim \mathcal{CN}(0, \sigma_{SR}^2)$ and $n_{SD} \sim \mathcal{CN}(0, \sigma_{SD}^2)$ being the samples of a complex additive white Gaussian noise (AWGN). During the second phase while the source is silent, the relay sends t, a transformation of y_R towards the destination.

1.1.1 Cooperative strategies

As stated above, in the second phase of the transmission, the relay will transmit towards the destination, a signal t based on its received signal. The received signal at the destination is then:

$$y_{D2} = h_{RD} t + n_{RD} \tag{1.4}$$

with $n_{RD} \sim \mathcal{CN}(0, \sigma_{RD}^2)$ being the AWGN noise. The choice of t determines the cooperative strategy. Several scenarios may be considered [7–9]:

- amplify-and-forward (AF)
- classic multi-hop
- compress-and-forward (CF)
- decode-and-forward (DF)
- multipath decode-and-forward (MDF)

In this dissertation we only use AF [10, 11] and DF [12–14] cases. In the AF case, t is only a scaled version of y_R to the available relay power E_R:

$$t = \sqrt{\frac{E_R}{\sigma_{SR}^2 + |h_{SR}|^2 E_s}}\, y_R \tag{1.5}$$

It can be easily proved [15] that using the AF protocol the equivalent SNR for the source-relay-destination path can be determined by:

$$\gamma_{SRD} = \frac{\gamma_{SR}\gamma_{RD}}{1 + \gamma_{SR} + \gamma_{RD}} \tag{1.6}$$

where γ_{SR} and γ_{RD} respectively denote the signal to noise ratios of source-relay and relay-destination.

In the DF scenario, the relay will first decode its received signal y_R, and finds the likeliest transmitted symbol $\hat{s}$ and then retransmits the signal $\hat{s}$ scaled to the available relay power:

$$t = \sqrt{E_R}\hat{s} \tag{1.7}$$

The advantage of DF relaying is that if the relay can successfully decode the received signal, the source-relay link noise effect is canceled out, while in the AF strategy, the relay amplifies also the noise.

1.1.2 Combination techniques

At the end of the second phase, the destination has two replicas of the received signal y_{D1} and y_{D2}. Obviously the expression of y_{D2} depends on the used protocol (AF or DF). Another classification of the system is based on the destination's choice on the demodulated symbol [16–19]:

- **SC:** The selection combining (SC) is used when the destination chooses to only use the strongest received signal [20–22]. In this case the signal received from the path with the highest equivalent SNR is used to decode the likeliest transmitted symbol.

- **EGC:** In the equal gain combining (EGC) case, destination will dephase y_{D1} and y_{D2} to remove the dephasing induced by the channel and combines them with equal gains [23].

- **MRC:** In maximum ratio combining (MRC) the destination combines y_{D1} and y_{D2} with weighting coefficients in order to take into account the SNR of each link [24–26]. Naturally the link with a stronger signal, is more likely to influence the destination's decision on the likeliest transmitted symbol. If MRC is used, the equivalent system SNR is the sum of the signal to noise ratios of both source-relay-destination and source-destination paths.

Figure 1.5 shows the bit error rate (BER) simulation of different combination techniques on AF and DF strategies over a Rayleigh flat fading channel. Transmission symbols are quadrature phase-shift keying (QPSK) with unit power. The source-relay link is assumed to have an average signal to noise ratio of 10 dB more than source-destination and relay-destination links. Figure 1.5 shows that MRC outperforms other combination techniques and in general AF relaying is more efficient that DF relaying.

In AF relaying scheme, the equivalent SNR of the source-relay-destination has a close form expression. As a result the method of applying different combination techniques (i.e. SC, EGC, MRC, ...) is more or less unanimous in the literature. On the other hand such equivalent SNR does not exist for the source-relay-destination path of a DF relaying scheme. As a result different authors propose different metrics to implement each combination technique in order to take into account the possible errors of the source-relay link.

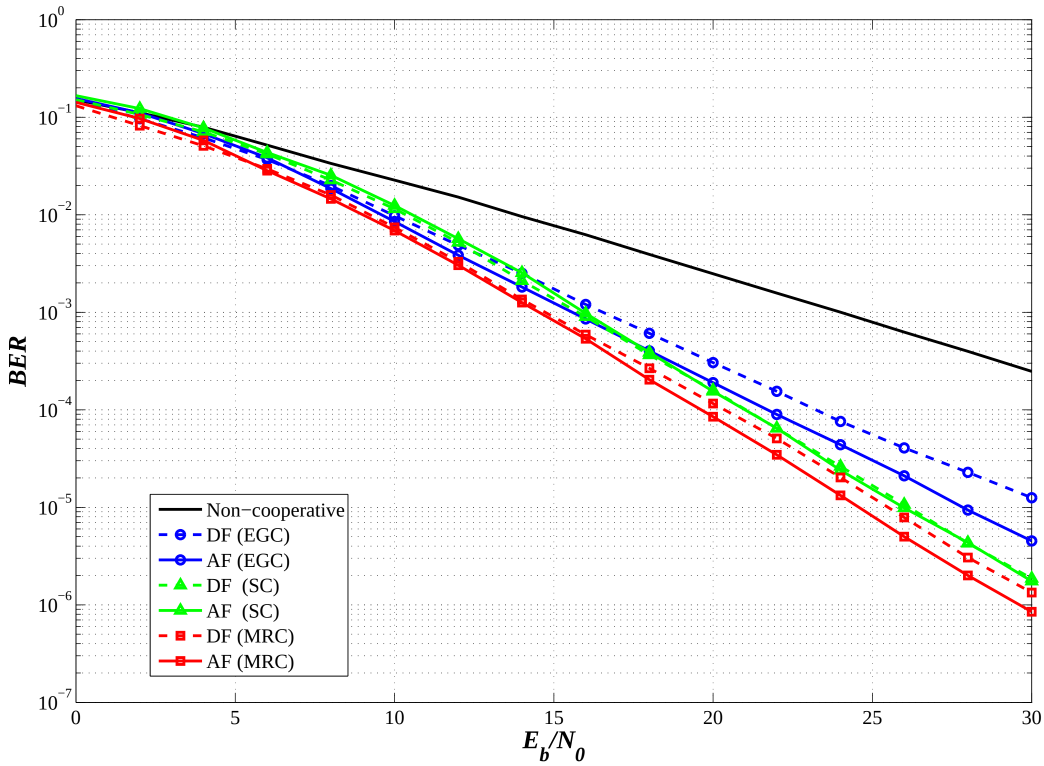


Figure 1.5: Performance of different cooperative strategies and combination techniques

1.2 Precoding in multiuser networks

During the past few years, some researchers proposed to use precoding in MIMO and cooperative communications in order to increase the performance of such systems. Precoding is usually used in multiuser networks where two or more concurrent users try to access the same network. In the multiuser systems involving precoding, the transmitter multiplies its symbols by some precoding coefficient in order to minimize the multiple access interference (MAI). Of course, these precoding coefficients must be calculated as a function of the channels and the type of receiver employed [27–30].

In this section we will first introduce distributed relay systems, the multi-access networks

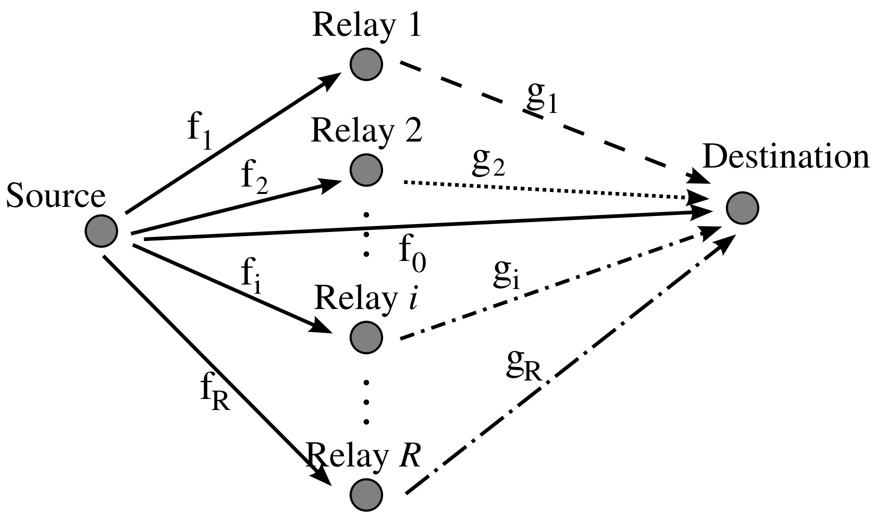


Figure 1.6: System model for a multiple relaying scheme

will be discussed, and finally the use of precoding vectors will be introduced.

1.2.1 Multiple relaying schemes

Let us consider the point-to-point transmission system depicted in Figure 1.6. The system is composed of one source node, R relays, and one destination node. Like the case addressed in Section 1.1, different transmission phases are represented by different line types. Note that here the transmission is composed of $R+1$ phases (e.g. time slots). In the first phase the source broadcasts the message symbol s to the relays and destination. In the next R phases, the relays transmit their symbols to the destination one at a time. Some works assume that the direct link between the source and the destination (f_0) is too weak and can be neglected [31, 32].

Assuming that all links are i.i.d flat fading Rayleigh channels, at the end of the first phase, the received signals by relays and destination are:

$$r_i = \sqrt{E_s} f_i s + v_i \qquad i = 1, 2, \cdots R \tag{1.8}$$

$$y_0 = \sqrt{E_s} f_0 s + v_0 \tag{1.9}$$

where v_i is the receiver noise. The amplify-and-forward and decode-and-forward strategies can be applied here, exactly like they were applied in the previous section:

$$t_{i_{AF}} = \sqrt{\frac{E_R}{\sigma_{v_i}^2 + |f_i|^2 E_s}} r_i \tag{1.10}$$

$$t_{i_{DF}} = \sqrt{E_r} \hat{s}_i \quad \text{with } \hat{s}_i \text{ decoded symbol at the } i\text{th relay} \tag{1.11}$$

At the end of all phases, the destination receives R more signals from relays:

$$y_i = g_i t_i + w_i, \qquad 1, 2, \cdots, R \tag{1.12}$$

with w_i being the AWGN. For the multiple relay case, as well as the simple cooperative case discussed in the previous section, the destination may use different combination techniques such as SC, EGC, or MRC. Figure 1.7 shows the simulation results for the AF strategy and different combining techniques. Note that all techniques produce the full diversity ($d = 3$) and that the maximal ratio combining is the most effective method.

Note that if AF is used along with MRC, the equivalent signal to noise ratio can be expressed as [15]:

$$\gamma = \sum_{i=0}^{R} \gamma_i \tag{1.13}$$

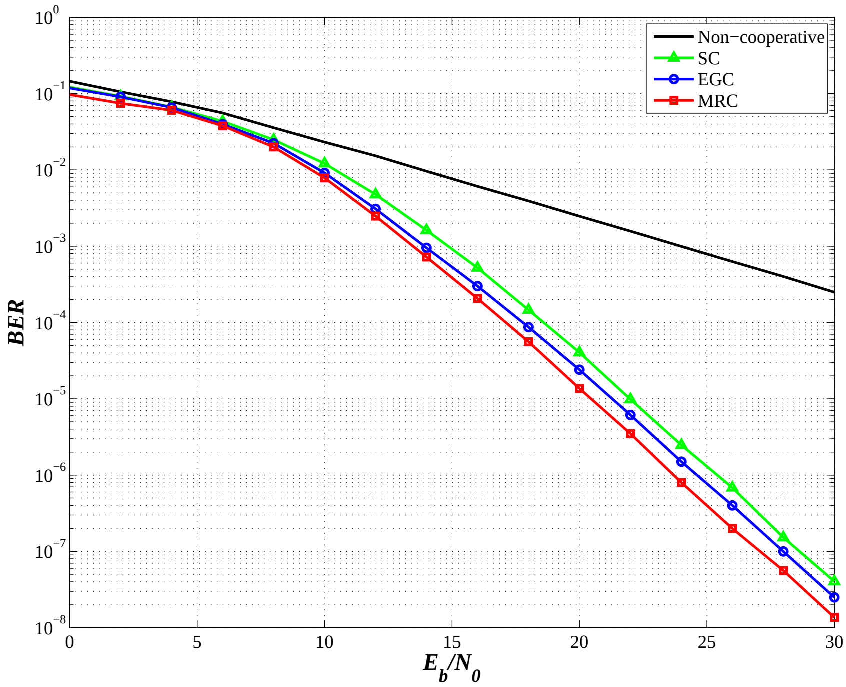


Figure 1.7: Simulation results for a distributed multi relay AF scheme without a direct link ($f_0 = 0$) and three relays

with:

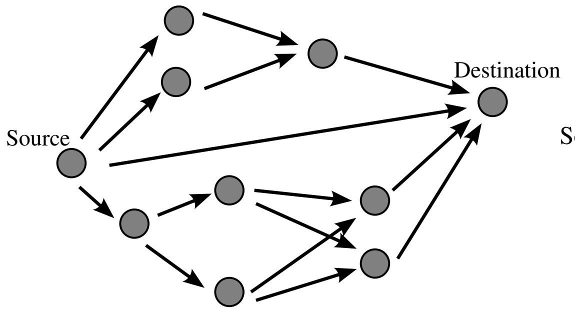
$$\gamma_0 = \frac{E_s \left|f_0\right|^2}{\sigma_{v0}^2} \tag{1.14}$$

$$\gamma_i = \frac{\frac{E_s|f_i|^2}{\sigma_{vi}^2} \frac{E_r|g_i|^2}{\sigma_{w0}^2}}{1 + \frac{E_s|f_i|^2}{\sigma_{vi}^2} + \frac{E_r|g_i|^2}{\sigma_{wi}^2}} \qquad i = 1, 2, \cdot R \tag{1.15}$$

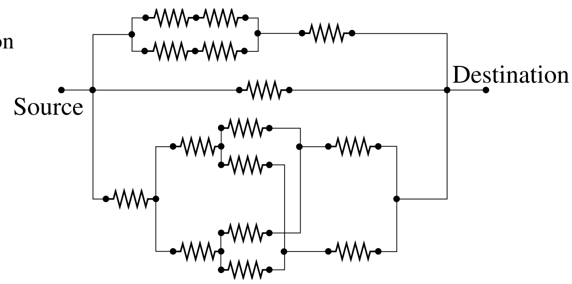
For high SNRs, the 1 in the denominator of (1.15) can be neglected, and γ_i can be expressed as:

$$\gamma_i \approx \frac{\gamma_{SRi}\gamma_{RiD}}{\gamma_{SRi} + \gamma_{RiD}} \tag{1.16}$$

with γ_{SRi} and γ_{RiD} being respectively the signal to noise ratios corresponding to the first and second hop of the ith branch. Equation (1.16) looks like the equation corresponding to



(a) cooperative system model



(b) equivalent electric system

Figure 1.8: System model and its equivalent electrical circuit for a given cooperative network

the equivalent conductance of two conductances connected in series. Since maximal ratio combining allows to sum up individual SNRs, it can be seen as parallel conductances. As a result, for an amplify-and-forward MRC scheme, the equivalent SNR of a given network can be calculated in the same way as the equivalent conductance of a circuit. Figure 1.8a shows the system model of a given cooperative scheme. If the system is operated with AF strategy using MRC, its equivalent SNR can be calculated from the equivalent conductance of the circuit depicted in Figure 1.8b where each conductance corresponds to the SNR of the corresponding link.

1.2.2 Precoding in relays

One problem of the system discussed in the previous subsection is that it leads to a loss of the system throughput due to different phases needed to implement in order to prevent interference from different relays. If we assume that the relays in DF mode have the knowledge of the forward channel (i.e. each relay knows the channel between itself and the destination), each relay can multiply its signal by the complex conjugate of its forward channel to ensure that the signals from all relays arrive at the destination with the same phase. This is a simple example of precoding to achieve beamforming. All relays could then send their symbols simultaneously towards the destination. Furthermore, in the case

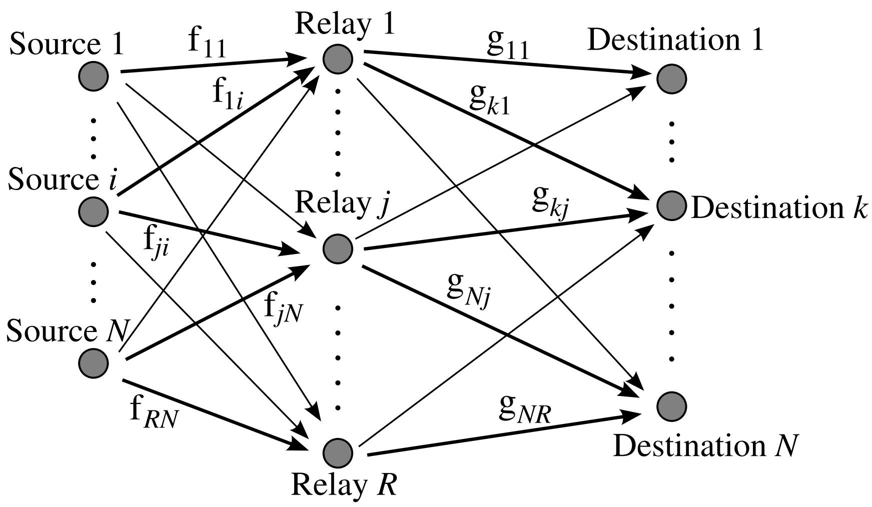


Figure 1.9: A multi-user multi-relay network. The thick lines represent the links with annotaions

of centralized relays (when the precoding coefficients are calculated in a central unit having access to the channel between all relays and the destination), the channel information may be used to maximize SNR at the destination by assigning more power to the relays that benefit from better links. In this case precoding is used to not only perform the beamforming, but also to use water-filling in order to increase the system performance [33, 34].

In the above examples precoding was used in a point-to-point single user scheme. In this dissertation we are more focused on the precoding techniques in the multiple access networks [35–39]. Let us consider the multiuser AF relaying system with N single antenna sources and N single antenna destinations as depicted in Figure 1.9. R single antenna relays assist the sources to send data to the destinations. The direct links between sources and destinations are assumed to be negligible. In every transmission period, the sources 1 to N wish to send the baseband signal s_1 to s_N to destinations 1 to N respectively. Note that we want each destination to receive only its intended symbol. It means that the effect of s_2 to s_N must be canceled out at the 1st destination, and so on. The transmit power of each source is fixed to P_0. In the first slot of every transmission period, the received signal vector at the relays equals:

$$\mathbf{r} = \sqrt{P_0}\mathbf{F}\mathbf{s} + \mathbf{n} \tag{1.17}$$

with $\mathbf{r} = [r_1, r_2, \cdots, r_N]^T$ and $\mathbf{n}$ being the column vector of the receiver noise at the relays. Matrix $\mathbf{F}$ is defined as an R by N matrix in which f_{ji}, the element at the (j, i)th position of the matrix, is the channel between the ith source and the jth relay. The relays multiply their received signals by some precoding coefficients prior to forwarding them toward the destinations: $t_i = w_i r_i$ for $i = 1, 2, \cdots, R$. In this equation w_i is the precoding coefficient of the ith relay and t_i is the transmitted symbol of this relay. Defining $\mathbf{W} = \mathrm{diag}([w_1, w_2, \cdots, w_R])$, the R by R diagonal matrix of precoding coefficients, we can rewrite this equation in the matrix form:

$$\mathbf{t} = \mathbf{W}\mathbf{R} = \sqrt{P_0}\mathbf{W}\mathbf{F}\mathbf{s} + \mathbf{W}\mathbf{n} \tag{1.18}$$

In this case, the received signals at the destination is expressed by:

$$\mathbf{y} = \sqrt{P_0}\mathbf{G}\mathbf{W}\mathbf{F}\mathbf{s} + \mathbf{G}\mathbf{W}\mathbf{n} + \mathbf{z} \tag{1.19}$$

where $\mathbf{z}$ is the vector of destination noises and $\mathbf{G}$ is an N by R matrix with g_{kj}, the (k, j)th element of $\mathbf{G}$, being the channel coefficient of the link between the jth relay and the kth destination node. Note that if we consider the centralized structure (i.e. the precoding coefficients are calculated by a central processing unit which has the complete channel state information (CSI) of the system and the received signals at all relays, and provides the relays with their respective symbol to transmit), the matrix $\mathbf{W}$ no longer requires to be diagonal and may have an arbitrary structure. This provides the system with more degrees of liberty at the cost of the network's more complex structure and protocol.

In any cases, if we want the MAI to be canceled out, we must guarantee that $\mathbf{GWF}$ is a diagonal matrix. Let us determine $\mathbf{W}$ as[1]:

$$\mathbf{W} = \underbrace{\mathbf{G}^{\dagger}\left(\mathbf{G}\mathbf{G}^{\dagger}\right)^{-1}}_{\mathbf{W}_{\mathbf{G}}}\boldsymbol{\Lambda}_R\underbrace{\left(\mathbf{F}^{\dagger}\mathbf{F}\right)^{-1}\mathbf{F}^{\dagger}}_{\mathbf{W}_{\mathbf{F}}} \tag{1.20}$$

where $(\cdot)^{\dagger}$ denotes the conjugate transpose of $(\cdot)$ and $\boldsymbol{\Lambda}_R$ is a diagonal matrix. The received signal can be thus written as:

$$\begin{aligned}\mathbf{y} &= \sqrt{P_0}\mathbf{G}\mathbf{G}^{\dagger}(\mathbf{G}\mathbf{G}^{\dagger})^{-1}\boldsymbol{\Lambda}_R(\mathbf{F}^{\dagger}\mathbf{F})^{-1}\mathbf{F}^{\dagger}\mathbf{F}\mathbf{s} + \mathbf{G}\mathbf{G}^{\dagger}(\mathbf{G}\mathbf{G}^{\dagger})^{-1}\boldsymbol{\Lambda}_R(\mathbf{F}^{\dagger}\mathbf{F})^{-1}\mathbf{F}^{\dagger}\mathbf{n} + \mathbf{z} \\ &= \sqrt{P_0}\boldsymbol{\Lambda}_R\mathbf{s} + \boldsymbol{\Lambda}_R\mathbf{W}_{\mathbf{F}}\mathbf{n} + \mathbf{z}\end{aligned} \tag{1.21}$$

Note that since $\boldsymbol{\Lambda}_R$ is a diagonal matrix, the received signal at the ith destination node depends only on s_i. The matrix $\mathbf{W}$ has three components: The relays first use $\mathbf{W}_{\mathbf{F}}$ to transform the received signal into parallel streams, then process power allocation through $\boldsymbol{\Lambda}_R$, and finally send the signal precoded by $\mathbf{W}_{\mathbf{G}}$ to cancel the inter destination interference caused by $\mathbf{G}$.

In this case, the precoding matrix has been used to remove the MAI at the transmitter and the receiver sides, and by optimizing $\boldsymbol{\Lambda}_R$ it can maximize the SNR at the destinations.

The following subsection consists of a brief survey on the use of precoding in the literature.

1.2.3 Precoding in the literature

A lot of works have been already published concerning the optimization of precoding vectors at relays for a single destination terminal [33, 40–43]. The optimization criterion is always the maximization of the SNR at destination. Zhihang et al [33] presented an outstanding work dealing with different realistic scenarios including the case where only second order

[1]For an explication of the method used to obtain $\mathbf{W}$, see Chapter 3

statistics of the channel are available. However, it does not cover the multi-user case. In general, the majority of works on the subject is related to the AF case with only one destination terminal. Some works like [44] deal with the particular case of two destination users. In fact, the use of precoding vectors for the case of arbitrary number of destination users is a very poor investigated aria. However, some recent articles have worked on this subject [36,45–47]. In [45], the authors propose optimal beamforming designs to maximize the SNR margin in a multiuser multi-relay network with AF relays. Two kinds of power constraints are considered: sum relay power constraint and per-relay power constraints. Simple iterative algorithms are given which converge to the optimal solution. However, no diversity study or analytical derivation of the average expected symbol error probability (SEP) is given. The authors in [36] calculate beamforming matrix for multiple independent sources, destinations and AF relays to minimize the sum transmit power at the relays while meeting signal to noise plus interference ratio (SINR) requirements at the destinations. Once again, no diversity study is available and theoretical system performance evaluation is not provided. In [46] the authors propose the application of SINR-Max cooperative beamforming for multiuser detector over flat MIMO fading channels and extend it in [47] to the case of multiuser MIMO-OFDM systems, but they did not provide any diversity study and SEP analytical calculations. Another recent study on the topic is the work of Shu et al. [48] where the authors try to maximize the system capacity using precoding vectors optimized by Particle Swarm Optimization (PSO) algorithm, however the proposed solution in [48] implies a complicated receiver structure at the mobile station (MS) side which limits the practical interest of the work by imposing expensive structure to the end line users. At the same time, due to the complicated equivalent channel equations, very little analytic predictions are produced.

1.3 Organization of the dissertation

This dissertation is organized as follows:

The second chapter uses mathematical optimization based on Lagrange multipliers method in order to optimize the performance of a multi-user multi-relay two-hop transmission scheme. The Lagrange multiplier method is modified to fit the problems involving vectors and matrices. The resulting algorithm is very flexible and under a vast variety of constraints can be implemented using linear matrix operations.

In chapter 3 a special case of the optimization problem addressed in chapter 2 is studied. In this chapter, since a special case is addressed, analytical performance analysis is possible. We use the Expectation-Maximization (EM) algorithm to approximate the probability distribution function (pdf) of the signal to noise ratio. Diversity gain and the symbol error probability are calculated theoretically.

Chapter 4 optimizes the relays transmit power in order to maximize the system performance. The Gram-Schmidt orthonormalization process is used to cancel out the MAI. Theoretical performance analysis is carried out and the diversity order of the system is derived mathematically.

Chapter 5 addresses a multi-user multi-relay case where only the second order statistics of the CSI is available. The system studied in this chapter features very simple single-antenna relays. The system performance is optimized under different types of power constraints.

Finally, general conclusions are derived in the last chapter of this dissertation.

Chapter 2

Mathematical Optimization

In this chapter we will use different mathematical optimization methods to optimize the performance of a given telecommunication system. We will use a modified version of Lagrange multipliers method to optimize our approach.

The proposed system is composed of one base station with M antennas, which sends N symbols s_1 to s_N respectively to N mobile stations MS_1 to MS_N via L not-moving relays each with R antennas (see 2.1)[1]. A two hop communication scheme is considered. In the first hop, the base station sends the signal to the relays. The relays will then decode the received signal and multiply it by some precoding vectors before transmitting them to mobile stations in the second hop. Since the base station and the relays are considered not to move, the communication between the BS and relays is assumed to be perfect. In fact a wide variety of low-noise communication media such as optical fibers may be used. Moreover cyclic redundancy codes (CRC) may be employed to detect any possible errors in relays and to ask the base station to resend the missing information. As a result BS to RS links are considered error-free [2]. In the remainder of this chapter we will focus on the second hop of the communication where L relays cooperate in sending each of the N data symbols to their intended mobile stations. The link from the ith relay to the jth mobile station is assumed to be a flat fading Rayleigh channel $\mathbf{h}_{ij} \sim \mathcal{CN}(0, \mathbf{I}_R)$ of size $1 \times R$. The channel coefficients are assumed to be known at the transmitter. The receivers however do not use any information of the channel.

For simplicity, at the first time we will consider the case that only two relay stations are used. The results may be generalized to the case of arbitrary number of relay stations[3]. In

[1] **Note that R denotes the number of antennas in a relay and not the number of antennas which is denoted by L**

[2] Even when this is not the case, we can instruct the relays that did not succeed to correctly detect the information to be silent while the other $L' \leq L$ relays with correctly decoded data will cooperate to send the signals to the MSs. The system can be seen as the same system with the number of relays decreased from L to L'

[3] See section2.4: Conclusion

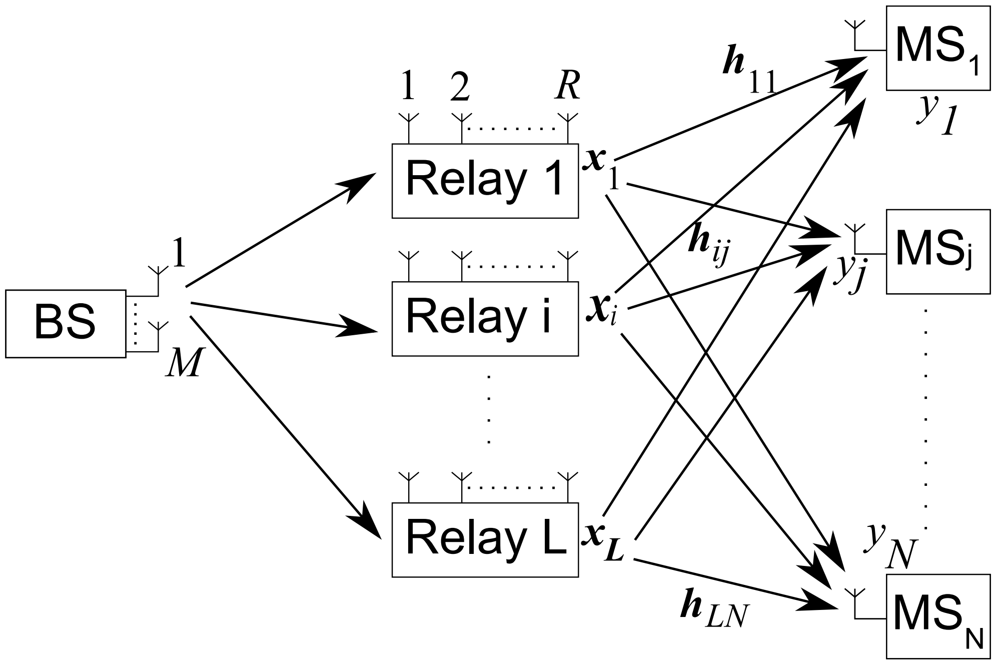


Figure 2.1: System model

this case the signals sent by RS_1 and RS_2 are given by:

$$\begin{aligned}\mathbf{x}_1 &= \sum_{j=1}^{N} s_j \mathbf{w}_j^1 \\ \mathbf{x}_2 &= \sum_{j=1}^{N} s_j \mathbf{w}_j^2\end{aligned} \tag{2.1}$$

Where $\mathbf{w}_j^i$ of size $R \times 1$ represents the precoding vectors of ith relay. $\mathbf{x}_i$ are then transmitted to mobile stations via the Rayleigh channels $\mathbf{h}_{ij}$, $i = 1, 2$, $j = 1 \cdots N$. The second hop channel being a Rayleigh channel of size $1 \times R$, the signal at the jth mobile station can thus be expressed as

$$y_j = \mathbf{h}_{1j} \cdot \mathbf{x}_1 + \mathbf{h}_{2j} \cdot \mathbf{x}_2 + n_j \quad , j = 1 \cdots N \tag{2.2}$$

Where $\mathbf{h}_{ij}$ is a $\mathcal{CN}(0, \mathbf{I}_R)$ and n_j is the additive white Gaussian noise (AWGN) at the jth mobile station. By substituting (2.1) in (2.2) we obtain:

$$y_j = \mathbf{h}_{1j} \cdot \sum_{k=1}^{N} s_k \mathbf{w}_k^1 + \mathbf{h}_{2j} \cdot \sum_{k=1}^{N} s_k \mathbf{w}_k^2 + n_j \tag{2.3}$$

As stated above, we want y_j to depend only on s_j, that is to say:

$$\sum_{k \neq j} s_k \mathbf{h}_{1j} \cdot \mathbf{w}_k^1 + \sum_{k \neq j} s_k \mathbf{h}_{2j} \cdot \mathbf{w}_k^2 = 0, \qquad j = 1, 2, \cdots N \tag{2.4}$$

If (2.4) is satisfied the resulting signal at each mobile station is given by (2.5).

$$y_j = s_j \left(\mathbf{h}_{1j} \cdot \mathbf{w}_j^1 + \mathbf{h}_{2j} \cdot \mathbf{w}_j^2 \right) + n_j \tag{2.5}$$

The precoding vectors must guarantee that the term in parentheses at the right hand side of (2.5) is a real positive number.

We will now examine the constraints that the precoding vectors must satisfy in order to:

- cancel out the interference between messages, i.e. MS_j receives only s_j
- guarantee a constructive superpositioning between the RS_1 and RS_2
- maintain the consumed power at relays below a given level.

These constraints and the equations related to them are discussed below.

2.1 Mathematical model to optimize

In this section we will write the equations leading to the system optimization. Note that two different scenarios may be considered depending on whether or not one relay has the channel state information (CSI) of the other relay. If both relays have the CSI the system will have more degrees of freedom and as a consequence it will show better performance, however this is at the cost of less practical interest caused by the necessity of some kind of dialog between relays in order to pass the CSI and the calculated precoding vectors. In this chapter we will consider the case where each relay has only the knowledge of its own link to the MSs.

2.1.1 Canceling interferences

In this subsection we will find the equations guaranteeing that the system will act like N parallel and independent links between the source and destinations Expanding the criterion in (2.4) for the first mobile station ($j = 1$) we obtain:

$$s_2 \mathbf{h}_{11} \mathbf{w}_2^1 + \cdots + s_N \mathbf{h}_{11} \mathbf{w}_N^1 + s_2 \mathbf{h}_{21} \mathbf{w}_2^2 + \cdots + s_N \mathbf{h}_{21} \mathbf{w}_N^2 = 0 \tag{2.6}$$

This equation requires the mutual CSI and the precoding vectors knowledge between the relays. We split it into two parts to overcome this knowledge:

$$\begin{cases} s_2 \mathbf{h}_{11} \mathbf{w}_2^1 + s_3 \mathbf{h}_{11} \mathbf{w}_3^1 + \cdots + s_N \mathbf{h}_{11} \mathbf{w}_N^1 = 0 \\ s_2 \mathbf{h}_{21} \mathbf{w}_2^2 + s_3 \mathbf{h}_{21} \mathbf{w}_3^2 + \cdots + s_N \mathbf{h}_{21} \mathbf{w}_N^2 = 0 \end{cases} \tag{2.7}$$

The same reasoning for MS_2 ($j = 2$) leads to:

$$\begin{cases} s_1 \mathbf{h}_{12} \mathbf{w}_1^1 + s_3 \mathbf{h}_{12} \mathbf{w}_3^1 \cdots + s_N \mathbf{h}_{12} \mathbf{w}_N^1 = 0 \\ s_1 \mathbf{h}_{22} \mathbf{w}_1^2 + s_3 \mathbf{h}_{22} \mathbf{w}_3^2 \cdots + s_N \mathbf{h}_{22} \mathbf{w}_N^2 = 0 \end{cases} \tag{2.8}$$

And finally for MS_N we have:

$$\begin{cases} s_1\mathbf{h}_{1N}\mathbf{w}_1^1 + s_2\mathbf{h}_{1N}\mathbf{w}_2^1 \cdots + s_{N-1}\mathbf{h}_{1N}\mathbf{w}_{N-1}^1 = 0 \\ s_1\mathbf{h}_{2N}\mathbf{w}_1^2 + s_2\mathbf{h}_{2N}\mathbf{w}_2^2 \cdots + s_{N-1}\mathbf{h}_{2N}\mathbf{w}_{N-1}^2 = 0 \end{cases} \tag{2.9}$$

Now for each relay we have a set of equations:

$$\text{Relay①}\begin{cases} 0 + s_2\mathbf{h}_{11}\mathbf{w}_2^1 + s_3\mathbf{h}_{11}\mathbf{w}_3^1 + \cdots + s_N\mathbf{h}_{11}\mathbf{w}_N^1 = 0 \\ s_1\mathbf{h}_{12}\mathbf{w}_1^1 + 0 + s_3\mathbf{h}_{12}\mathbf{w}_3^1 + \cdots + s_N\mathbf{h}_{12}\mathbf{w}_N^1 = 0 \\ \vdots + \vdots + \ddots + \ddots + \vdots = \vdots \\ s_1\mathbf{h}_{1N}\mathbf{w}_1^1 + s_2\mathbf{h}_{1N}\mathbf{w}_2^1 + \cdots + s_{N-1}\mathbf{h}_{1N}\mathbf{w}_{N-1}^1 + 0 = 0 \end{cases} \tag{2.10}$$

$$\text{Relay②}\begin{cases} 0 + s_2\mathbf{h}_{21}\mathbf{w}_2^2 + s_3\mathbf{h}_{21}\mathbf{w}_3^2 + \cdots + s_N\mathbf{h}_{21}\mathbf{w}_N^2 = 0 \\ s_1\mathbf{h}_{22}\mathbf{w}_1^2 + 0 + s_3\mathbf{h}_{22}\mathbf{w}_3^2 + \cdots + s_N\mathbf{h}_{22}\mathbf{w}_N^2 = 0 \\ \vdots + \vdots + \ddots + \ddots + \vdots = \vdots \\ s_1\mathbf{h}_{2N}\mathbf{w}_1^2 + s_2\mathbf{h}_{2N}\mathbf{w}_2^2 + \cdots + s_{N-1}\mathbf{h}_{2N}\mathbf{w}_{N-1}^2 + 0 = 0 \end{cases} \tag{2.11}$$

Equations (2.10) and (2.11) are two sets of N linear equations each set having $N \times R$ unknown scalar values of N vectors $\mathbf{w}_1^i$ to $\mathbf{w}_N^i$ to be determined.

Each row in (2.10) and (2.11) can be factorized with $\mathbf{h}_{ij}$ as factor:

$$\text{Relay①}\begin{cases} \mathbf{h}_{11} \cdot (0 + s_2\mathbf{w}_2^1 + s_3\mathbf{w}_3^1 + \cdots + s_N\mathbf{w}_N^1) = 0 \\ \mathbf{h}_{12} \cdot (s_1\mathbf{w}_1^1 + 0 + s_3\mathbf{w}_3^1 + \cdots + s_N\mathbf{w}_N^1) = 0 \\ \vdots \\ \mathbf{h}_{1N} \cdot \big(s_1\mathbf{w}_1^1 + s_2\mathbf{w}_2^1 + \cdots + s_{N-1}\mathbf{w}_{N-1}^1 + 0\big) = 0 \end{cases} \tag{2.12}$$

$$\text{Relay②}\begin{cases}\mathbf{h}_{21}\cdot(0+s_2\mathbf{w}_2^2+s_3\mathbf{w}_3^2+\cdots+s_N\mathbf{w}_N^2)=0\\ \mathbf{h}_{22}\cdot(s_1\mathbf{w}_1^2+0+s_3\mathbf{w}_3^2+\cdots+s_N\mathbf{w}_N^2)=0\\ \vdots\\ \mathbf{h}_{2N}\cdot\left(s_1\mathbf{w}_1^2+s_2\mathbf{w}_2^2+\cdots+s_{N-1}\mathbf{w}_{N-1}^2+0\right)=0\end{cases}\tag{2.13}$$

Or:

$$\mathbf{h}_{ij}\cdot\sum_{\substack{k=1\\k\neq j}}^{k=N}s_k\mathbf{w}_k^i=0\qquad i=1,2\text{ and }j=1,\cdots,N\tag{2.14}$$

Note that (2.12) and (2.13) have the same structure. As a result, we will only write the equations for the first relay, knowing that the second relay is bound to an equation set of the same form. Denoting the real and imaginary parts of a complex number (or vector) c by c_R and c_I respectively, we can decompose the terms $s_j\mathbf{w}_\mathbf{j}^\mathbf{1}$ in (2.12) into real and imaginary parts:

$$\begin{aligned}\left(s_j\mathbf{w}_j^1\right)_R&=s_{j_R}\mathbf{w}_{j_R}^1-s_{j_I}\mathbf{w}_{j_I}^1\\ \left(s_j\mathbf{w}_j^1\right)_I&=s_{j_R}\mathbf{w}_{j_I}^1+s_{j_I}\mathbf{w}_{j_R}^1\end{aligned}\tag{2.15}$$

Now (2.12) becomes:

$$\begin{aligned}\left(\mathbf{h}_{1j}\cdot\sum_{i\neq j}s_i\mathbf{w}_i^1\right)_R&=\mathbf{h}_{1j_R}\cdot\left(\sum_{i\neq j}s_i\mathbf{w}_i^1\right)_R-\mathbf{h}_{1j_I}\cdot\left(\sum_{i\neq j}s_i\mathbf{w}_i^1\right)_I\\&=\mathbf{h}_{1j_R}\cdot\sum_{i\neq j}\left(s_{i_R}\mathbf{w}_{i_R}^1-s_{i_I}\mathbf{w}_{i_I}^1\right)\\&\quad-\mathbf{h}_{1j_I}\cdot\sum_{i\neq j}\left(s_{i_R}\mathbf{w}_{i_I}^1+s_{i_I}\mathbf{w}_{i_R}^1\right)\\&=0,\qquad j=1\cdots N\end{aligned}\tag{2.16}$$

$$
\begin{aligned}
\left(\mathbf{h}_{1j} \cdot \sum_{i \neq j} s_i \mathbf{w}_i^1\right)_I &= \mathbf{h}_{1j_R} \cdot \left(\sum_{i \neq j} s_i \mathbf{w}_i^1\right)_I + \mathbf{h}_{1j_I} \cdot \left(\sum_{i \neq j} s_i \mathbf{w}_i^1\right)_R \\
&= \mathbf{h}_{1j_R} \cdot \sum_{i \neq j} \left(s_{i_R} \mathbf{w}_{i_I}^1 + s_{i_I} \mathbf{w}_{i_R}^1\right) \\
&\quad + \mathbf{h}_{1j_I} \cdot \sum_{i \neq j} \left(s_{i_R} \mathbf{w}_{i_R}^1 - s_{i_I} \mathbf{w}_{i_I}^1\right) \\
&= 0, \qquad j = 1 \cdots N
\end{aligned}
\tag{2.17}
$$

These are $2N$ scalar valued linear equations bounding the $2NR$ real values of the precoding vectors corresponding to first relay station $\mathbf{w}_1^1$ to $\mathbf{w}_j^1$.

2.1.2 Coherent addition

If the interference cancellation is carried out, the received signal at the MSs given in (2.3) reduces to:

$$
\begin{aligned}
\mathbf{y}_1 &= s_1 \mathbf{h}_{11} \mathbf{w}_1^1 + s_1 \mathbf{h}_{21} \mathbf{w}_1^2 + n_1 \\
\mathbf{y}_2 &= s_2 \mathbf{h}_{12} \mathbf{w}_2^1 + s_1 \mathbf{h}_{22} \mathbf{w}_2^2 + n_2 \\
&\vdots \\
\mathbf{y}_N &= s_N \mathbf{h}_{1N} \mathbf{w}_N^1 + s_N \mathbf{h}_{2N} \mathbf{w}_N^2 + n_N
\end{aligned}
\tag{2.18}
$$

We want the first and the second terms in (2.18), i.e. the parts of the intended message sent from first and second relays, to arrive in phase at the mobile stations. We thus require that:

$$
\begin{aligned}
\angle\left(\mathbf{h}_{11}\mathbf{w}_1^1\right) &= \angle\left(\mathbf{h}_{21}\mathbf{w}_1^2\right)\\
\angle\left(\mathbf{h}_{12}\mathbf{w}_2^1\right) &= \angle\left(\mathbf{h}_{22}\mathbf{w}_2^2\right)\\
&\vdots\\
\angle\left(\mathbf{h}_{1N}\mathbf{w}_N^1\right) &= \angle\left(\mathbf{h}_{2N}\mathbf{w}_N^2\right)
\end{aligned}
\tag{2.19}
$$

In which $\angle(c)$ denotes the phase of the complex number c. Note that since $\mathbf{h}_{ij}$ is of size $1\times R$ and $\mathbf{w}_j^i$ is of size $R\times 1$, the term $\mathbf{h}_{ij}\mathbf{w}_j^i$ is a complex scalar. The problem with (2.19) is that it requires each relay station to know the precoding vectors and the channel coefficient of other relay. To overcome this problem we further impose that:

$$
\angle\left(\mathbf{h}_{11}\mathbf{w}_1^1\right) = \angle\left(\mathbf{h}_{21}\mathbf{w}_1^2\right)\angle\left(\mathbf{h}_{12}\mathbf{w}_2^1\right) = \cdots = \angle\left(\mathbf{h}_{2N}\mathbf{w}_N^2\right) = 0 \tag{2.20}
$$

Or:

$$
\angle\left(\mathbf{h}_{ij}\mathbf{w}_j^i\right) = 0, \qquad i = 1,2 \text{ and } j = 1,\cdots,N \tag{2.21}
$$

By imposing (2.21), relay ① no longer needs to know the channel coefficients of relay ②. This equation can be formulated as:

$$
\begin{cases}
\left(\mathbf{h}_{ij}\mathbf{w}_j^i\right)_I = 0 & i = 1,2 \text{ and } j = 1,\cdots,N\\
\left(\mathbf{h}_{ij}\mathbf{w}_j^i\right)_R > 0 & i = 1,2 \text{ and } j = 1,\cdots,N
\end{cases}
\tag{2.22}
$$

It is straight forward to derive following equations for the real and imaginary parts of $\mathbf{h}_{ij}\mathbf{w}_j^i$:

$$
\left(\mathbf{h}_{ij}\mathbf{w}_j^i\right)_R = \mathbf{h}_{ij_R}\mathbf{w}_{j_R}^i - \mathbf{h}_{ij_I}\mathbf{w}_{j_I}^i \tag{2.23}
$$

$$\left(\mathbf{h}_{ij}\mathbf{w}_j^i\right)_I = \mathbf{h}_{ij_R}\mathbf{w}_{j_I}^i + \mathbf{h}_{ij_I}\mathbf{w}_{j_R}^i \tag{2.24}$$

In this case, equation (2.21) can be expressed as:

These are a set of N linear scalar valued equations and a set of N linear scalar valued inequalities.

2.1.3 Power constraint

We assume that the relay RS_k is bounded to a maximum power of p_k. Transmission power of each relay can be written as:

$$P_k = \|\mathbf{x}_k\|^2 = \|\sum_{j=1}^{N} s_j \mathbf{w}_j^k\|^2 \leq p_k \qquad k = 1, 2 \tag{2.25}$$

By developing (2.25) we obtain:

$$\left(\sum_{i=1}^{N} s_i \mathbf{w}_i^k\right)^\dagger \cdot \left(\sum_{j=1}^{N} s_j \mathbf{w}_j^k\right) = \sum_{i=1}^{N}\sum_{j=1}^{N} s_i^* s_j \left(\mathbf{w}_i^k\right)^\dagger \cdot \mathbf{w}_j^k \leq p_k \qquad k = 1, 2 \tag{2.26}$$

where $(.)^*$ and $(.)^\dagger$ denote respectively the complex conjugate and the conjugate transpose. Again, if we write a complex number c as $c_R + jc_I$ we have:

$$\begin{aligned}
\left(\mathbf{w}_i^k\right)^\dagger \cdot \mathbf{w}_j^k &= \left(\mathbf{w}_{i_R}^k + j\mathbf{w}_{i_I}^k\right)^\dagger \cdot \left(\mathbf{w}_{j_R}^k + j\mathbf{w}_{j_I}^k\right) \\
&= \left(\mathbf{w}_{i_R}^k - j\mathbf{w}_{i_I}^k\right)^T \cdot \left(\mathbf{w}_{j_R}^k + j\mathbf{w}_{j_I}^k\right) \\
&= \mathbf{w}_{i_R}^{k^T} \cdot \mathbf{w}_{j_R}^k + j\mathbf{w}_{i_R}^{k^T} \cdot \mathbf{w}_{j_I}^k - j\mathbf{w}_{i_I}^{k^T} \cdot \mathbf{w}_{j_R}^k + \mathbf{w}_{i_I}^{k^T} \cdot \mathbf{w}_{j_I}^k \\
&= \mathbf{w}_{i_R}^{k^T} \cdot \mathbf{w}_{j_R}^k + \mathbf{w}_{i_I}^{k^T} \cdot \mathbf{w}_{j_I}^k + j\left(\mathbf{w}_{i_R}^{k^T} \cdot \mathbf{w}_{j_I}^k - \mathbf{w}_{i_I}^{k^T} \cdot \mathbf{w}_{j_R}^k\right)
\end{aligned} \tag{2.27}$$

and:

$$
\begin{aligned}
s_i^* s_j &= \left(s_{i_R} + j s_{i_I}\right)^* \left(s_{j_R} + j s_{j_I}\right) \\
&= \left(s_{i_R} - j s_{i_I}\right) \left(s_{j_R} + j s_{j_I}\right) \\
&= s_{i_R} s_{j_R} + j s_{i_R} s_{j_I} - j s_{i_I} s_{j_R} + s_{i_I} s_{j_I} \\
&= s_{i_R} s_{j_R} + s_{i_I} s_{j_I} + j \left(s_{i_R} s_{j_I} - s_{i_I} s_{j_R}\right)
\end{aligned} \tag{2.28}
$$

Thus (2.26) can be written as:

$$
\begin{aligned}
P_k = \quad & \sum_{i=1}^{N} \sum_{j=1}^{N} \left(s_{i_R} s_{j_R} + s_{i_I} s_{j_I}\right) \left(\mathbf{w}_{i_R}^{k^T} \cdot \mathbf{w}_{j_R}^{k} + \mathbf{w}_{i_I}^{k^T} \cdot \mathbf{w}_{j_I}^{k}\right) \\
& - \sum_{i=1}^{N} \sum_{j=1}^{N} \left(s_{i_R} s_{j_I} - s_{i_I} s_{j_R}\right) \left(\mathbf{w}_{i_R}^{k^T} \cdot \mathbf{w}_{j_I}^{k} - \mathbf{w}_{i_I}^{k^T} \cdot \mathbf{w}_{j_R}^{k}\right) \\
\leq p_k & \qquad\qquad k = 1, 2
\end{aligned} \tag{2.29}
$$

These are two quadratic inequalities bounding the value of precoding vectors.

2.1.4 Objective function and problem formulation

The system objective function is a function that will quantify the performance of the system. Choosing the objective function is the reflection of our choice of the system feature that we wish to improve while satisfying all of the above-mentioned constraints. The objective goal of the optimization may be to minimize the overall bit error rate (BER) or outage probability or to maximize the overall capacity of the system. We may also consider privileging one or some of the mobile stations over others, e.g. minimizing the BER of one mobile station while maintaining all other BERs below an acceptable threshold.

The problem with optimal solution is that we must be able to express the objective function (e.g. the system capacity) in terms of the problem variables and parameters, the precoding vectors and the channel coefficients in this case. Another possible method is to use suboptimal solutions instead. We may consider using the signal to noise ratio (SNR) at the destinations. The SNR is much simpler to express in terms of system variables. SNR at a given mobile station is simply derived from (2.5) as:

$$\mathrm{SNR}_{ij} = \frac{|\mathbf{h}_{ij}\mathbf{w}_j^i|^2}{\mathbb{E}\left\{|n_j|^2\right\}} = \frac{|\mathbf{h}_{ij}\mathbf{w}_j^i|^2}{N_0} \tag{2.30}$$

where $\mathbb{E}\left\{X\right\}$ represents the mathematical expectation of the random variable X and SNR_{ij} is the contribution of the ith relay in the signal to noise ratio at the jth mobile station. Here also, several strategies and possibilities may be considered. One possible approach is maximizing $\sum \mathrm{SNR}_{ij}$. Other possibility may be to impose the same SNR at all mobile stations and to maximize the signal to noise ratio at one mobile station (note that we may have multiple constraints, but only one objective function is conceivable). Other possible strategy is to impose a different SNR at each mobile station. For example we may use water-filling to assign a SNR proportional to equivalent channel at each mobile station.

Let us choose the following scenario as an example: we will make sure that all mobile stations benefit from the same SNR and then maximize one of them, say the first mobile station. In this case optimization problem for the first relay will be:

- Maximize $\mathbf{h}_{11}\mathbf{w}_1^1$
- subject to
  - Canceling interference

$$\mathbf{h}_{1j_R} \cdot \sum_{i \neq j} \left(s_{i_R}\mathbf{w}_{i_R}^1 - s_{i_I}\mathbf{w}_{i_I}^1 \right) - \mathbf{h}_{1j_I} \cdot \sum_{i \neq j} \left(s_{i_R}\mathbf{w}_{i_I}^1 + s_{i_I}\mathbf{w}_{i_R}^1 \right) = 0$$

$$\mathbf{h}_{1j_R} \cdot \sum_{i \neq j} \left(s_{i_R}\mathbf{w}_{i_I}^1 + s_{i_I}\mathbf{w}_{i_R}^1 \right) + \mathbf{h}_{1j_I} \cdot \sum_{i \neq j} \left(s_{i_R}\mathbf{w}_{i_R}^1 - s_{i_I}\mathbf{w}_{i_I}^1 \right) = 0$$

for all j which is $2N$ equations in total.

– Coherent addition

$$\begin{aligned} \mathbf{h}_{1j_R}\mathbf{w}^1_{j_I} + \mathbf{h}_{1j_I}\mathbf{w}^1_{j_R} &= 0, \qquad j = 1, \cdots, N \\ \mathbf{h}_{11_R}\mathbf{w}^1_{1_R} - \mathbf{h}_{11_I}\mathbf{w}^1_{1_I} &= \mathbf{h}_{12_R}\mathbf{w}^1_{2_R} - \mathbf{h}_{12_I}\mathbf{w}^1_{2_I} \\ &= \cdots = \mathbf{h}_{1N_R}\mathbf{w}^1_{N_R} - \mathbf{h}_{1N_I}\mathbf{w}^1_{N_I} > 0 \end{aligned}$$

Note that the lower equations are the result of imposing the same SNR for all mobile stations.

– power constraint

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N}(s_{i_R}s_{j_R} + s_{i_I}s_{j_I})\left(\mathbf{w}^{k^T}_{i_R}\cdot\mathbf{w}^k_{j_R} + \mathbf{w}^{k^T}_{i_I}\cdot\mathbf{w}^k_{j_I}\right) \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{N}(s_{i_R}s_{j_I} - s_{i_I}s_{j_R})\left(\mathbf{w}^{k^T}_{i_R}\cdot\mathbf{w}^k_{j_I} - \mathbf{w}^{k^T}_{i_I}\cdot\mathbf{w}^k_{j_R}\right) \le p_1 \end{aligned}$$

This is a non-linear problem that is not easy to solve. In the following section we will develop a solution to this problem using the Lagrange multipliers method.

2.2 Lagrange multipliers method for matrices and vectors

Named after Joseph Louis Lagrange, the method of Lagrange multipliers is a method for finding the local extrema of a function subject to constraints. For example if we want to maximize $f(x, y)$ subject to $g(x, y) = c$, we introduce a variable λ called Lagrange multiplier and study the Lagrange function $\Lambda(x, y, \lambda) = f(x, y) - \lambda\big(g(x, y) - c\big)$. It is proved that If $f(x, y)$ is an extrema of the original constrained problem, then there exists a λ such that

(x, y, λ) is a stationary point for the Lagrange function $\Lambda(x, y, \lambda)$, i.e. a point where all the partial derivatives of Λ are zero.

In this section we will develop a derivation of Lagrange multipliers method which fits the solution of problems similar to our case. When this method is elaborated we will come back at our specific problem to solve it using the developed method.

2.2.1 Background calculus prerequisites

Before we start developing our modified version of Lagrange multipliers method we will need some basic vectors and matrices calculus.

The first step in our approach will be to write every equation in a vector or matrix form. The following equation will be useful:

$$\sum_{i=1}^{N} a_i b_i \equiv \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} \quad \text{with } \mathbf{a} = [a_1 \cdots a_N]^T, \mathbf{b} = [b_1 \cdots b_N]^T \tag{2.31}$$

$$\sum_{i=1}^{N} a_i \mathbf{b}_i \equiv \mathbf{B} \cdot \mathbf{a} \quad \text{with } \mathbf{a} = [a_1 \cdots a_N]^T, \mathbf{B} = [\mathbf{b}_1 \cdots \mathbf{b}_N] \tag{2.32}$$

Once the optimizing equation and the criteria have been written in vectorial representation we will find the stationary points of the target function. We will have to derivate vectors and matrices with respect to vectors. Here are some of the most useful equations:

$$\frac{\partial\left(\mathbf{A}^T\mathbf{x}\right)}{\partial\mathbf{x}} = \mathbf{A} \tag{2.33}$$

$$\frac{\partial\left(\mathbf{x}^T\mathbf{A}\right)}{\partial\mathbf{x}} = \mathbf{A} \tag{2.34}$$

$$\frac{\partial\left(\mathbf{x}^T\mathbf{x}\right)}{\partial\mathbf{x}} = 2\mathbf{x} \tag{2.35}$$

$$\frac{\partial\left(\mathbf{x}^T\mathbf{A}\mathbf{x}\right)}{\partial\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{A}^T\mathbf{x} \tag{2.36}$$

Note that (2.35) is a special case of (2.36) where $\mathbf{A} = \mathbf{I}$.

Another complication of optimizing the proposed system in this chapter is that we must separate real and imaginary parts of complex equations. This is important because we have some constraints that require to set only the imaginary part of an equation to zero. It is thus preferred to break each complex variable into two real variables and deal with real variables.

Let us write a complex number c as $c_R + jc_I$. In this case the equation $a \cdot b = c$, with a, b, and c being complex scalars will be written as:

$$\begin{bmatrix} c_R \\ c_I \end{bmatrix} = \begin{bmatrix} a_R & -a_I \\ a_I & a_R \end{bmatrix} \begin{bmatrix} b_R \\ b_I \end{bmatrix} \tag{2.37}$$

Using the same principle, a complex matrix equation such as $\mathbf{Ab} = \mathbf{c}$ where $\mathbf{A}$ is a complex matrix and $\mathbf{b}$ and $\mathbf{c}$ are complex column vectors can be written as follows:

$$\begin{bmatrix} \mathbf{A}_R & -\mathbf{A}_I \\ \mathbf{A}_I & \mathbf{A}_R \end{bmatrix} \begin{bmatrix} \mathbf{b}_R \\ \mathbf{b}_I \end{bmatrix} = \begin{bmatrix} \mathbf{c}_R \\ \mathbf{c}_I \end{bmatrix} \tag{2.38}$$

Or:

$$\left(\begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix} \otimes \mathbf{A} \right)_R \left(\begin{bmatrix} 1 \\ -j \end{bmatrix} \otimes \mathbf{b} \right)_R = \left(\begin{bmatrix} 1 \\ -j \end{bmatrix} \otimes \mathbf{c} \right)_R \tag{2.39}$$

where $\otimes$ denotes the Kronecker product defined as follows: If $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{B}$ is a $p \times q$ matrix, then the Kronecker product $\mathbf{A} \otimes \mathbf{B}$ is the $mp \times nq$ block matrix such that:

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\mathbf{B} & \cdots & a_{mn}\mathbf{B} \end{bmatrix}$$

For the further simplicity of the application intended in this work, we prefer to use (2.40) instead of (2.39) which leads to the same equations:

$$\left(\mathbf{A}\otimes\begin{bmatrix}1 & j\\ -j & 1\end{bmatrix}\right)_R\left(\mathbf{b}\otimes\begin{bmatrix}1\\ -j\end{bmatrix}\right)_R=\left(\mathbf{c}\otimes\begin{bmatrix}1\\ -j\end{bmatrix}\right)_R \tag{2.40}$$

While (2.40) and (2.40) result in same set of equations, (2.40) has the advantage of having the real and imaginary parts of an element of a matrix next together, which simplifies the practical programming of the system.

We will now start to develop our method using some examples.

2.2.2 Examples

We will use two examples to demonstrate the vectorial use of Lagrange multipliers method. Each example is solved using two approaches: the conventional method and the method using vectors and matrices.

Example 1. Let us consider that we want to minimize the function $f(x,y) = x^2 + y^2$ respecting the constraint $g(x,y) = x + y = 2$.

Conventional method. We define $\Lambda(x,y,\lambda) = f(x,y) + \lambda g(x,y)$ where λ is the introduced Lagrange multiplier. The solution is $\overrightarrow{\nabla\Lambda} = \vec{0}$ which yields:

$$\frac{\partial\Lambda}{\partial x} = 2x + \lambda = 0 \tag{2.41}$$

$$\frac{\partial\Lambda}{\partial y} = 2y + \lambda = 0 \tag{2.42}$$

$$\frac{\partial\Lambda}{\partial\lambda} = x + y - 2 = 0 \tag{2.43}$$

Which has the unique solution $x = y = -\lambda/2 = 1$ or $x^2 + y^2 = 2$. □

Vectorial method. Let us define $\mathbf{a} = [1 \quad 1]^T$, $\mathbf{u} = [x \quad y]$, $f(\mathbf{u}) = \mathbf{u}^T\mathbf{u}$, and $g(\mathbf{u}) = \mathbf{a}^T\mathbf{u} - 2$. The problem reduces to minimizing $f(\mathbf{u})$ respecting $g(\mathbf{u}) = 0$.
We define $\Lambda(\mathbf{u}, \lambda) = f(\mathbf{u}) + \lambda g(\mathbf{u})$ and set $\overrightarrow{\nabla\Lambda} = \vec{0}$:

$$\Lambda(\mathbf{u}, \lambda) = f(\mathbf{u}) + \lambda g(\mathbf{u}) = \mathbf{u}^T\mathbf{u} + \lambda\left(\mathbf{a}^T\mathbf{u} - 2\right) \tag{2.44}$$

$$\begin{aligned}
\overrightarrow{\nabla\Lambda} &= \begin{bmatrix} \dfrac{\partial\Lambda}{\partial\mathbf{u}} \\ \hdashline \dfrac{\partial\Lambda}{\partial\lambda} \end{bmatrix} = \begin{bmatrix} 2\mathbf{I_2}\mathbf{u} + \lambda\mathbf{a} \\ \hdashline \mathbf{a}^T\mathbf{u} - 2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hdashline 0 \end{bmatrix} \\
\Rightarrow & \left[\begin{array}{c:c} 2\mathbf{I}_2 & \mathbf{a} \\ \hdashline \mathbf{a}^T & 0 \end{array}\right] \begin{bmatrix} \mathbf{u} \\ \hdashline \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hdashline 2 \end{bmatrix} \\
\Rightarrow & \left[\begin{array}{cc:c} 2 & 0 & 1 \\ 0 & 2 & 1 \\ \hdashline 1 & 1 & 0 \end{array}\right] \begin{bmatrix} x \\ y \\ \hdashline \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \hdashline 2 \end{bmatrix}
\end{aligned} \tag{2.45}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \tag{2.46}$$

Which is obviously the same result. $\square$

It is easy to perceive that the final equations will be linear, if and only if the constraints and the objective functions are respectively linear and quadratic functions of variables.

Example 2. Minimize the quantity $u_1^2+u_2^2+u_3^2+u_4^2$ subject to constraints $u_1+u_2=4$ and $2u_1+u_3-u_4=0$.

Conventional method. We will try to maximize:

$$f(u_1,u_2,u_3,u_4)=u_1^2+u_2^2+u_3^2+u_4^2$$

respecting:

$$\begin{aligned} g_1(u_1,u_2)=u_1+u_2-4=0 \qquad \text{and} \\ g_2(u_1,u_3,u_4)=2u_1+u_3-u_4=0. \end{aligned}$$

We define

$$\begin{aligned} \Lambda(u_1,u_2,u_3,u_4,\lambda_1,\lambda_2) = \quad & f(u_1,u_2,u_3,u_4)+\lambda_1 g_1(u_1,u_2)+\lambda_2 g_2(u_1,u_3,u_4) \\ = \quad & u_1^2+u_2^2+u_3^2+u_4^2 \\ & +\lambda_1\left(u_1+u_2-4\right) \\ & +\lambda_2\left(2u_1+u_3-u_4\right) \end{aligned} \tag{2.47}$$

$\overrightarrow{\nabla\Lambda}=\vec{0}$ imposes that:

$$\frac{\partial\Lambda}{\partial u_1}=2u_1+\lambda_1+2\lambda_2=0 \tag{2.48}$$
$$\frac{\partial\Lambda}{\partial u_2}=2u_2+\lambda_1=0 \tag{2.49}$$
$$\frac{\partial\Lambda}{\partial u_3}=2u_3+\lambda_2=0 \tag{2.50}$$
$$\frac{\partial\Lambda}{\partial u_4}=2u_4-\lambda_2=0 \tag{2.51}$$

$$\frac{\partial \Lambda}{\partial \lambda_1} = u_1 + u_2 - 4 = 0 \tag{2.52}$$

$$\frac{\partial \Lambda}{\partial \lambda_1} = 2u_1 + u_3 - u_4 = 0 \tag{2.53}$$

The set of equations (2.48) to (2.53) has the unique answer $u_1 = 1$, $u_2 = 3$, and $u_4 = -u_3 = 1$. □

Vectorial method. Let us define:

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T \tag{2.54}$$

$$\boldsymbol{\lambda} = [\lambda_1 \quad \lambda_2]^T \tag{2.55}$$

$$\mathbf{a} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & -1 \end{bmatrix}^T \tag{2.56}$$

$$\mathbf{c} = [4 \quad 0]^T \tag{2.57}$$

$$f(\mathbf{u}) = \mathbf{u}^T\mathbf{u} \tag{2.58}$$

$$\mathbf{g}(\mathbf{u}) = \mathbf{a}^T\mathbf{u} \tag{2.59}$$

$$\Lambda(\mathbf{u}, \boldsymbol{\lambda}) = f(\mathbf{u}) + \boldsymbol{\lambda}^T (\mathbf{g}(\mathbf{u}) - \mathbf{c}) = \mathbf{u}^T\mathbf{u} + \boldsymbol{\lambda}^T \left(\mathbf{a}^T\mathbf{u} - \mathbf{c}\right) \tag{2.60}$$

Using the matrix derivation we have:

$$\frac{\partial \Lambda(\mathbf{u}, \boldsymbol{\lambda})}{\partial \mathbf{u}} = \frac{\partial}{\partial \mathbf{u}}\mathbf{u}^T\mathbf{u} + \frac{\partial}{\partial \mathbf{u}}\boldsymbol{\lambda}^T\mathbf{a}^T\mathbf{u} = 2\mathbf{u} + \left(\boldsymbol{\lambda}^T\mathbf{a}^T\right)^T = 2\mathbf{I_4}\mathbf{u} + \mathbf{a}\boldsymbol{\lambda} \tag{2.61}$$

Now we can write:

$$\overrightarrow{\nabla}\Lambda = \begin{bmatrix} \frac{\partial \Lambda}{\partial \mathbf{u}} \\ \text{- - - -} \\ \frac{\partial \Lambda}{\partial \boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} 2\mathbf{I_4}\mathbf{u} + \mathbf{a}\boldsymbol{\lambda} \\ \text{- - - - - - - - -} \\ \mathbf{a}^T\mathbf{u} - \mathbf{c} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \text{- - -} \\ \mathbf{0} \end{bmatrix} \tag{2.62}$$

$$\Rightarrow \left[\begin{array}{c:c} 2\mathbf{I}_4 & \mathbf{a} \\ \hdashline \mathbf{a}^T & \mathbf{0} \end{array}\right] \left[\begin{array}{c} \mathbf{u} \\ \hdashline \boldsymbol{\lambda} \end{array}\right] = \left[\begin{array}{c} \mathbf{0} \\ \hdashline \mathbf{c} \end{array}\right] \tag{2.63}$$

$$\Rightarrow \left[\begin{array}{cccc:cc} 2 & 0 & 0 & 0 & 1 & 2 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & -1 \\ \hdashline 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & -1 & 0 & 0 \end{array}\right] \left[\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \hdashline \lambda_1 \\ \lambda_2 \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ \hdashline 4 \\ 0 \end{array}\right] \tag{2.64}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 & 1 & 2 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 1 \\ -6 \\ 2 \end{bmatrix} \tag{2.65}$$

which is the same result as before. □

2.2.3 General formulation

We are now ready to introduce the step by step algorithm to solve an optimization problem based on vectorial application of Lagrange multipliers method:

- Make sure that the optimizing problem meets the following criteria:

  - the objective function is a quadratic function of N variables x_1 to x_N
  - all of the M constraints are linear functions of the variables

- Express all the constraints in a single vectorial form: $\mathbf{A}_{M\times N}\mathbf{x}_{N\times 1} = \mathbf{c}_{M\times 1}$ where $\mathbf{x} = [\begin{array}{cccc} x_1 & x_2 & \cdots & x_N \end{array}]^T$.

- Form the intermediate matrix $\mathcal{A}_{(M+N)\times(M+N)} = \left[\begin{array}{c:c} 2\mathbf{I}_N & \mathbf{A}^T \\ \hdashline \mathbf{A} & \mathbf{0}_M \end{array}\right]$

- Form the intermediate vector $\mathbf{d}_{(M+N)\times 1} = \left[\begin{array}{cc} 0_{1\times N} & \mathbf{c}^T \end{array}\right]^T$

- Solve the system $\mathcal{A}\mathbf{u} = \mathbf{d}$ to find the $N+M$ unknown coefficient of $\mathbf{u}$.

- Take the first N elements of $\mathbf{u}$ for the vector $\mathbf{x}$.

Note that if the variables and consequently the equations are complex-valued, we can use (2.40) to separate them into $2N$ real-valued variables.

2.3 Optimizing the performance of proposed system

In this section we will use the method elaborated in section 2.2 to optimize the problem posed under 2.1.4.

2.3.1 Forming the equations

It is obvious that the problem discussed under 2.1.4 must be modified to fit the criteria given in 2.2.3. The reason is that the problem in 2.1.4 involves a quadratic constraint.

Note that when all other constraints are verified, maximizing SNR with respect to a fixed transmission power reduces to minimizing the transmission power while maintaining a desired signal to noise ratio at the receiver. The only difference is a scaling factor that will amplify (or attenuate) the precoding vectors to the desired power level. Since all other constraints are linear, this scaling will not cause the precoding vectors fail to satisfy a constraint that they had satisfied before being scaled.

Now that the problem meets the criteria of 2.2.3, we will proceed with writing the equations of the system in vectorial form. We will use these notations:

$$\mathbf{H}_i = \left[\begin{array}{c|c|c|c} \mathbf{h}_{i1}{}^T & \mathbf{h}_{i2}{}^T & \dots & \mathbf{h}_{iN}{}^T \end{array}\right]^T_{N\times R} \qquad ,i=1,2 \tag{2.66}$$

$$\boldsymbol{\mathcal{W}}_i = \left[\begin{array}{cccc} \mathbf{w}_1^{i\,T} & \mathbf{w}_2^{i\,T} & \dots & \mathbf{w}_N^{i\,T} \end{array}\right]^T_{NR\times 1} \qquad ,i=1,2 \tag{2.67}$$

<u>Interference cancellation</u>. Equation (2.14) maybe rewritten in vector form using (2.66) and (2.67):

$$\mathbf{A}_{i1}\boldsymbol{\mathcal{W}}_i = \mathbf{0} \quad \text{with } \mathbf{A}_{i1} = \left(\mathbf{s}^T \otimes \mathbf{1}_{N\times 1} - \operatorname{diag}(\mathbf{s})\right) \circledast \mathbf{H}_i \tag{2.68}$$

where $\otimes$ and $\circledast$ respectively denote Kronecker and row-wise Kronecker products[4] and $\mathbf{s} = [s_1, s_2, \cdots, s_N]^T$. Equation (2.68) is a set of N linear complex equations for each relay that guarantee the cancellation of intersymbol interferences. In order to obtain the equivalent real equations we use the method introduced under 2.2.1. Using (2.40) we will obtain a set of $2N$ real equations for each relay station:

$$\left(\mathbf{A}_{i1} \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}\right)_R \left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix}\right)_R = \mathbf{0}_{2N\times 1} \tag{2.69}$$

[4]Row-wise Kronecker product of matrices **A** and **B** is a matrix each line of which is the Kronecker product of corresponding lines in **A** and **B**.

This equation has two rows per mobile station, the first row is the real part of (2.68) and the second row is the imaginary part of (2.68). We will rewrite this equation to simplify future developments.

$$\hat{\mathbf{A}}_{i1}\hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{0}_{2N\times 1} \tag{2.70}$$

with:

$$\hat{\mathbf{A}}_{i1} = \left(\mathbf{A}_{i1} \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}\right)_R \qquad \text{and} \qquad \hat{\boldsymbol{\mathcal{W}}}_i = \left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix}\right)_R \tag{2.71}$$

Note that $\hat{\mathbf{A}}_{i1}$ and $\hat{\boldsymbol{\mathcal{W}}}_i$ are only composed of real values.

<u>Coherent addition</u>. According to 2.1.2, in order to guarantee the coherent addition, we must have:

$$\mathbf{h}_{1j_R}\mathbf{w}^1_{j_I} + \mathbf{h}_{1j_I}\mathbf{w}^1_{j_R} = 0, \qquad j = 1, \cdots, N \tag{2.72}$$

The criterion bounding the real parts to a positive value is no longer necessary because we will fix a given value for $\left(\mathbf{h}_{ij}\mathbf{w}^i_j\right)_R$. The criteria for the imaginary part of the received signal in (2.72) can be written in compact form as:

$$\mathbf{A}_{i2}\boldsymbol{\mathcal{W}}_i = 0 \quad \text{with } \mathbf{A}_{i2} = \left(\mathbf{I}_N \circledast \mathbf{H}_i\right), \qquad i = 1, 2 \tag{2.73}$$

with $\circledast$ denoting again the row-wise Kronecker product. Using (2.40), this equation can be written as:

$$\left(\mathbf{A}_{i2} \otimes \begin{bmatrix} -j & 1 \end{bmatrix}\right)_R \left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix}\right)_R = \mathbf{0}_{N\times 1} \tag{2.74}$$

$$\Rightarrow \qquad \hat{\mathbf{A}}_{i2}\hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{0}_{N\times 1} \tag{2.75}$$

with $\hat{\mathbf{A}}_{i2} = \left(\mathbf{A}_{i2} \otimes \begin{bmatrix} -j & 1 \end{bmatrix}\right)_R$. Since (2.75) is only about the imaginary part of $\mathbf{h}_{ij} \cdot \mathbf{w}^i_j$, the matrix $\hat{\mathbf{A}}_{i2}$ has a single row per mobile station.

Fixing the SNR. We will consider the case where the same SNR is required for all mobile stations. From (2.5), we know that the contribution of the first relay in the signal received by the jth mobile station is $\mathbf{h}_{1j}\mathbf{w}_j^1$. As a result the real part of this contribution may be written as $\mathbf{h}_{1j_R}\mathbf{w}_{j_R}^1 - \mathbf{h}_{1j_I}\mathbf{w}_{j_I}^1$. Imposing the same SNR for all mobile stations requires that:

$$\mathbf{h}_{11_R}\mathbf{w}_{1_R}^1 - \mathbf{h}_{11_I}\mathbf{w}_{1_I}^1 = \mathbf{h}_{12_R}\mathbf{w}_{2_R}^1 - \mathbf{h}_{12_I}\mathbf{w}_{2_I}^1 = \cdots = \mathbf{h}_{1N_R}\mathbf{w}_{N_R}^1 - \mathbf{h}_{1N_I}\mathbf{w}_{N_I}^1 = 1 \qquad (2.76)$$

Note that the real parts are set equal so that all mobile stations benefit from the same SNR. We could omit these constraints if we would like to have different SNRs at different MSs. Knowing that we will later scale the precoding vectors to the available relay power, the '1' at the right hand side of (2.76) is just an arbitrary value and could be replaced by any other positive number. This equation can be rewritten as:

$$\hat{\mathbf{A}}_{i3}\hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{1}_{N\times 1} \qquad (2.77)$$

with

$$\hat{\mathbf{A}}_{i3} = \left((\mathbf{I}_N \circledast \mathbf{H}_i) \otimes \begin{bmatrix} 1 & j \end{bmatrix} \right)_R, \qquad i = 1,2 \qquad (2.78)$$

$$\text{and} \qquad \hat{\boldsymbol{\mathcal{W}}}_i = \left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix} \right)_R, \qquad i = 1,2 \qquad (2.79)$$

Note that $\hat{\mathbf{A}}_{i3}$ in (2.77) has one row per mobile station.

$\square$

Now the optimization problem has been simplified to:

- minimize the transmission power $\hat{\boldsymbol{\mathcal{W}}}_i^T \hat{\boldsymbol{\mathcal{W}}}_i$

- subject to

$$\hat{\mathbf{A}}_i \hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{c} \tag{2.80}$$

with $\hat{\mathbf{A}}_i = \begin{bmatrix} \hat{\mathbf{A}}_{i1}^T & \hat{\mathbf{A}}_{i2}^T & \hat{\mathbf{A}}_{i3}^T \end{bmatrix}^T_{4N \times 2RN}$
and $\mathbf{c} = \begin{bmatrix} \mathbf{0}_{1\times 2N}^T & \mathbf{0}_{1\times N}^T & \mathbf{1}_{1\times N}^T \end{bmatrix}^T_{4N\times 1}$.

2.3.2 Solving the system and simulation results

In order to solve the optimization problem in (2.80) using the method introduced under 2.2.3 it suffices to write for each relay $(i = 1, 2)$, the equation to be solved, $\mathbf{A}_i \mathbf{u}_i = \mathbf{b}$, with

$$\mathbf{A}_i = \left[\begin{array}{c:c} 2\mathbf{I}_{2N} & \hat{\mathbf{A}}_i^T \\ \hdashline \hat{\mathbf{A}}_i & \mathbf{0} \end{array}\right], \ \mathbf{u}_i = \left[\begin{array}{c} \hat{\boldsymbol{\mathcal{W}}}_i \\ \hdashline \boldsymbol{\lambda} \end{array}\right], \ \mathbf{b} = \left[\begin{array}{c} \mathbf{0}_{2N\times 1} \\ \hdashline \mathbf{c} \end{array}\right] \tag{2.81}$$

and find the solution as $\mathbf{u}_i = \mathbf{A}_i^{-1}\mathbf{b}$ and take the first $2N$ elements of $\mathbf{u}_i$ for $\hat{\boldsymbol{\mathcal{W}}}_i$.

Since $\hat{\boldsymbol{\mathcal{W}}}_i$ is the $\boldsymbol{\mathcal{W}}_i$ with the real and imaginary parts separated, we can combine every two consecutive elements of $\hat{\boldsymbol{\mathcal{W}}}_i$ into a complex number and form $\boldsymbol{\mathcal{W}}_i$. We can then find precoding vectors $\mathbf{w}_j^i$ with respect to (2.67). We will then scale the precoding vectors to available relay power: If $\boldsymbol{\mathcal{W}}'_i$ is the set of precoding vectors which minimizes the transmission power subject to a desired SNR value, the scaled precoding vector can be easily calculated by $\boldsymbol{\mathcal{W}}_i = \boldsymbol{\mathcal{W}}'_i \sqrt{\frac{P_i}{|\boldsymbol{\mathcal{W}}'^H_i \boldsymbol{\mathcal{W}}'_i|}}$.

Note that in this case we have $4N$ equations and $2RN$ unknown precoding coefficients. As a result the system needs to satisfy $R \geq 2$ to be solvable. In general when the same SNR is required at mobile stations, and when the relays are independent (i.e. each relay knows only its own CSI), the relay station antennas must outnumber the mobile stations.

2.4 Conclusion

In this chapter a modified version of the Lagrange multiplier method applicable to the vectors and matrices was developed. It has been demonstrated that if certain criteria are met, the resulting equation may be solved using linear matrix operations. The proposed method was only used for a case of two relay stations, but it can be easily generalized to the case of arbitrary relay station numbers. Since the system calculates the precoding vectors of each relay independently, there is absolutely no difference when the number of relay stations is increased. The equations (2.80) and (2.81) are still applicable for i from 1 to L where L is the number of mobile stations.

The main advantage of the proposed method is its flexibility. A large variety of optimization constraints and objectives may be considered. The only restriction is that the system must minimize the transmission power while satisfying a set of linear constraints. It means that the constraints must be expressed as a linear combination of the precoding vectors coefficients. Any linear constraints on the SNR at the destination is applicable. For instance we may give explicit values for all MSs or make the priority on some or one of the MSs. We can also use SNR values that are directly proportional to the channel coefficients.

The main inconvenience of the proposed system is that due to the nature of the proposed solution, no or very little theoretical prediction of system performance may be produced. For some special cases like when the same SNR is imposed for all destinations some analytical predictions may be envisaged[5], but for general cases there is no possible BER or system capacity calculations.

[5]This issue will be covered in the following chapters

Chapter 3

Calculating Precoding Vectors Using Pseudo Inverse

This chapter covers a special case of the previous chapter for which the same signal to noise ratio (SNR) is imposed at each mobile station. The multiple-input multiple-output (MIMO) systems have proved to be very promising in order to overcome the random fading attenuation of the channel and to obtain reliable point to point communications. MIMO systems may be used to both extend the coverage of existing cellular networks and to improve the quality of wireless links. However the relative closeness of the antennas make the MIMO channels correlated. Therefore in a large scale fading case or shadowing, all the MIMO channels can experience deep fading. One promising solution is to use distributed MIMO [49, 50] or cooperative systems which separate MIMO elements in space or time and therefore making MIMO channels independent. Since the proposition of this technique, it has been a very hot research area and a large number of works addressed the problem deeply.

The first hop of the proposed system is the link between the BS and the RSs. We are assuming that a good high speed link exists between the base station and relay stations. This assumption is quite realistic since the BS and RSs are considered to be fixed. For example a good optical link can be used between BS and RSs. Furthermore not only error correcting codes may be used in relays, cyclic redundancy codes (CRC) codes may be also used at relays to detect any errors and to ask the BS to resend the defected information. When the data is decoded correctly at the RSs, the noise effect of this link is removed [9]. As a result, the first hop of our channel is assumed ideal.

In the second hop of the system, the relays will transmit the decoded data to the MSs. The problem is now to make the relays cooperate to eliminate the multiple access interference and to maximize the signal to noise ratio at each MS. To achieve this purpose, a kind of beamforming is used in relays to make the intended signal for the jth MS to be sum up coherently and all the other signals to be canceled out at this MS. In the proposed scheme the beamforming and multiple access interference (MAI) cancellation are performed by means of some precoding vectors at RSs.

The rest of this chapter is organized as follows: The system model is explained under Section 3.1. The precoding vectors are calculated in Section 3.2. Two cases are studied based on whether or not a given relay has the channel state information (CSI) of other relays. The system performance is theoretically evaluated under Section 3.3. The pdf of the SNR for the case of two mobiles and arbitrary relay numbers is analytically calculated under 3.3.1. Using this pdf, the diversity gain is also derived. The diversity gain of the system is discussed under 3.3.2. For the case of more mobile stations, the SNR distribution is approximated by a mixture of Nakagami laws. This approximation, leading to the calculation of SEP is covered by 3.3.3. Section 3.4 provides some simulation results and confirms the theoretical predictions of the system performance.

3.1 System model

As depicted in Figure 3.1, the system is composed of one base station with M antennas, which sends N symbols s_1 to s_N respectively to N mobile stations MS_1 to MS_N via L not-moving relays each with R antennas[1]. A two hop communication scheme is considered. In the first hop, the base station sends the signal to the relays. The relays will then decode the received signal and multiply it by some precoding vectors before transmitting them to mobile stations in the second hop. The BS to RS links are considered to be error-free. In the remainder of this chapter we will focus on the second hop of the communication where L relays cooperate in sending each of the N data symbols to their intended mobile stations. The link from the ith relay to the jth mobile station is assumed to be a flat fading Rayleigh

[1] **Note that R denotes the number of antennas in a relay and not the number of antennas which is denoted by L**

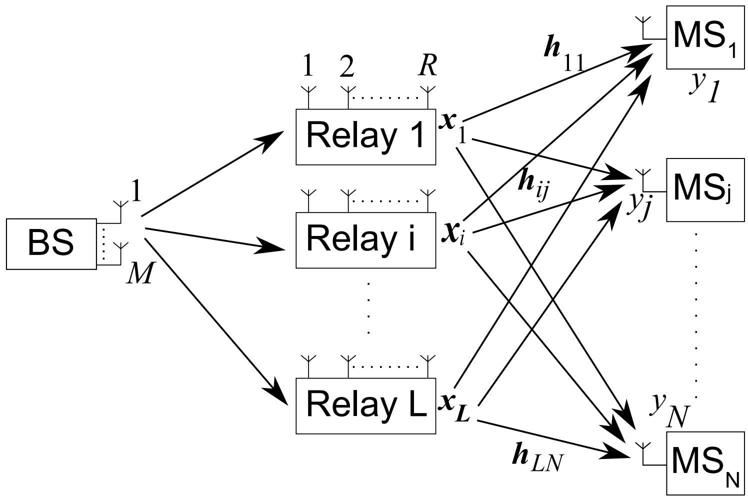


Figure 3.1: System model

channel $\mathbf{h}_{ij} \sim \mathcal{CN}(0, \mathbf{I}_R)$ of size $1 \times R$. The channel state information is assumed to be known at the transmitter. The receivers however do not use any information of the channel.

We define $\mathbf{s} = [s_1, s_2, \cdots s_N]^T$, with s_1 to s_N being N M-ary Phase Shift Keying (M-PSK) unitary symbols (i.e $|s_j|^2 = 1$). Each relay RS_i will then multiply its received signals by a set of precoding vectors $\mathbf{w}_j^i$ each of size $R \times 1$:

$$\mathbf{x}_i = \sum_{j=1}^{N} s_j \mathbf{w}_j^i \quad , \qquad i = 1, \cdots, L \tag{3.1}$$

The relays will then send these signals to mobile stations, the received signal at MS_j (the jth mobile station) is given by:

$$\begin{aligned} y_j &= \sum_{i=1}^{L} \mathbf{h}_{ij} \cdot \mathbf{x}_i + n_j \\ &= \sum_{i=1}^{L} \left(\mathbf{h}_{ij} \cdot \sum_{j'=1}^{N} s_{j'} \mathbf{w}_{j'}^i \right) + n_j \quad , j = 1, \cdots, N \end{aligned} \tag{3.2}$$

with $n_j \sim \mathcal{CN}(0, N_0)$ being a white additive Gaussian noise. This equation may be rewritten as:

$$\mathbf{y} = \left(\sum_{i=1}^{L} \mathbf{H}_i \mathbf{W}_i \right) \mathbf{s} + \mathbf{n} \tag{3.3}$$

with $\mathbf{y} = [y_1, y_2, \cdots y_N]^T$ and

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{h}_{i1} \\ \mathbf{h}_{i2} \\ \vdots \\ \mathbf{h}_{iN} \end{bmatrix}_{N \times R} \qquad , i = 1, \cdots, L \tag{3.4}$$

and

$$\mathbf{W}_i = \left[\begin{array}{c|c|c|c} \mathbf{w}_1^i & \mathbf{w}_2^i & \ldots & \mathbf{w}_N^i \end{array} \right]_{R \times N} \qquad , i = 1, \cdots, L \tag{3.5}$$

Since we want the MAI to be canceled in (3.3), that is to say we want y_j to depend only on s_j, we must guarantee that $\left(\sum_{i=1}^{L} \mathbf{H}_i \mathbf{W}_i \right)$ be a diagonal matrix.

Now we define $\mathbf{H}$ and $\mathbf{W}$ as:

$$\begin{aligned} \mathbf{H} &= \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \cdots & \mathbf{H}_L \end{bmatrix} \qquad (3.6) \\ &= \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{21} & \cdots & \mathbf{h}_{L1} \\ \mathbf{h}_{12} & \mathbf{h}_{22} & \cdots & \mathbf{h}_{L2} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{h}_{1N} & \mathbf{h}_{2N} & \cdots & \mathbf{h}_{LN} \end{bmatrix}_{N \times RL} \end{aligned}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \vdots \\ \mathbf{W}_L \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^1 & \mathbf{w}_2^1 & \cdots & \mathbf{w}_N^1 \\ \mathbf{w}_1^2 & \mathbf{w}_2^2 & \cdots & \mathbf{w}_N^2 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{w}_1^L & \mathbf{w}_2^L & \cdots & \mathbf{w}_N^L \end{bmatrix}_{RL \times N} \tag{3.7}$$

Using (3.6) and (3.7) we can rewrite (3.3):

$$\mathbf{y} = \mathbf{HWs} + \mathbf{n} \tag{3.8}$$

We note that $\mathbf{HW}$ is an N by N matrix that we require to be diagonal.

3.2 Calculation of precoding vectors

In this section we will calculate the precoding vectors that allow a coherent detection and eliminate the multiple access interference at the mobile stations. As stated above the precoding vectors must guarantee that each MS receives only its intended information. To do this we will calculate the precoding vectors such that $\mathbf{HW}$ be a diagonal matrix. The jth element on the diagonal of $\mathbf{HW}$ determines the signal to noise ratio of the jth MS. In this chapter we focus on the case where all MSs have the same SNR (i.e $\mathbf{HW} = g\mathbf{I}_N$ with g being a positive real number indicating the system gain). The case where different SNRs may be fixed for each MS is addressed in the previous chapter, where due to the complex set of equations no theoretical prediction of the system performance is provided. Two scenarios may be considered. In the first scenario we will assume that the complete CSI is known at each RS. It means that each relay knows not only its own channel to the MSs, but also the channels between other relays and MSs. In the second scenario we assume that each relay knows only its own CSI. Second scenario is more flexible in the way that it does not require relays to have the CSI knowledge of other relays. In this case each relay can calculate its relative precoding vectors independently.

3.2.1 Case I: Complete CSI for all relays

When all relays have the complete CSI, mathematically the system would be the same as one relay with RL antennas. However having L relays each with R antennas as presented here will have the advantage of preventing the entire system to fail if one of the relays (i.e. all antennas of it) falls in a deep fading region. As stated before, in order to cancel the MAI while imposing equal received power for all MSs, we must find a $\mathbf{W}$ that satisfies:

$$\mathbf{H}_{N\times LR}\mathbf{W}_{LR\times N} = g\mathbf{I}_N \tag{3.9}$$

with g denoting the system gain which is determined by available relaying power. For simplicity we will first solve the system for $g = 1$ and find $\mathbf{W}'$ (i.e. $\mathbf{H}_{N\times LR}\mathbf{W}'_{LR\times N} = \mathbf{I}_N$), then we will scale the answer to available power to find $\mathbf{W}$. The solution to (3.9) exists if and only if $\mathbf{H}$ has full row rank (i.e. $RL \geq N$). If such is the case, $\mathbf{W}'$ will be the Moore-Penrose pseudo inverse of $\mathbf{H}$:

$$\mathbf{W} = \mathbf{H}^{\dagger}\left(\mathbf{H}\mathbf{H}^{\dagger}\right)^{-1} \tag{3.10}$$

When the total number of relay antennas (LR) is equal to the number of mobile stations (N), pseudo inverse reduces to an expensive way to calculate inverse and we will have $\mathbf{W}' = \mathbf{H}^{-1}$.

Now we will scale the precoding vectors to available transmission power P:

$$\mathbf{W} = \frac{\sqrt{P}\,\mathbf{W}'}{\|\mathbf{W}'\|} \tag{3.11}$$

By substituting (3.10) and (3.11) in (3.8) we will obtain:

$$\mathbf{y} = \frac{\sqrt{P}}{\|\mathbf{H}^{\dagger}\left(\mathbf{H}\mathbf{H}^{\dagger}\right)^{-1}\|}\mathbf{s} + \mathbf{n} \tag{3.12}$$

It is obvious from (3.12) that the jth element of $\mathbf{y}$ depends only on s_j and not on the other data symbols. As a result the system can be considered as N parallel channels each one transmitting a symbol s_j.

3.2.2 Case II: Independent relays

When each relay knows only the channel coefficients of the links between itself and the MSs, the precoding vectors of the ith relay (i.e. $\mathbf{W}_i$) must be calculated only as a function of the channel between RS_i and MSs (i.e. $\mathbf{H}_i$). That is to say for all $i = 1 \cdots L$, $\mathbf{W}_i$ must satisfy:

$$\mathbf{H}_{i(N\times R)}\mathbf{W}_{i(R\times N)} = g_i\mathbf{I}_N \quad , \qquad i = 1, \cdots, L \tag{3.13}$$

The equation in (3.13) has an answer if $\mathbf{H}_i$ has full row rank, thus the number of antennas in each relay must be greater than or equal to the number of receivers ($R \geq N$). Of course this criterion is much more difficult to obtain than its counterpart in the first case, but on the other hand relays do not need to be inter-connected to each other. Precoding vectors of the ith relay are given by:

$$\mathbf{W}_i = \frac{\sqrt{P_i}\,\mathbf{W}'_i}{\|\mathbf{W}'_i\|} \quad , \qquad i = 1, \cdots, L \tag{3.14}$$

with P_i being available transmission power in RS_i such that $\sum P_i = P$ and

$$\mathbf{W}_i = \mathbf{H}_i^{\dagger}\left(\mathbf{H}_i\mathbf{H}_i^{\dagger}\right)^{-1} \quad , \qquad i = 1, \cdots, L \tag{3.15}$$

By substituting (3.14) and (3.15) in (3.8) we obtain

$$\mathbf{y} = \sum_{i=1}^{L} \mathbf{y}_i + \mathbf{n} \tag{3.16}$$

In this equation, $\mathbf{y}_i$ is the contribution of the ith relay in the received signal and is given by:

$$\mathbf{y}_i = \frac{\sqrt{P_i}}{\|\mathbf{H}_i^\dagger \left(\mathbf{H}_i \mathbf{H}_i^\dagger\right)^{-1}\|}\mathbf{s} \quad , \qquad i = 1, \cdots, L \tag{3.17}$$

Note that $\mathbf{y}_i$ (printed in bold letters) in (3.16) and (3.17) is different from y_j (printed in italic letters) in (3.2). y_j is a single symbol received by the jth mobile station while $\mathbf{y}_i$ is an $N \times 1$ vector that denotes the contribution of the ith relay in the received signal of all MSs. Once again, (3.16) and (3.17) show clearly that MAI has been canceled out and the system can be seen as N separate channels.

3.3 System performance

In this section we will evaluate the performance of the proposed scheme. We will develop an analytic expression for the diversity gain for the case of two mobile stations (i.e $N = 2$). For higher number of mobile stations a semi-analytic approach will be presented.

3.3.1 Analytic calculation of the diversity order for $N = 2$

Let us consider the instantaneous SNR of a given telecommunication link as a random variable $\gamma = \overline{\gamma}\beta$, where $\overline{\gamma}$ is a deterministic positive quantity, and β is a channel-dependent nonnegative random variable with its pdf denoted by $f(\beta)$. Suppose that $f(\beta)$ can be approximated by a single polynomial term for $\beta \to 0^+$ as $f(\beta) = a\beta^t + o(\beta^{t+\epsilon})$, where $\epsilon > 0$, and a is a positive constant. It has been proved [51] that the diversity order of the given system will be $G_d = t + 1$. Note that if $f(\beta)$ is well-behaved around $\beta = 0$ so that it accepts a Taylor series expansion at $\beta = 0$, then t is just the first nonzero derivative order of $f(\beta)$ at $\beta = 0$. As a result, we need to evaluate $f(\gamma)$, the pdf of the SNR of the proposed system, for the the low-probability event that the instantaneous SNR becomes small (i.e $\gamma \to 0^+$ or equivalently $\beta \to 0^+$), in order to derive the diversity gain of the proposed system.

From (3.12), we can write the received signal at the jth MS as:

$$y_j = \frac{\sqrt{P}}{\|\mathbf{H}^\dagger\left(\mathbf{H}\mathbf{H}^\dagger\right)^{-1}\|} s_j + n_j \tag{3.18}$$

For an MSPK modulation, the instantaneous SNR of (3.18) can be written as:

$$\gamma = \frac{P}{N_0 \|\mathbf{H}^\dagger\left(\mathbf{H}\mathbf{H}^\dagger\right)^{-1}\|^2} \tag{3.19}$$

Knowing that $\|\mathbf{A}\|^2 = \text{trace}\left(\mathbf{A}^\dagger\mathbf{A}\right)$, the quantity in the denominator of (3.19) can be further simplified as:

$$\begin{aligned}
\|\mathbf{H}^\dagger\left(\mathbf{H}\mathbf{H}^\dagger\right)^{-1}\|^2 &= \text{trace}\left(\left(\mathbf{H}^\dagger(\mathbf{H}\mathbf{H}^\dagger)^{-1}\right)^\dagger \mathbf{H}^\dagger(\mathbf{H}\mathbf{H}^\dagger)^{-1}\right) \\
&= \text{trace}\left((\mathbf{H}\mathbf{H}^\dagger)^{-\dagger}\mathbf{H}\mathbf{H}^\dagger(\mathbf{H}\mathbf{H}^\dagger)^{-1}\right) \\
&= \text{trace}(\mathbf{H}\mathbf{H}^\dagger)^{-1}
\end{aligned} \tag{3.20}$$

where in this development we have used the properties $\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$ and $\text{trace}(\mathbf{A}) = \text{trace}(\mathbf{A}^\dagger)$. Thus we can calculate γ as:

$$\gamma = \frac{P}{N_0} \cdot \frac{1}{\text{trace}\left((\mathbf{H}\mathbf{H}^\dagger)^{-1}\right)} = \frac{P}{N_0}\beta \tag{3.21}$$

where $\beta = 1/\text{trace}\left((\mathbf{H}\mathbf{H}^\dagger)^{-1}\right)$. The term $\mathbf{Hh}^\dagger$ in (3.21) is called the Wishart Matrix [2]. We will now calculate $\lim\limits_{\beta \to 0^+} f(\beta)$.

The joint pdf of λ_1 and λ_2, the unordered strictly positive eigenvalues of the Wishart matrix $\mathbf{H}\mathbf{H}^\dagger$ equals [52]:

$$f(\lambda_1, \lambda_2) = \frac{1}{2} K_{2,n}^{-1} (\lambda_1 \lambda_2)^{n-2} e^{-(\lambda_1+\lambda_2)} (\lambda_1 - \lambda_2)^2 \tag{3.22}$$

where $K_{2,n}$ is a normalization factor and we have posed $n = RL$ for simplicity. Since $f(\lambda_1, \lambda_2)$ is a pdf function, the normalization condition $\iint f(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 = 1$, yields $K_{2,n} = (n-1)!(n-2)!$.

Considering that $\text{trace}(\mathbf{A}^{-1}) = \sum \frac{1}{\lambda_i}$ with λ_i being the eigenvalues of $\mathbf{A}$, we can express β as a function of the eigenvalues of the Wishart matrix:

$$\beta = \frac{1}{\text{trace}\left((\mathbf{H}\mathbf{H}^\dagger)^{-1}\right)} = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}} \tag{3.23}$$

We set $x = \frac{1}{\lambda_1}$ and $y = \frac{1}{\lambda_1}$, the Jacobian of this transform is:

$$J(\lambda_1 = \frac{1}{x}, \lambda_2 = \frac{1}{y}) = \begin{vmatrix} \frac{\partial \lambda_1}{\partial x} & \frac{\partial \lambda_1}{\partial y} \\ \frac{\partial \lambda_2}{\partial x} & \frac{\partial \lambda_2}{\partial y} \end{vmatrix} = \begin{vmatrix} -\frac{1}{x^2} & 0 \\ 0 & -\frac{1}{y^2} \end{vmatrix} = \frac{1}{x^2 y^2} \tag{3.24}$$

Thus the joint pdf of the new random variables X and Y is given by:

$$\begin{aligned} f_{XY}(x,y) &= \frac{1}{2} K_{2,n}^{-1} \left(\frac{1}{xy}\right)^{n-2} e^{-(\frac{1}{x}+\frac{1}{y})} \left(\frac{1}{x} - \frac{1}{y}\right)^2 \frac{1}{x^2 y^2} \\ &= \frac{1}{2} K_{2,n}^{-1} \left(\frac{1}{xy}\right)^{n} e^{-(\frac{1}{x}+\frac{1}{y})} \left(\frac{1}{x} - \frac{1}{y}\right)^2 \end{aligned} \tag{3.25}$$

[2] In statistics, the Wishart distribution is a generalization to multiple dimensions of the chi-squared distribution, or, in the case of non-integer degrees of freedom, of the gamma distribution. It is named in honor of John Wishart, who first formulated the distribution in 1928.

Now we will calculate the pdf of the random variable $U = \frac{1}{\beta} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = X + Y$. The first step is to calculate the probability of $U < u$:

$$\begin{aligned} P(U < u) = P(X + Y < u) &= \int_0^{+\infty} \int_0^{u-x} f_{XY}(x, y) dydx \\ &= \int_0^{+\infty} \int_0^{u-x} \frac{1}{2} K_{2,n}^{-1} \left(\frac{1}{xy} \right)^n e^{-(\frac{1}{x}+\frac{1}{y})} \left(\frac{1}{x} - \frac{1}{y} \right)^2 dydx \end{aligned} \tag{3.26}$$

Now $f_U(u)$ is obtained by differentiating this equation with respect to u:

$$\begin{aligned} f_U(u) &= \frac{d}{du} P(U < u) = \frac{d}{du} \int_0^{+\infty} \int_0^{u-x} \frac{1}{2} K_{2,n}^{-1} \left(\frac{1}{xy} \right)^n e^{-(\frac{1}{x}+\frac{1}{y})} \left(\frac{1}{x} - \frac{1}{y} \right)^2 dydx \\ &= \frac{1}{2} K_{2,n}^{-1} \int_0^u \frac{1}{x^n (u-x)^n} (\frac{1}{x} - \frac{1}{u-x})^2 e^{-u/x(u-x)} dx \\ &= \frac{1}{2} K_{2,n}^{-1} \int_0^u \frac{(u-2x)^2}{x^{n+2}(u-x)^{n+2}} e^{-u/x(u-x)} dx \\ &\triangleq \frac{1}{2} K_{2,n}^{-1} M_n(u) \end{aligned} \tag{3.27}$$

Noting that the integrand in (3.27) is symmetrical around the point $x = u/2$ and using the substitution $t = \frac{u^2}{4x(u-x)}$ we obtain:

$$\begin{aligned} M_n(u) &= 2 \int_0^{u/2} \frac{(u-2x)^2}{x^{n+2}(u-x)^{n+2}} e^{-\frac{u}{x(u-x)}} dx \\ &= 2 \int_1^{+\infty} 4 \times \left(\frac{4t}{u^2} \right)^n \times \frac{1}{u^2} \sqrt{u^2 - u^2/t} \times e^{-4t/u} dt \\ &= \frac{2^{2n+3}}{u^{2n+1}} \int_1^{+\infty} t^n \sqrt{1 - \frac{1}{t}} e^{-4t/u} dt \end{aligned} \tag{3.28}$$

To evaluate (3.28), we expand $\sqrt{1 - \frac{1}{t}}$ for $t > 1$:

$$(1 - 1/t)^{\frac{1}{2}} = 1 - \sum_{k=1}^{+\infty} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} . \frac{1}{t^k} \qquad \text{for } |1/t| < 1 \tag{3.29}$$

In this case, we obtain:

$$
\begin{aligned}
M_n(u) &= \frac{2^{2n+3}}{u^{2n+1}} \int_1^{+\infty} t^n \sqrt{1-\frac{1}{t}} e^{-4t/u} dt \\
&= \frac{2^{2n+3}}{u^{2n+1}} \int_1^{+\infty} t^n [1 - \sum_{k=1}^{+\infty} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} \frac{1}{t^k}] e^{-4t/u} dt \\
&= \frac{2^{2n+3}}{u^{2n+1}} \left[\int_1^{+\infty} t^n e^{-4t/u} dt - \sum_{k=1}^{+\infty} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} \int_1^{+\infty} t^{n-k} e^{-4t/u} dt \right] \\
&= \frac{2^{2n+3}}{u^{2n+1}} \left[\int_1^{+\infty} t^n e^{-4t/u} dt - \sum_{k=1}^{n} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} \int_1^{+\infty} t^{n-k} e^{-4t/u} dt \right. \\
&\qquad \left. - \sum_{k=n+1}^{+\infty} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} \int_1^{+\infty} \frac{e^{-4t/u}}{t^{k-n}} dt \right] \\
&= \frac{2^{2n+3}}{u^{2n+1}} \left[F_n(u) - \sum_{k=1}^{n} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} F_{n-k}(u) - \sum_{k=n+1}^{+\infty} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} G_{k-n}(u) \right]
\end{aligned}
\tag{3.30}
$$

with (by setting $t = x + 1$):

$$
\begin{aligned}
F_m(u) &= \int_1^{+\infty} t^m e^{-4t/u} dt = \int_0^{+\infty} (1+x)^m e^{-4(x+1)/u} dx \\
&= e^{-4/u} \sum_{p=0}^{m} C_m^p \int_0^{+\infty} x^p e^{-4x/u} dx = e^{-4/u} \sum_{p=0}^{m} C_m^p \int_0^{+\infty} \left[\frac{ut}{4}\right]^p e^{-t} \frac{u}{4} dt \\
&= e^{-4/u} \sum_{p=0}^{m} C_m^p \left[\frac{u}{4}\right]^{p+1} \int_0^{+\infty} t^p e^{-t} dt = e^{-4/u} \sum_{p=0}^{m} C_m^p \left[\frac{u}{4}\right]^{p+1} \Gamma(p+1) \\
&= e^{-4/u} \sum_{p=0}^{m} \frac{m!}{p!(m-p)!} \frac{u^{p+1}}{4^{p+1}} p! \\
&= e^{-4/u} \sum_{p=0}^{m} \frac{m!}{(m-p)!} . \frac{u^{p+1}}{4^{p+1}}
\end{aligned}
\tag{3.31}
$$

and

$$\begin{aligned} G_m(u) &= \int_1^{+\infty} \frac{e^{-4t/u}}{t^m} dt \\ &= e^{-\frac{4}{u}} \sum_{p=0}^{m-2} (-1)^p \frac{4^p}{u^p} . \frac{(m-p-2)!}{(m-1)!} + (-1)^{m-1} \frac{4^{m-1}}{u^{m-1}(m-1)!} \mathrm{Ei}(\frac{4}{u}) \end{aligned} \tag{3.32}$$

with Ei denoting the exponential integral:

$$\mathrm{Ei}(x) = \int_x^{+\infty} \frac{e^{-t}dt}{t} = -\ln x - \gamma + \sum_{n=1}^{+\infty} \frac{(-1)^{n-1} x^n}{n n!}$$

Now that we have calculated $f_U(u)$, the pdf of β is straightforward to obtain:

$$\beta = \frac{1}{U} \Longrightarrow f(\beta) = \frac{1}{\beta^2} f_U(\frac{1}{\beta}) \tag{3.33}$$

which yields:

$$\begin{aligned} f(\beta) = K_{2,n}^{-1} 2^{2n+3} \beta^{2n-1} \Bigg[& F_n(1/\beta) - \sum_{k=1}^{n} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} F_{n-k}(1/\beta) \\ & - \sum_{k=n+1}^{+\infty} \frac{(2k-2)!}{2^{2k-1}(k-1)!k!} G_{k-n}(1/\beta) \Bigg] \end{aligned} \tag{3.34}$$

with

$$F_m(\frac{1}{\beta}) = e^{-4\beta} \sum_{p=0}^{m} \frac{m!}{(m-p)!} \frac{1}{4^{p+1}\beta^{p+1}} \tag{3.35}$$

and

$$G_m(\frac{1}{\beta}) = e^{-\frac{4}{u}} \sum_{p=0}^{m-2} (-1)^p \frac{(4\beta)^p (m-p-2)!}{(m-1)!} + (-1)^{m-1} \frac{4^{m-1}}{u^{m-1}(m-1)!} \mathrm{Ei}(4\beta) \tag{3.36}$$

Now that the expression of the pdf of the SNR is obtained, we can calculate the SEP by [53]:

$$\bar{P}_s = \int_0^\infty 2Q(\sqrt{k\beta\bar{\gamma}})f(\beta)d\beta \tag{3.37}$$

where Q denotes the Q-function and $k = 2\sin^2(\frac{\pi}{M})$ is a constant determined by the modulation. In order to calculate the diversity order of the system we need to determine the first nonzero derivative order of $f(\beta)$ at $\beta = 0$ [54]. It is obvious that the smallest nonzero exponent of β in (3.34) is obtained from the term $\beta^{2n-1}F_n(\frac{1}{\beta})$ and equals $t = 2n - 1 - n - 1 = n - 2 = RL - N$. As a result diversity order is given by $G_d = t + 1 = n - 1 = RL - N + 1$. In the next section we will discuss why this result is intuitively correct.

3.3.2 Disscussing the diversity of the system

In this section we will discuss the diversity gain of proposed scheme. We will argue for both scenarios, i) when all relays have the complete CSI of the system, ii) when each relay knows only its own channel to the MSs.

Case I: CSI Known to all relays

When the channel coefficients of all links are known to all relays, mathematically the system is equivalent of one relay with LR antennas. There are thus LR antennas cooperating to send a signal s_j to its destination MS$_j$, the maximum diversity order is therefore RL. But the difference with a usual MIMO system is that here we have multiple access network and we want to cancel out the MAI. At each mobile station there are $N-1$ undesired symbols to be canceled out. Thus the system has less degrees of freedom and the diversity order is expected to be:

$$G_d = LR - (N-1) = LR - N + 1 \tag{3.38}$$

Case II: Independent relays

Let us consider a single R-antenna relay sending information toward N single antenna users, using (3.38) we can say that the diversity order is $R-N+1$. Now assume that we have L relays cooperating to send the information thus the diversity gain is multiplied by L:

$$G_d = L(R-N+1) \tag{3.39}$$

This is also intuitively correct, in the second case when each relay knows only its own link to mobile stations, lower diversity is obtained. In this case, like the former, there are LR replicas of the message at each mobile station. The maximum achievable diversity is therefore LR. The difference in this case is that each relay performs independently of other relays. As a result, each relay must cancel out MAI (i.e. $N-1$ undesired symbols) by itself. There are $N-1$ limiting constraints per relay. Thus the diversity would be $LR - L(N-1) = L(R-N+1)$.

These results are verified under the section 3.4.

3.3.3 SEP semi-analytic calculation for arbitrary N

Analytical calculation of the pdf of γ for arbitrary number of mobile stations if not impossible is very difficult. An alternative method is to approximate the distribution of SNR. In this work we have used a mixture of Nakagami distributions to approximate the pdf of the instantaneous SNR, γ.

The pdf of a Nakagami distribution, parametered by a shape parameter μ and a scaling factor Ω, is given by:

$$f(x) = \frac{2\mu^{\mu}}{\Gamma(\mu)\Omega^{\mu}} x^{2\mu-1} \exp\left(-\frac{\mu}{\Omega}x^2\right) \tag{3.40}$$

A mixture of J Nakagami distributions is defined as a random variable whose value is selected randomly from one of the given Nakagami distributions $f_1(x)$ to $f_J(x)$:

$$X|(Z=j) \sim Nakagami(\mu_j, \Omega_j) \tag{3.41}$$

where Z, called latent variable, is a random variable that takes a value from $1, 2, \cdots, J$ respectively with probabilities $\pi_1, \pi_2, \cdots, \pi_J$. Thus the mixture of J Nakagami distributions has the following distribution:

$$f_C(x) = \sum_{j=1}^{J} \pi_j \frac{2\mu_j^{\mu_j}}{\Gamma(\mu_j)\Omega_j^{\mu_j}} x^{2\mu_j - 1} \exp\left(-\frac{\mu_j}{\Omega_j} x^2\right) \tag{3.42}$$

The problem is now to find the most likely values of parameters in our probabilistic model (i.e. $\pi_1 \cdots \pi_J$, $\mu_1 \cdots \mu_J$, and $\Omega_1 \cdots \Omega_J$). It is obvious that a maximization algorithm by searching all of the possible values of π_j, μ_j, and Ω_j is too complicated. As a result Expectation Maximization (EM) [55–59] is used. EM is an iterative algorithm which alternates between performing an expectation step and a maximization step. Given a likelihood function $L(\boldsymbol{\theta}; \mathbf{x}, \mathbf{z})$, where $\boldsymbol{\theta}$ is the parameter vector, $\mathbf{x}$ is the n samples of observed data and $\mathbf{z}$ represents the unobserved latent variable, the maximum likelihood estimation (MLE) is determined by the marginal likelihood of the observed data $L(\boldsymbol{\theta}; \mathbf{x})$. The EM algorithm seeks to find the MLE by iteratively applying the following two steps:

- **Expectation step:** Calculate the expected value of the log-likelihood function, with respect to the conditional distribution of $\mathbf{z}$ given $\mathbf{x}$ under the current estimate of the parameters $\boldsymbol{\theta}^{(t)}$. With $\mathbb{I}$ being an indicator function:

$$\mathbb{I}(z_i = j) = \begin{cases} 1, & z_i = j \\ 0, & z_i \neq j \end{cases}, \tag{3.43}$$

  The log-likelihood function is determined by:

$$
\begin{aligned}
\log L(\boldsymbol{\theta}; \mathbf{x}, \mathbf{z}) = \sum_{i=1}^{n} \sum_{j=1}^{J} \mathbb{I}(z_i = j) \Big[\log \pi_j + \log(2\mu_j^{\mu_j}) \\
- \log \Gamma(\mu_j) - \log \Omega_j^{\mu_j} + \log x_i^{2\mu_j - 1} - \frac{\mu_j}{\Omega_j} x_i^2 \Big]
\end{aligned}
\tag{3.44}
$$

Given the current estimate of the parameters, the conditional distribution of the Z_i is determined by Bayes theorem to be the proportional height of the Nakagami density function weighted by π:

$$
\begin{aligned}
T_{j,i}^{(t)} &\triangleq \mathrm{P}(Z_i = j | X_i = \mathbf{x}_i; \boldsymbol{\theta}^{(t)}) \\
&= \frac{\pi_j^{(t)} \ f(\mathbf{x}_i; \mu_j^{(t)}, \Omega_j^{(t)})}{\sum_{l=1}^{J} \pi_l^{(t)} \ f(\mathbf{x}_i; \mu_l^{(t)}, \Omega_l^{(t)})}
\end{aligned}
\tag{3.45}
$$

Finally the expectation step results in:

$$
\begin{aligned}
Q(\theta | \theta^{(t)}) \triangleq \mathbb{E} \left\{ \log L(\theta; \mathbf{x}, \mathbf{Z}) \right\} = \sum_{i=1}^{n} \sum_{j=1}^{J} T_{j,i}^{(t)} \\
\Big[\log \pi_j + \log(2\mu_j^{\mu_j}) - \log \Gamma(\mu_j) \\
- \mu_j \log \Omega_j + (2\mu_j - 1) \log x_i - \frac{\mu_j}{\Omega_j} x_i^2 \Big]
\end{aligned}
\tag{3.46}
$$

- **Maximization step:** Find the parameters which maximizes the quantity $Q(\theta|\theta^{(t)})$. In order to maximize $Q(\theta|\theta^{(t)})$, the partial derivatives of this function must be calculated:

$$
\frac{\partial Q(\theta|\theta^{(t)})}{\partial \Omega_j} = 0 \Longrightarrow \Omega_j = \frac{\sum_{i=1}^{n} x_i^2 T_{j,i}^{(t)}}{\sum_{i=1}^{n} T_{j,i}^{(t)}}
\tag{3.47}
$$

and

$$\frac{\partial Q(\theta|\theta^{(t)})}{\partial \mu_j} = 0 \Longrightarrow \log \mu_j \sum_{i=1}^{n} T_{j,i}^{(t)} = \sum_{i=1}^{n} T_{j,i}^{(t)} \left(\log \Omega_j - 2 \log x_i + \frac{x_i^2}{\Omega_j} + 1 - \psi(\mu_j) \right) \tag{3.48}$$

where $\psi(x)$ denotes the polygamma function of order 0: $\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$.

Even though (3.48) may be solved iteratively, since the ψ function has the values around 1 in the domain of interest, without the loss of precision the equation in (3.48) may be approximated by:

$$\log \mu_j = \frac{\sum_{i=1}^{n} T_{j,i}^{(t)} \left(\log \Omega_j - 2 \log x_i + \frac{x_i^2}{\Omega_j} \right)}{\sum_{i=1}^{n} T_{j,i}^{(t)}} \tag{3.49}$$

In this work a mixture of 6 Nakagami distribution is used to fit the pdf of random variable $1/\text{trace}((\mathbf{H}\mathbf{H}^{\dagger})^{-1})$ of the matrix $\mathbf{H}$ of size $N \times P$ with $h_{np} \sim \mathcal{N}(0, 1)$. The mixture parameters for some values of N and P are given in Table 3.1.

N	P	$\boldsymbol{\pi}$	$\boldsymbol{\mu}$	$\boldsymbol{\Omega}$
3	4	0.14 0.30 0.18 0.22 0.11 0.05	1.2 0.9 1.8 1.3 1.0 3.2	0.17 0.2 0.4 0.5 0.7 0.7
3	5	0.12 0.18 0.22 0.24 0.11 0.13	1.5 1.9 2.3 2.1 2.0 3.2	0.29 0.5 0.7 0.9 1.3 1.6
3	6	0.05 0.09 0.16 0.26 0.16 0.28	2.5 2.9 3.3 3.1 3.0 3.2	0.34 0.8 1.1 1.4 1.6 2.4
3	7	0.01 0.03 0.07 0.17 0.18 0.55	3.5 3.9 4.3 4.1 4.0 3.2	0.42 0.9 1.3 1.8 2.3 3.0
3	8	0.00 0.00 0.01 0.06 0.13 0.80	4.5 4.9 5.3 5.1 5.0 3.2	0.68 1.2 1.7 2.5 3.0 3.8
3	10	0.06 0.11 0.17 0.33 0.12 0.21	5.5 5.9 6.3 6.1 6.0 3.2	4.18 5.1 5.9 7.2 6.6 7.2

Table 3.1: Parameters of Nakagami mixture

We must now calculate the SEP for a receiver with a signal to noise ratio following the Nakagami distribution:

$$f(x) = \frac{2\mu^{\mu}}{\Gamma(\mu)\Omega^{\mu}} x^{2\mu - 1} \exp\left(-\frac{\mu}{\Omega} x^2 \right) \tag{3.50}$$

For an M-PSK modulation with an SNR of γ, instantaneous symbol error probability (SEP) is given in [60]: $P_e(\gamma) = 2\alpha\text{erfc}(\sqrt{\beta\gamma}) - [\alpha\text{erfc}(\sqrt{\beta\gamma})]^2$ where $\alpha = 1 - (1/\sqrt{M})$ and $\beta = 3/\left[2(M-1)\right]$ and $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{+\infty} e^{-u^2}du$. A very tight approximation of this formula is $P_e(\gamma) \approx 2\alpha\text{erfc}(\sqrt{\beta\gamma})$. Averaging this probability over different values of γ yields:

$$\bar{P}_e = \int_0^{+\infty} P_e(u)f(u)du \tag{3.51}$$

with $f(u)$ denoting the probability density function of SNR. Assuming that the SNR follows a Nakagami distribution and using σ^2 to denote the average noise variance we obtain:

$$\begin{aligned} f(u) = \sigma^2 f_N(\sigma^2 u) &= \sigma^2 \frac{2\mu^\mu}{\Gamma(\mu)\Omega^\mu}(\sigma^2 u)^{2\mu-1}e^{-\frac{\mu}{\Omega}(\sigma^2 u)^2} \\ &= \frac{2\mu^\mu\sigma^{4\mu}}{\Gamma(\mu)\Omega^\mu}u^{2\mu-1}e^{-\frac{\mu\sigma^4}{\Omega}u^2} \end{aligned} \tag{3.52}$$

Substituting (3.52) in (3.51) results in:

$$\bar{P}_e = \int_0^{+\infty} P_e(u)f(u)du = \frac{4\alpha\mu^\mu\sigma^{4\mu}}{\Gamma(\mu)\Omega^\mu}L \tag{3.53}$$

with:

$$L = \int_0^{+\infty} u^{2\mu-1}e^{-\frac{\mu\sigma^4}{\Omega}u^2}\text{erfc}(\sqrt{\beta u})du \tag{3.54}$$

In order to evaluate (3.53), we set $\mu = n+\xi$ with n being a non negative integer number and $0 < \xi < 1$. First, we will integrate the following function:

$$f(x) = x^{2\mu-1}\exp(-\frac{\mu\sigma^4}{\Omega}x^2) = x^{2n-1+2\xi}e^{-\lambda x^2} \tag{3.55}$$

with $\lambda = \frac{\mu\sigma^4}{\Omega}$. Using the variable substitution $u = \lambda t^2$, we can write:

$$F(x) = \int_0^x t^{2n-1+2\xi} e^{-\lambda t^2} dt = \frac{1}{2\lambda^{n+\xi}} \int_0^{\lambda x^2} u^{n-1+\xi} e^{-u} du \tag{3.56}$$

The integrals of form $I_m(x) = \int_0^x u^{m+\xi} e^{-u} du$ need to be calculated. Integrating by parts yields:

$$I_m(x) = \int_0^x u^{m+\xi} e^{-u} du = -x^{m+\xi} e^{-x} + (m+\xi) I_{m-1}(x) \tag{3.57}$$

This is a recursive expression of I_m with $I_0 = \Gamma(x, \xi+1) = \int_0^x u^{\xi} e^{-u} du$ being the lower incomplete Gamma function. As a result $F(x)$ can be calculated by:

$$\begin{aligned}
F(x) &= \frac{1}{2\lambda^{n+\xi}} I_{m-1}(\lambda x^2) \\
&= \frac{1}{2\lambda^{n+\xi}} \left[-x^{m-1+\xi} + (m-1+\xi) I_{m-2}(\lambda x^2) \right] \\
&= \frac{1}{2\lambda^{n+\xi}} \left\{ -x^{m-1+\xi} + (m-1+\xi) \left[-x^{m-2+\xi} + (m-2+\xi) I_{m-3}(\lambda x^2) \right] \right\} \\
&= \cdots \\
&= \frac{1}{2\lambda^{n+\xi}} \Big[-(\lambda x^2)^{m-1+\xi} e^{-\lambda x^2} - (m-1+\xi)(\lambda x^2)^{m-2+\xi} e^{-\lambda x^2} \\
&\qquad + \cdots \\
&\qquad - (m-1+\xi)(m-2+\xi)\cdots(m-k+1+\xi)(\lambda x^2)^{m-k+\xi} e^{-\lambda x^2} \\
&\qquad + ... \\
&\qquad - (m-1+\xi)(m-2+\xi)\cdots(2+\xi)(\lambda x^2)^{1+\xi} e^{-\lambda x^2} \\
&\qquad + (m-1+\xi)(m-2+\xi)\cdots(1+\xi)\Gamma(\lambda x^2, \xi+1) \Big]
\end{aligned} \tag{3.58}$$

Using this result and noting that:

$$\frac{d}{du} \mathrm{erfc}(\sqrt{\beta u}) = -\frac{\sqrt{\beta} e^{-\beta u}}{\sqrt{\pi u}} \tag{3.59}$$

the quantity L in (3.54) can be calculated as:

$$\begin{aligned}
L &= \int_0^{+\infty} u^{2\mu-1} e^{-\frac{\mu\sigma^4}{\Omega}u^2} \mathrm{erfc}(\sqrt{\beta u}) du \\
&= \left[F(u).\mathrm{erfc}(\sqrt{\beta u})\right]_0^{+\infty} + \sqrt{\frac{\beta}{\pi}} \int_0^{+\infty} F(u) \frac{e^{-\beta u}}{\sqrt{u}} du \\
&= \sqrt{\frac{\beta}{\pi}} \int_0^{+\infty} F(u) \frac{e{-}\beta u}{\sqrt{u}} du \\
&= \sqrt{\frac{\beta}{\pi}} \int_0^{+\infty} \frac{1}{2\lambda^{n+\xi}} \Big[-(\lambda u^2)^{m-1+\xi} e^{-\lambda u^2} - (m-1+\xi)(\lambda u^2)^{m-2+\xi} e^{-\lambda u^2} \\
&\quad + \cdots \\
&\quad - (m-1+\xi)(m-2+\xi)\cdots(m-k+1+\xi)(\lambda u^2)^{m-k+\xi} e^{-\lambda u^2} \\
&\quad + ... \\
&\quad - (m-1+\xi)(m-2+\xi)\cdots(2+\xi)(\lambda u^2)^{1+\xi} e^{-\lambda u^2} \\
&\quad +(m-1+\xi)(m-1+\xi)\cdots(2+\xi)\Gamma(\lambda u^2, \xi+1)\Big] \frac{e^{-\beta u}}{\sqrt{u}} du
\end{aligned} \tag{3.60}$$

We start by calculating the integral containing $\Gamma(\lambda u^2, \xi + 1)$:

$$\begin{aligned}
L_0 &= \int_0^{+\infty} \Gamma(\lambda u^2, \xi+1) \frac{e^{-\beta u}}{\sqrt{u}} du \\
&= \sqrt{\frac{\pi}{\beta}} \int_0^{+\infty} \Gamma(\lambda u^2, \xi+1) \frac{d\left[\mathrm{erf}(\sqrt{\beta u})\right]}{du} du \\
&= \sqrt{\frac{\pi}{\beta}} \left[[\Gamma(\lambda u^2, \xi+1)\mathrm{erf}(\sqrt{\beta u})]_0^{+\infty} - \int_0^{+\infty} \frac{d\Gamma}{du}(\lambda u^2, \xi+1)\mathrm{erf}(\sqrt{\beta u}) du \right] \\
&= \sqrt{\frac{\pi}{\beta}} \left[\Gamma(\xi+1) - \int_0^{+\infty} 2\lambda u (\lambda u^2)^{\xi} e^{-\lambda u^2} \mathrm{erf}(\sqrt{\beta u}) du \right] \\
&= \sqrt{\frac{\pi}{\beta}} \left[\Gamma(\xi+1) - 2\lambda^{\xi+1} \int_0^{+\infty} u^{2\xi+1} e^{-\lambda u^2} \mathrm{erf}(\sqrt{\beta u}) du \right]
\end{aligned} \tag{3.61}$$

We will use the series development of the error function:

$$
\begin{aligned}
\operatorname{erf}(\sqrt{\beta u}) &= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\beta u}} e^{-x^2} dx = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\beta u}} \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{n!} dx \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2} u^{n+1/2}}{(2n+1)}
\end{aligned}
\tag{3.62}
$$

We can now calculate the integral in (3.61):

$$
\begin{aligned}
\int_0^{+\infty} u^{2\xi+1} e^{-\lambda u^2} \operatorname{erf}(\sqrt{\beta u}) du &= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \int_0^{+\infty} u^{2\xi+1} e^{-\lambda u^2} u^{n+1/2} du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \int_0^{+\infty} u^{n+2\xi+3/2} e^{-\lambda u^2} du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \frac{1}{\sqrt{\lambda}} \int_0^{+\infty} (\frac{x}{\lambda})^{\xi+n/2+3/4} e^{-x} \frac{dx}{2\sqrt{x}} \\
&= \frac{1}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)\lambda^{\beta+n/2+5/4}} \Gamma(n/2+\xi+5/4)
\end{aligned}
\tag{3.63}
$$

Thus, we obtain:

$$
\begin{aligned}
L_0 &= \sqrt{\frac{\pi}{\beta}} \left[\Gamma(\xi+1) - 2\lambda^{\xi+1} \int_0^{+\infty} u^{2\xi+1} e^{-\lambda u^2} \operatorname{erf}(\sqrt{\beta u}) du \right] \\
&= \sqrt{\frac{\pi}{\beta}} [\Gamma(\xi+1) - \frac{2\lambda^{\xi+1}}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n \beta^{n+\frac{1}{2}} \Gamma(\frac{n}{2}+\xi+\frac{5}{4})}{n!(2n+1)\lambda^{\xi+\frac{n}{2}+\frac{5}{4}}}]
\end{aligned}
\tag{3.64}
$$

Now, we will calculate other integrals in (3.60); the integrals of form L_r need to be calculated:

$$
\begin{aligned}
L_r &= \int_0^{+\infty} u^{2r+2\xi} \frac{e^{-\beta u} e^{-\lambda u^2}}{\sqrt{u}} du = \sqrt{\frac{\pi}{\xi}} \int_0^{+\infty} u^{2r+2\xi} e^{-\lambda u^2} \frac{d\left[\operatorname{erf}(\sqrt{\beta u})\right]}{du} du \\
&= -2\sqrt{\frac{\pi}{\xi}} \int_0^{+\infty} \operatorname{erf}(\sqrt{\beta u}) e^{-\lambda u^2} \left[(r+\xi) u^{2r+2\xi-1} - \lambda u^{2r+2\xi+1}\right]
\end{aligned}
\tag{3.65}
$$

The integral in (3.65) can be expanded into two integrals L_{r_1} and L_{r_2} both of which will be calculated using the series development of the error function:

$$
\begin{aligned}
L_{r_1} &= \int_0^{+\infty} u^{2r+2\xi-1} e^{-\lambda u^2} \operatorname{erf}(\sqrt{\beta u}) du \\
&= \int_0^{+\infty} u^{2r+2\xi-1} e^{-\lambda u^2} (\frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\xi^{n+1/2} u^{n+1/2}}{(2n+1)}) du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\xi^{n+1/2}}{(2n+1)} \int_0^{+\infty} u^{2r+2\xi-1} u^{n+1/2} e^{-\lambda u^2} du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\xi^{n+1/2}}{(2n+1)} \int_0^{+\infty} u^{2r+2\xi+n-1/2} e^{-\lambda u^2} du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\xi^{n+1/2}}{(2n+1)} \frac{1}{\sqrt{\lambda}} \int_0^{+\infty} (\frac{x}{\lambda})^{r+\xi+n/2-1/4} e^{-x} \frac{dx}{2\sqrt{x}} \\
&= \frac{1}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\xi^{n+1/2}}{(2n+1)} \frac{\Gamma(r+\xi+n/2+1/4)}{\lambda^{r+\xi+n/2+1/4}}
\end{aligned}
\tag{3.66}
$$

And:

$$
\begin{aligned}
L_{r_2} &= \int_0^{+\infty} u^{2r+2\xi+1} e^{-\lambda u^2} \operatorname{erf}(\sqrt{\beta u}) du \\
&= \int_0^{+\infty} u^{2r+2\xi+1} e^{-\lambda u^2} (\frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2} u^{n+1/2}}{(2n+1)}) du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \int_0^{+\infty} u^{2r+2\xi+1} u^{n+1/2} e^{-\lambda u^2} du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \int_0^{+\infty} u^{2r+2\xi+n+3/2} e^{-\lambda u^2} du \\
&= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \frac{1}{\sqrt{\lambda}} \int_0^{+\infty} (\frac{x}{\lambda})^{r+\xi+n/2+3/4} e^{-x} \frac{dx}{2\sqrt{x}} \\
&= \frac{1}{\sqrt{\pi}} \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \frac{\beta^{n+1/2}}{(2n+1)} \frac{\Gamma(r+\xi+n/2+5/4)}{\lambda^{r+\xi+n/2+5/4}}
\end{aligned}
\tag{3.67}
$$

L_r is calculated by summing up L_{r_1} and L_{r_2}:

$$\begin{aligned} L_r &= -2\sum_{n=0}^{+\infty}\frac{(-1)^n}{n!}\frac{\beta^n}{(2n+1)\lambda^{r+\xi+\frac{n}{2}+\frac{1}{4}}}\left[(r+\xi)\Gamma(r+\xi+\frac{n}{2}+\frac{1}{4})-\Gamma(r+\xi+\frac{n}{2}+\frac{5}{4})\right] \\ &= \frac{1}{2\lambda^{\frac{1}{4}}}\sum_{n=0}^{+\infty}\left(\frac{(-\beta)}{\sqrt{\lambda}}\right)^n\frac{\Gamma(r+\xi+n/2+1/4)}{n!} \end{aligned} \tag{3.68}$$

In the last development we have used the well known property of Gamma function: $\Gamma(z+1)=z\Gamma(z)$.

We can thus calculate the SEP of a system for which the SNR of the received signal has the Nakagami distribution:

$$\begin{aligned} \bar{P}_e &= \frac{4\alpha\mu^\mu\sigma^{4\mu}}{\Gamma(\mu)\Omega^\mu}L \\ &= \frac{2\alpha}{\Gamma(\mu)}\sqrt{\frac{\beta}{\pi}}\Big[L_{n-1}-(n-1+\xi)L_{n-2}+... \\ &\quad -(n-1+\xi)(n-2+\xi)..(n-k+1+\xi)L_{n-k}+ \\ &\quad ...-(n-1+\xi)(n-2+\xi)..(2+\xi)L_1+ \\ &\quad +(n-1+\xi)(n-2+\xi)..(1+\xi)L_0\Big] \end{aligned} \tag{3.69}$$

Where L_0 is given in (3.64) and L_r for $r \geq 1$ is given in(3.68)

- Special Case: SEP for higher SNR

Although the series in (3.69) is strictly convergent for all SNR values, we can note that for higher SNR, since $\sigma^2 \rightarrow 0$ (i.e. $\lambda \rightarrow 0$), the series will converge more slowly. We have developed an alternative formula to overcome this problem. Starting from (3.53) and applying integration by parts, we obtain:

$$\begin{aligned}
\bar{P}_{e,highSNR} &= \frac{4\alpha\mu^{\mu}\sigma^{4\mu}}{\Gamma(\mu)\Omega^{\mu}} \int_{0}^{+\infty} u^{2\mu-1} e^{-\frac{\mu\sigma^4}{\Omega}u^2} \mathrm{erfc}(\sqrt{\beta u}) du \\
&= \frac{4\alpha\mu^{\mu}\sigma^{4\mu}}{\Gamma(\mu)\Omega^{\mu}} \sum_{n=0}^{+\infty} \frac{(-1)^n \mu^n \sigma^{4n}}{n!\Omega^n} \sqrt{\frac{\beta}{\pi}} \frac{1}{2(\mu+n)} \int_{0}^{+\infty} u^{2(\mu+n)-\frac{1}{2}} e^{-\beta u} du \\
&= \frac{4\alpha\mu^{\mu}\sigma^{4\mu}}{\Gamma(\mu)\Omega^{\mu}} \sum_{n=0}^{+\infty} \frac{(-1)^n \mu^n \sigma^{4n}}{n!\Omega^n} \sqrt{\frac{\beta}{\pi}} \frac{1}{2\beta^{2\mu+2n+\frac{1}{2}}(\mu+n)} \int_{0}^{+\infty} t^{2(\mu+n)-\frac{1}{2}} e^{t} dt \qquad (3.70) \\
&= \frac{4\alpha\mu^{\mu}\sigma^{4\mu}}{\Gamma(\mu)\Omega^{\mu}} \sum_{n=0}^{+\infty} \frac{(-1)^n \mu^n \sigma^{4n}}{n!\Omega^n} \sqrt{\frac{\beta}{\pi}} \frac{1}{2\beta^{2\mu+2n+\frac{1}{2}}(\mu+n)} \Gamma(2(\mu+n)+\frac{1}{2}) \\
&= \frac{2\alpha}{\sqrt{\pi}\Gamma(\mu)} \sum_{n=0}^{+\infty} \frac{(-1)^n \mu^{n+\mu} \sigma^{4(n+\mu)}}{n!\Omega^{(n+\mu)}} \frac{\Gamma(2(\mu+n)+1/2)}{\beta^{2(\mu+n)}(\mu+n)}
\end{aligned}$$

In order to obtain (3.70) we have used the derivative of error function (3.59) and the series development of the exponential function as well as the definition of Gamma function.

The series in (3.70) is divergent, however if only a few terms are used, and for high values of SNR (i.e. $\sigma^2 \to 0$), it can give a very tight evaluation of the SEP. In fact for high values of SNR, (3.70) diverges very slowly (and not within the first few terms) just like (3.69) converges very slowly.

Now that we calculated the SEP over a single Nakagami distributed SNR, we can proceed to the calculation of SEP of a system with a signal to noise ratio following a mixture of Nakagami distributions. If the SEP corresponding to the jth Nakagami law is denoted by $\bar{P}_{e_j}(\mu_j, \Omega_j)$, the overall symbol error probability of the mixture will be given by:

$$\bar{P}_{e_C} = \sum_{j=1}^{J} \pi_j \bar{P}_{e_j}(\mu_j, \Omega_j) \qquad (3.71)$$

This calculation is confirmed by the simulation results.

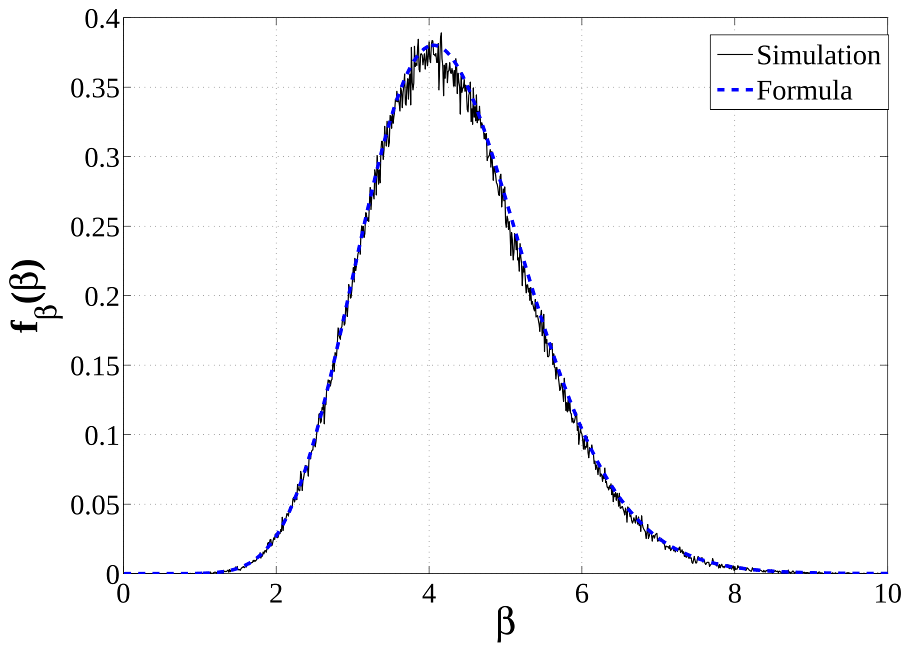


Figure 3.2: The theoretical value of the pdf of SNR for the case N=2, RL=10 based on equation given in (3.34)

3.4 Simulation Results

This section introduces some simulation results that confirm the equations in the previous sections. All simulations are obtained for quadrature phase-shift keying (QPSK) modulation using Monte Carlo method in MATLAB.

We have derived an analytic expression in (3.34) for the distribution of SNR of proposed system for the case of two mobile stations (i.e. $N = 2$) and arbitrary antennas. As can be seen in Figure 3.2, this result is verified by simulation. This figure shows that for the case $N = 2$ and $LR = 10$, the theoretical and simulated curves are very close. Since the most significant part of the $f_\beta(\beta)$ for system performance evaluation is the part where β is small, it is more interesting to see the behavior of $\log(f_\beta(\beta))$ for small β. This behavior is shown in Figure 3.3 where logarithmic scale is used.

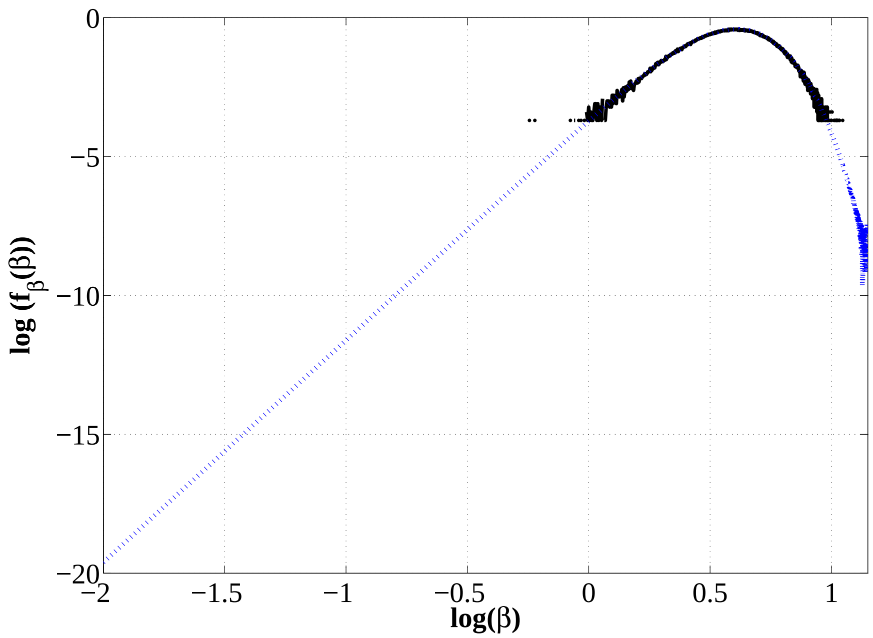


Figure 3.3: The theoretical value of the pdf of SNR for the case N=2, RL=10 in logarithmic scale

Under section 3.3.3 we have used a mixture of Nakagami distributions to approximate the SNR. Figure 3.4 shows that the Nakagami mixture is a very good approximation of the probability density function of the signal to noise ratio. The curves are obtained for the case of one four-antenna relay and 3 mobile stations. Since the system performance for high SNR depends only on the behavior of this curve near the origin, the same curve has been traced in logarithmic scale in Figure 3.5.

As stated under section 3.2, there are two possible scenarios depending on whether or not the relay stations are provided with the knowledge of channel state information of other relays. Figure 3.6 shows the system performance in both cases when 2 three-antenna relay stations cooperate in sending a message toward two mobile stations. It shows that if the channel information is available to both relays, lower bit error rate and higher diversity is obtained $(2\times3-2+1=5)$. This is at the cost of more complexity in the

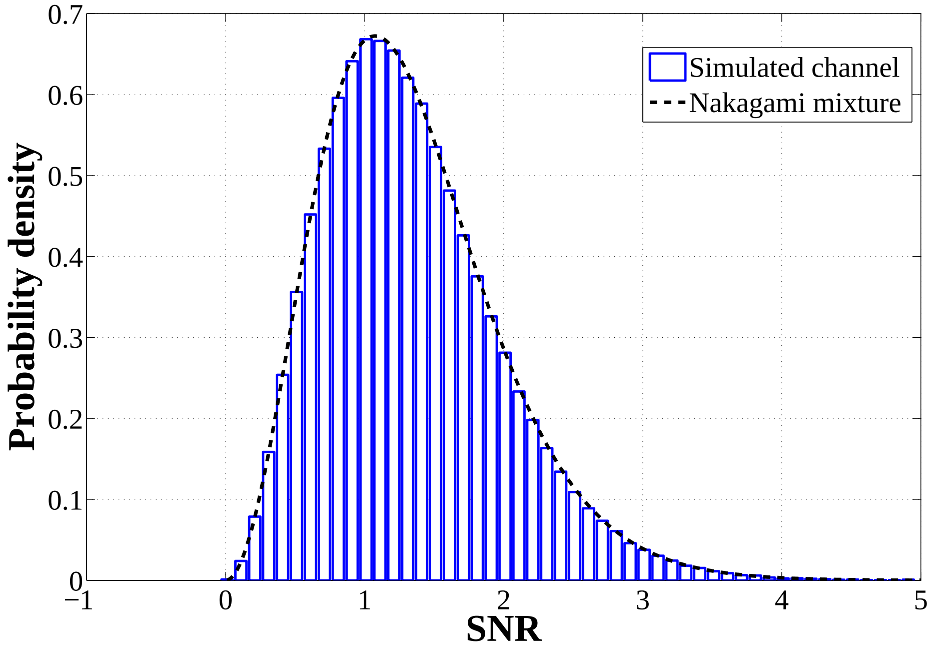


Figure 3.4: Comparison between the Nakagami mixture approximation and the Monte-Carlo simulated values of SNR

transmission protocol. On the other hand, if each of the relays knows only its respective channel information, the system is more practical at the cost of higher Bit Error Rate (BER) and lower diversity $(2 \times (3-1) = 4)$.

Under section 3.3 a theoretical expression was given for the SEP of the proposed system in (3.69) and (3.70). Figure 3.7 shows the system performance for the case of 3 mobile stations and different number of relays. The relays are assumed to be inter connected (i.e. CSI from all relays is known to all relays). Obviously, when complete CSI is considered at relays, there is no mathematical difference between the system equations of a system with two two-antenna relay stations and that of a system with four single-antenna relay stations. The only difference is that in practice, four separated enough single antenna relays have a better chance to benefit from independent channel fading than two two-antenna relays. It is clear that the theoretical expressions in (3.69) and (3.70) provide a precise evaluation of

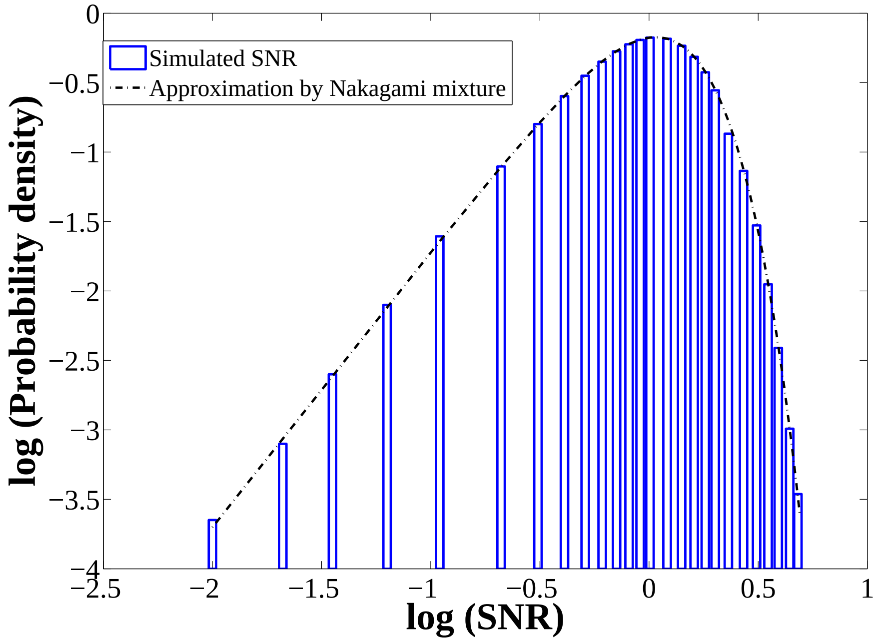


Figure 3.5: Comparison between the Nakagami mixture approximation and the Monte-Carlo simulated values of SNR on logarithmic scale

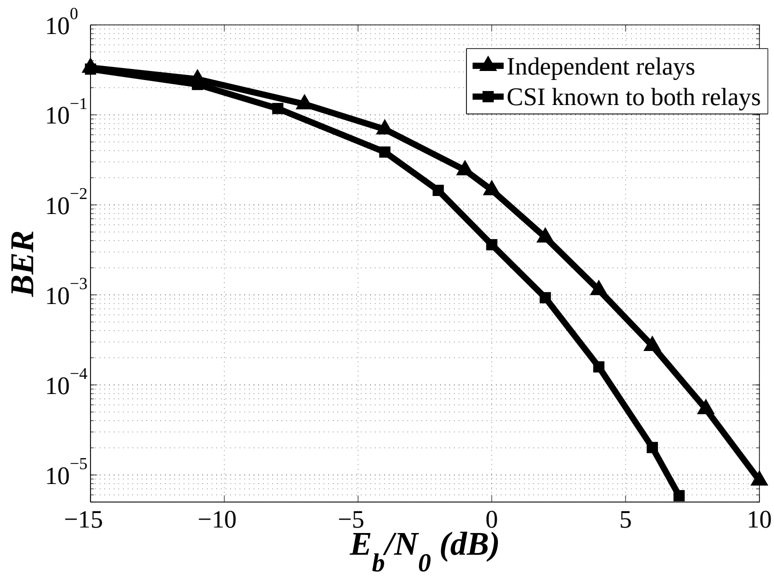


Figure 3.6: System performance ($L = 2$, $N = 2$, and $R = 3$) for i) when CSI is known to both relays and ii) when each relay only has its own relative CSI

system performance. In the following figures, only the simulated data is traced.

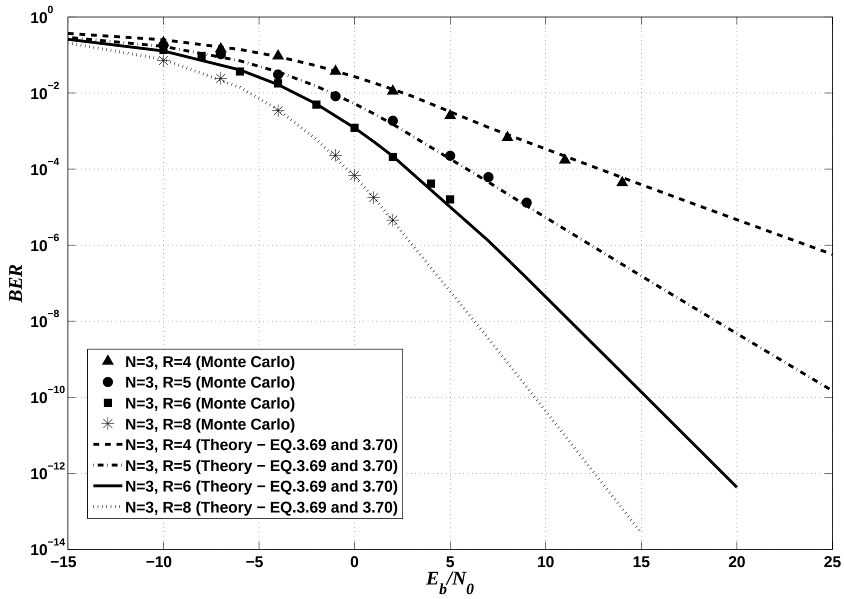


Figure 3.7: System performance when the CSI is known to all relays for 3 mobile stations ($RL = 3$) and 4, 5, 6, and 8 single-antenna relay station ($RL = 4, 5, 6, 8$)

Figure 3.8 shows the BER as a function of E_b/N_0 for different number of mobile stations ($N = 2 \cdots 5$), all for a given number of relays ($L = 2$) and relay antennas ($R = 4$). Using more relay antennas compared to mobile stations results in lower BER and higher diversity gain. We can see that when the mobile stations outnumber relay antennas, an error floor appears in the curves. Note that in this figure relays are considered to have no knowledge of the link between other relay and mobile stations. The diversity is thus $2 \times 4 - 2(2 - 1) = 6$, 4, and 2 when number of mobile stations is respectively 2, 3, and 4.

Figure 3.9 depicts the system performance for different number of relay stations. All curves are obtained for 3-antenna relay stations and two mobile stations. The only difference is the number of relay stations contributing in signal transmission ($L = 1, 2, 3, 4$). As we can see, higher relay numbers results in better system performance. The relays are considered

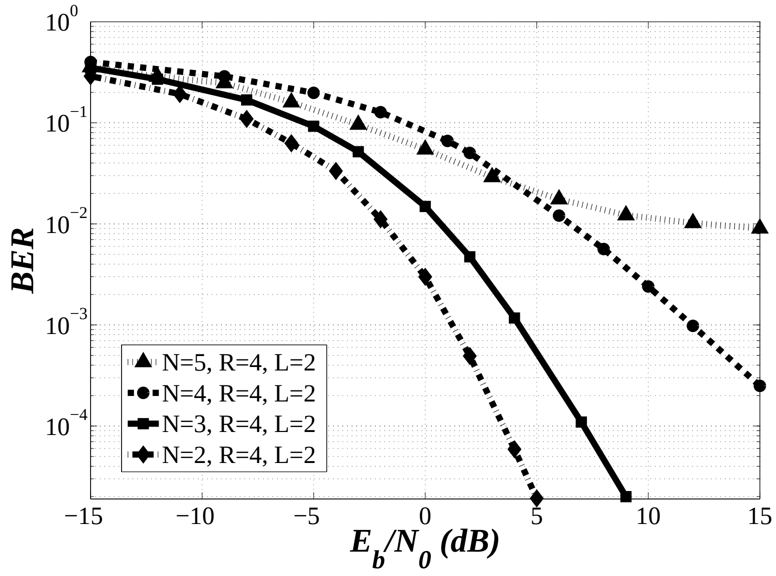


Figure 3.8: System performance for two independent 4-antenna relay stations with different mobile station numbers

to be independent, thus the diversity is of order $LR - L(N-1)$.

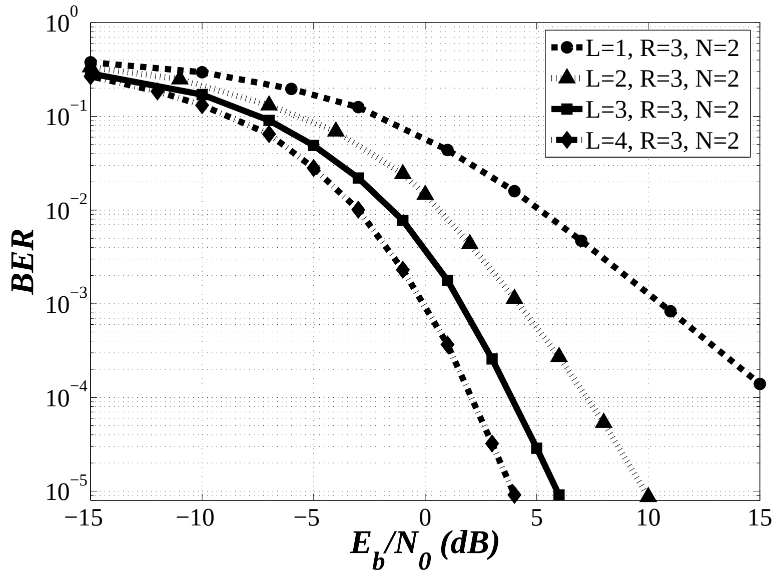


Figure 3.9: System performance for different number of relay stations (independent relays)

We have disscussed under section 3.3.2 that the diversity gain of the system is $LR-N+1$

if all relays have the complete CSI, and $L(R-N+1)$ if each relay has only its own channel coefficients. These results are verified by Figure 3.10. The points are obtained by simulation while the lines are traced using the approximation given in (3.69). The figure shows that the approximation is very tight for high SNRs and that the assumption under section 3.3.2 is correct.

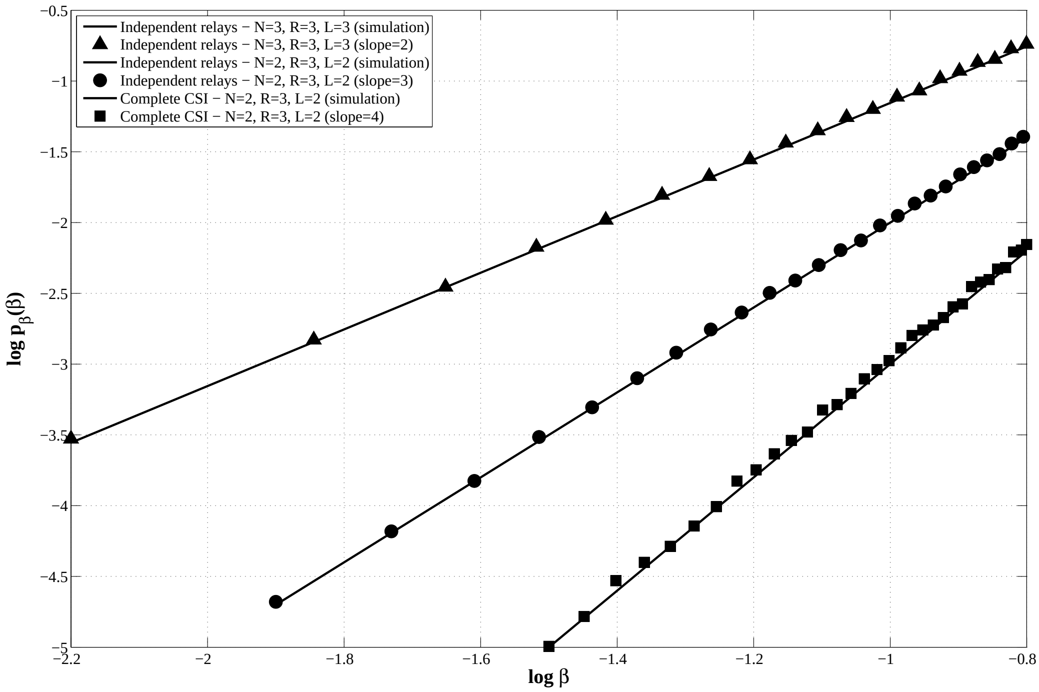


Figure 3.10: The correlation between the system architecture and the diversity gain

3.5 Conclusion

A scheme of multi-antenna multi-relay telecommunication system was addressed in this chapter. Multiple access interference was canceled out by means of multiplying the signal by proper precoding vectors at relay stations. Precoding vectors were calculated in a way to meet the requirements of two different scenarios. In the first scenario the complete CSI is supposed to be known by all RSs, while in the second one, each relay knows only its own relative channel coefficients. The former case has a better system performance and higher diversity while the later benefits from simpler design and protocol.

Using the eigenvalues of the Wishart matrix, the exact distribution of the SNR was calculated for the case of two mobile stations and arbitrary transmitter antennas. For larger numbers of transmitters, the exact value of pdf of SNR is very difficult, if not impossible, to calculate. We have thus used a mixture of 6 Nakagami distributions to approximate the pdf of SNR and to evaluate the system performance. Diversity order of the proposed scheme is $LR - N + 1$ if all relays have the complete CSI, and $L(R - N + 1)$ if each relay has only its own channel coefficients. The results were confirmed by Monte Carlo simulation.

Chapter 4

Precoding Algorithms Using Gram-Schmidt Orthonormalization Process

In this chapter we propose a new simple precoding solution based on the Gram-Schmidt orthonormalization to be used at the relay station (RS) of a multi-relay wireless network where different mobile stations use the same network, in order to mitigate the multi-user interference at each mobile station (MS). The strength of this method is that each relay only requires the channel information from itself to all of the mobile stations. In other words relays do not need to know the channel information of other relays to calculate their precoding vectors. Unlike the method discussed in the previous chapters where all mobile stations benefited from the same diversity gain, using this algorithm, some mobile stations can improve their diversity gain at the cost of a loss in the diversity order of other users. This can be used as a simple method to supply different user privileges in the case of a multi-service network. Furthermore, we perform the power allocation optimization for the relays. Analytical and accurate performance analyses for the different studied contexts are provided.

In this chapter, like the previous chapters concentrates on the link between relays and mobile stations[1]. Furthermore, we will consider the case of slowly non-frequency-selective fading channels between RSs and MSs, permitting the mobiles to feed back their channel state information (CSI) to the relays. We will study the configuration where the relays, using multiple transmit antennas, send the information corresponding to all the mobiles at the same time and at the same frequency carrier using optimized precoding vectors. The objective is to maximize the signal to noise ratio (SNR) at each MS and to mitigate the multiple access interference (MAI). This is done using the Gram-Schmidt orthonormalization process. Using this algorithm the diversity gain is not the same for all receivers and depends on the ranking of the mobile stations used during the orthogonalization process. The diversity gain for the first MS can be equal to the total number of transmit antennas

[1]The links from base station to relay stations are assumed to be ideal. See previous chapters for the arguments

in the network (i.e. number of relays multiplied by the number of antennas per relay). Obviously this is obtained at the price of a loss in the diversity order for the lower ranked MSs.

Two scenarios are addressed: i) when the transmit power of all relays are chosen randomly and ii) when the transmit power of relays is optimized. For the case of a low number of relaying stations, the optimization process substantially increases the system performance. This is done at the cost of increased complexity due to the centralized strategy. In other words in this case the relays need the CSI of all relays to mobile stations. In the case of a higher number of relaying stations, a uniform power allocation strategy will be the best solution. This strategy also preserves the advantage of only requiring the knowledge of the channel coefficients of the concerned relay.

The rest of this chapter is organized as follows: the system model is described in Section 4.1 together with the orthonormalization algorithm. The diversity gain is studied under the Section 4.2, power allocation is optimized in Section 4.3. Theoretical symbol error probability (SEP) derivations for three different power allocation criteria are given in Section 4.4. Simulation results illustrating the performances of the proposed system are provided under Section 4.5. Finally the main results are highlighted in the Section 4.6.

4.1 System model and precoding vectors

In this section we will discuss the system model, and calculate the precoding vectors using the Gram-Schmidt orthonormalization process.

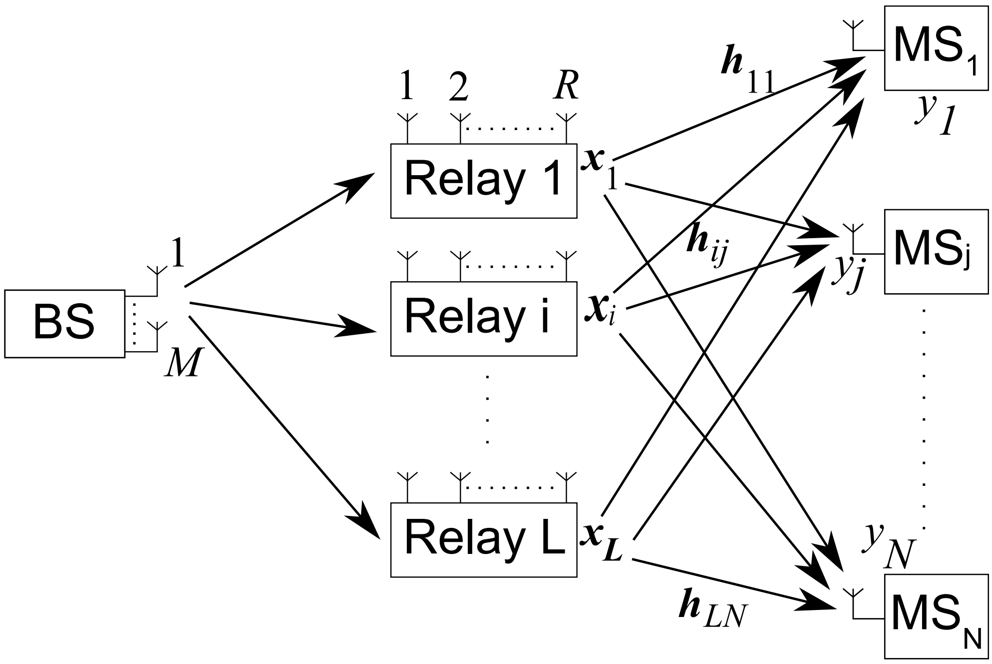


Figure 4.1: System model

4.1.1 System model

The system model is the same as the one discussed in Chapters 2 and 3. As shown in the system consists of one base station with M antennas which sends N symbols s_1 to s_N to N mobile stations MS_1 to MS_N via L fixed relays, each equipped with R antennas[2]. A two hop communication scheme is considered. In the first hop the base station sends the message to the relays. The relays will then decode the received signal and multiply it by some precoding vectors before transmitting them to mobile stations in the second hop. We will focus only on the second hop of the communication where L relays cooperate in sending each of the N data symbols to their intended mobile stations. The link from the ith relay to the jth mobile station is a flat fading Rayleigh channel $\mathbf{h}_{i,j} \sim \mathcal{CN}(0, \mathbf{I}_R)$ of size $1 \times R$. The channel state information is assumed to be known at the relays. We define the vector

[2]**Note that R denotes the number of antennas in a relay and not the number of antennas which is denoted by L**

$\boldsymbol{s}$ of transmitted symbols as: $\mathbf{s} = [s_1, s_2, ..., s_N]^T$ with s_1 to s_N being N M-ary phase-shift keying (PSK) unitary symbols ($\mid s_j \mid^2 = 1$). Since the BS to RS links are considered to be error-free, we will assume that by the end of the first hop $\mathbf{s}$ is received correctly by all relays. Each relay will then multiply its received signals by a set of precoding vectors $\mathbf{w}_i^j$ each of size $R \times 1$ and then sends it toward the mobile stations ($\mathbf{x}_i = \sum_{j=1}^{N} s_j \mathbf{w}_j^i$). Taking the same steps as (3.3) and by concatenating all received signals at the MSs in a column vector $\mathbf{y}$ we obtain:

$$\mathbf{y} = \left(\sum_{i=1}^{L} \mathbf{H}_i \mathbf{W}_i \right) \mathbf{s} + \mathbf{n} \tag{4.1}$$

with $\mathbf{y} = [y_1, y_2, \cdots y_N]^T$ and

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{h}_{i1} \\ \mathbf{h}_{i2} \\ \vdots \\ \mathbf{h}_{iN} \end{bmatrix}_{N \times R} \qquad , i = 1, \cdots, L \tag{4.2}$$

and

$$\mathbf{W}_i = \left[\begin{array}{c|c|c|c} \mathbf{w}_1^i & \mathbf{w}_2^i & \ldots & \mathbf{w}_N^i \end{array} \right]_{R \times N} \qquad , i = 1, \cdots, L \tag{4.3}$$

By defining $\mathbf{H}$ and $\mathbf{W}$ as:

$$\begin{aligned} \mathbf{H} &= \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \cdots & \mathbf{H}_L \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{21} & \cdots & \mathbf{h}_{L1} \\ \mathbf{h}_{12} & \mathbf{h}_{22} & \cdots & \mathbf{h}_{L2} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{h}_{1N} & \mathbf{h}_{2N} & \cdots & \mathbf{h}_{LN} \end{bmatrix}_{N \times RL} \end{aligned} \tag{4.4}$$

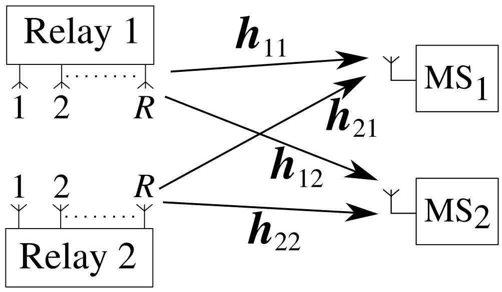


Figure 4.2: System model for two relays and two mobiles

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \vdots \\ \mathbf{W}_L \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^1 & \mathbf{w}_2^1 & \cdots & \mathbf{w}_N^1 \\ \mathbf{w}_1^2 & \mathbf{w}_2^2 & \cdots & \mathbf{w}_N^2 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{w}_1^L & \mathbf{w}_2^L & \cdots & \mathbf{w}_N^L \end{bmatrix}_{RL\times N} \tag{4.5}$$

Using we can rewrite (4.1):

$$\mathbf{y} = \mathbf{HWs} + \mathbf{n} \tag{4.6}$$

4.1.2 Precoding vectors for simplified system

For simplicity reasons at first we study the case of two RS communicating with two MS as depicted on Figure 4.2. The results will be then generalized to the case of arbitrary numbers of relays and mobile stations. In this case the received signals at the mobile stations can be written as:

$$y_1 = (\mathbf{h}_{11}^T\mathbf{w}_1^1 + \mathbf{h}_{21}^T\mathbf{w}_1^2)s_1 + (\mathbf{h}_{11}^T\mathbf{w}_2^1 + \mathbf{h}_{21}^T\mathbf{w}_2^2)s_2 + n_1 \tag{4.7}$$

$$y_2 = (\mathbf{h}_{12}^T\mathbf{w}_1^1 + \mathbf{h}_{22}^T\mathbf{w}_1^2)s_1 + (\mathbf{h}_{12}^T\mathbf{w}_2^1 + \mathbf{h}_{22}^T\mathbf{w}_2^2)s_2 + n_2 \tag{4.8}$$

The objective is to calculate the precoding vectors $\mathbf{w}_1^1$, $\mathbf{w}_1^2$, $\mathbf{w}_2^1$ and $\mathbf{w}_2^2$ in order to cancel out interference. This means that we must choose the precoding vectors of so that the coefficient ofs_2 in (4.7) and the coefficient of s_1 in (4.8) be zero. This is done by imposing $\mathbf{w}_2^1 \perp \mathbf{h}_{11}^T$, $\mathbf{w}_2^2 \perp \mathbf{h}_{21}^T$, $\mathbf{w}_1^1 \perp \mathbf{h}_{12}^T$ and $\mathbf{w}_1^2 \perp \mathbf{h}_{22}^T$. In order to maximize the signal to noise ratios at the mobile stations we must maximize the coefficient of s_1 in (4.7) and that of s_2 in (4.8); i.e. maximize $\mathbf{h}_{11}^T \mathbf{w}_1^1 + \mathbf{h}_{21}^T \mathbf{w}_1^2$ and $\mathbf{h}_{12}^T \mathbf{w}_2^1 + \mathbf{h}_{22}^T \mathbf{w}_2^2$. This optimization problem is solved using the orthogonal projection theorem as follows [61]:

Theorem 1. Among all vectors $\mathbf{z}$ satisfying $\|\mathbf{z}\| = 1$ and $\mathbf{X}^\dagger \cdot \mathbf{z} = 0$, the vector $\hat{\mathbf{z}} = \mathbf{\Pi}_{\mathbf{X}}^\perp \cdot \mathbf{y}$ with $\mathbf{\Pi}_{\mathbf{X}}^\perp = \mathbf{I} - \mathbf{X}(\mathbf{X}^\dagger \mathbf{X})^{-1} \mathbf{X}^\dagger$ maximizes the quantity $\left| \mathbf{z}^\dagger \cdot \mathbf{y} \right|$. $\square$

Applying this theorem to our optimization criteria yields:

$$\mathbf{w}_2^1 = \frac{\mathbf{\Pi}_{\mathbf{h}_{11}^*}^\perp . \mathbf{h}_{12}^*}{\left\| \mathbf{\Pi}_{\mathbf{h}_{11}^*}^\perp . \mathbf{h}_{12}^* \right\|}, \mathbf{w}_1^2 = \frac{\mathbf{\Pi}_{\mathbf{h}_{22}^*}^\perp . \mathbf{h}_{21}^*}{\left\| \mathbf{\Pi}_{\mathbf{h}_{22}^*}^\perp . \mathbf{h}_{21}^* \right\|}, \mathbf{w}_1^1 = \frac{\mathbf{\Pi}_{\mathbf{h}_{12}^*}^\perp . \mathbf{h}_{11}^*}{\left\| \mathbf{\Pi}_{\mathbf{h}_{12}^*}^\perp . \mathbf{h}_{11}^* \right\|}, \mathbf{w}_2^2 = \frac{\mathbf{\Pi}_{\mathbf{h}_{21}^*}^\perp . \mathbf{h}_{22}^*}{\left\| \mathbf{\Pi}_{\mathbf{h}_{21}^*}^\perp . \mathbf{h}_{22}^* \right\|} \tag{4.9}$$

where $\mathbf{h}^*$ denotes the complex conjugate of $\mathbf{h}$. Using the definitions in (4.9), it is straightforward to verify that $\mathbf{w}_2^1 \perp \mathbf{h}_{11}^T$:

$$\begin{aligned} \mathbf{h}_{11}^T \mathbf{w}_2^1 &= \mathbf{h}_{11}^T \frac{\mathbf{\Pi}_{\mathbf{h}_{11}^*}^\perp \mathbf{h}_{12}^*}{\left\| \mathbf{\Pi}_{\mathbf{h}_{11}^*}^\perp \mathbf{h}_{12}^* \right\|} \\ &= \mathbf{h}_{11}^T \frac{(\mathbf{I} - \mathbf{h}_{11}^* (\mathbf{h}_{11}^{*\dagger} \mathbf{h}_{11}^*)^{-1} \mathbf{h}_{11}^{*\dagger})}{\left\| \mathbf{\Pi}_{\mathbf{h}_{11}^*}^\perp \mathbf{h}_{12}^* \right\|} \mathbf{h}_{12}^* \\ &= \frac{\mathbf{h}_{11}^T - \mathbf{h}_{11}^T \mathbf{h}_{11}^* (\mathbf{h}_{11}^T \mathbf{h}_{11}^*)^{-1} \mathbf{h}_{11}^T}{\left\| \mathbf{\Pi}_{\mathbf{h}_{11}^*}^\perp \mathbf{h}_{12}^* \right\|} \mathbf{h}_{12}^* \\ &= 0 \end{aligned} \tag{4.10}$$

Similarly, we obtain $\mathbf{h}_{21}^T \mathbf{w}_2^2 = 0$, $\mathbf{h}_{12}^T \mathbf{w}_1^1 = 0$ and $\mathbf{h}_{22}^T \mathbf{w}_1^2 = 0$. This shows that $\mathbf{h}_{11}^T \mathbf{w}_1^1 + \mathbf{h}_{21}^T \mathbf{w}_1^2$ and $\mathbf{h}_{1,2}^T \mathbf{w}_2^1 + \mathbf{h}_{2,2}^T \mathbf{w}_2^2$ are maximized while the MAI cancellation criteria is satisfied.

4.1.3 Precoding vectors for arbitrary number of relays and mobiles

It is possible to generalize this case to the proposed cooperative relaying system of Figure 4.1 with arbitrary number of relays and mobiles. Denoting by $\mathbf{x}_k$, the signal sent by the kth relay , We have the following equations:

$$\begin{aligned} y_i &= \sum_{k=1}^{L} \mathbf{h}_{ki|_{1\times R}}^T \mathbf{x}_{k|_{R\times 1}} + n_i \qquad\qquad \text{for } i = 1, 2, \cdots, N \\ &= \left(\mathbf{h}_{1i}^T \mathbf{w}_1^1 + \mathbf{h}_{2i}^T \mathbf{w}_1^2 + \cdots + \mathbf{h}_{ji}^T \mathbf{w}_1^j + \cdots + \mathbf{h}_{Li}^T \mathbf{w}_1^L\right) s_1 \\ &\quad + \left(\mathbf{h}_{1i}^T \mathbf{w}_2^1 + \mathbf{h}_{2i}^T \mathbf{w}_2^2 + \cdots + \mathbf{h}_{ji}^T \mathbf{w}_2^j + \cdots + \mathbf{h}_{Li}^T \mathbf{w}_2^L\right) s_2 + \\ &\quad + \cdots + \left(\mathbf{h}_{1i}^T \mathbf{w}_i^1 + \mathbf{h}_{2i}^T \mathbf{w}_i^2 + \cdots + \mathbf{h}_{ji}^T \mathbf{w}_i^j + \cdots + \mathbf{h}_{Li}^T \mathbf{w}_i^L\right) s_i + \cdots \\ &\quad + \left(\mathbf{h}_{1i}^T \mathbf{w}_N^1 + \mathbf{h}_{2i}^T \mathbf{w}_N^2 + \cdots + \mathbf{h}_{ji}^T \mathbf{w}_N^j + \cdots + \mathbf{h}_{Li}^T \mathbf{w}_N^L\right) s_N + n_i \end{aligned} \tag{4.11}$$

Interference cancellation can be achieved with $\mathbf{w}_k^l \perp V_{\mathbf{h}}^{(l,k)}$ with $V_{\mathbf{h}}^{(l,k)}$ being the set of vectors $\{\mathbf{h}_{l1}, \mathbf{h}_{l2}, ..., \mathbf{h}_{l,k-1}, \mathbf{h}_{l,k+1}, ..., \mathbf{h}_{lN}\}$ for $k \in [1, N]$ and $l \in [1, L]$. The solution is calculated using the orthonormalization process of Gram-Schmidt [62]. It suffices to span the subspace defined by $V_{\mathbf{h}}^{(l,k)}$ with a set of $N-1$ orthonormal vectors (i.e. Gram-Schmidt process), and then find the $\mathbf{w}_k^l$ so that it spans the vector space formed by $\left\{V_{\mathbf{h}}^{(l,k)}, \mathbf{h}_{lk}^*\right\}$ among with the vectors produced by the Gram-Schmidt process.

The Gram-Schmidt process. We define the projection of the vector $\mathbf{v}$ orthogonally onto the vector $\mathbf{u}$ (see Figure 4.3):

$$\operatorname{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\langle \mathbf{v}, \mathbf{u} \rangle}{\langle \mathbf{u}, \mathbf{u} \rangle} \mathbf{u} = \frac{\langle \mathbf{v}, \mathbf{u} \rangle}{\|\mathbf{u}\|} \mathbf{u} \tag{4.12}$$

where $\langle \mathbf{v}, \mathbf{u} \rangle$ denotes the inner product of vectors $\mathbf{v}$ and $\mathbf{u}$.

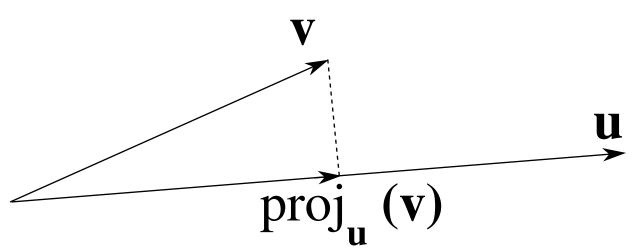


Figure 4.3: The projection of $\mathbf{v}$ onto the vector $\mathbf{u}$

The Gram-Schmidt process then works as follows for $l = 1, 2, \cdots L$:

$$
\begin{aligned}
&\mathbf{u}_1 = \mathbf{h}_{l1}, & \mathbf{e}_{l,1} &= \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} \\
&\mathbf{u}_2 = \mathbf{h}_{l2} - \mathrm{proj}_{\,\mathbf{u}_1}(\mathbf{h}_{l2}), & \mathbf{e}_{l,2} &= \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} \\
&\mathbf{u}_3 = \mathbf{h}_{l3} - \mathrm{proj}_{\,\mathbf{u}_1}(\mathbf{h}_{l3}) - \mathrm{proj}_{\,\mathbf{u}_2}(\mathbf{h}_{l3}), & \mathbf{e}_{l,3} &= \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|} \\
&\vdots & &\vdots \\
&\mathbf{u}_m = \mathbf{h}_{lm} - \sum_{j=1}^{m-1} \mathrm{proj}_{\,\mathbf{u}_j}(\mathbf{h}_{lm}), & \mathbf{e}_{l,m} &= \frac{\mathbf{u}_m}{\|\mathbf{u}_m\|} \\
&\vdots & &\vdots \\
&\mathbf{u}_N = \mathbf{h}_{lN} - \sum_{j=1}^{N-1} \mathrm{proj}_{\,\mathbf{u}_j}(\mathbf{h}_{lN}), & \mathbf{e}_{l,N} &= \frac{\mathbf{u}_N}{\|\mathbf{u}_N\|}
\end{aligned}
\tag{4.13}
$$

□

Using this new vector set it is straightforward to calculate the normalized precoding vectors. What we need to do is to project the vector $\mathbf{h}_{lk}^*$ to a direction that is orthogonal to the set $\{\mathbf{u}_{l,1}, \mathbf{u}_{l,2}, \cdots, \mathbf{u}_{l,k-1}, \mathbf{u}_{l,k+1}, \cdots, \mathbf{u}_{l,N}\}$. This can be seen as another step of Gram-Schmidt orthonormalization. Note that each step of the Gram-Schmidt process takes a vector ($\mathbf{h}_{lm}$ in the process) and transforms it to a vector which is orthogonal to all previous vectors. The received signal at MS_i is then given by:

$$\begin{aligned}
y_i &= \left(\mathbf{h}_{1i}^T\mathbf{w}_i^1 + \mathbf{h}_{2,i}^T\mathbf{w}_i^2 + \cdots + \mathbf{h}_{ki}^T\mathbf{w}_i^k + \cdots + \mathbf{h}_{Li}^T\mathbf{w}_i^L\right) s_i + n_i \\
&= \left(\mathbf{h}_{1i}^T \frac{\mathbf{h}_{1i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}}{\left\|\mathbf{h}_{1i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}\right\|} + \mathbf{h}_{2i}^T \frac{\mathbf{h}_{2i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{2i}^*, \mathbf{e}_{2,j}\rangle \mathbf{e}_{2,j}}{\left\|\mathbf{h}_{2i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{2i}^*, \mathbf{e}_{2,j}\rangle \mathbf{e}_{2,j}\right\|}\right. \\
&\quad \left. + \cdots + \mathbf{h}_{Li}^T \frac{\mathbf{h}_{Li}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}}{\left\|\mathbf{h}_{Li}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}\right\|}\right) s_i + n_i
\end{aligned} \tag{4.14}$$

Using $R-N$ additional vectors to complete the vector set produced from the Gram-Schmidt process $\{\mathbf{e}_{l,1}, \mathbf{e}_{l,2}, ..., \mathbf{e}_{l,k-1}, \mathbf{e}_{l,k+1}, ..., \mathbf{e}_{l,N}\}$, the complete basis can be expressed as $\{\mathbf{e}_{l,1}, \mathbf{e}_{l,2}, ..., \mathbf{e}_{l,k-1}, \mathbf{e}_{l,k}, \mathbf{e}_{l,k+1}, ..., \mathbf{e}_{l,N}, \mathbf{e}_{l,N+1}, ..., \mathbf{e}_{l,R}\}$. We can express $\mathbf{h}_{li}^*$ in this new basis to obtain $\mathbf{h}_{li}^* = \sum_{j=1}^{R} \langle \mathbf{h}_{li}^*, \mathbf{e}_{lj}\rangle \mathbf{e}_{l,j}$.

Then:

$$\begin{aligned}
y_i &= \left(\mathbf{h}_{1i}^T \frac{\sum_{j=1}^{R} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j} - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}}{\left\|\mathbf{h}_{1i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}\right\|}\right. \\
&\quad \left. + \cdots + \mathbf{h}_{Li}^T \frac{\sum_{j=1}^{R} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j} - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}}{\left\|\mathbf{h}_{Li}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}\right\|}\right) s_i + n_i \\
&= \left(\mathbf{h}_{1i}^T \frac{\langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,i}\rangle \mathbf{e}_{1,i} + \sum_{j=N+1}^{R} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}}{\left\|\mathbf{h}_{1i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}\right\|}\right. \\
&\quad \left. + \cdots + \mathbf{h}_{Li}^T \frac{\langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,i}\rangle \mathbf{e}_{L,i} + \sum_{j=N+1}^{R} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}}{\left\|\mathbf{h}_{Li}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}\right\|}\right) s_i + n_i \\
&= (\left\|\mathbf{h}_{1i}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{1i}^*, \mathbf{e}_{1,j}\rangle \mathbf{e}_{1,j}\right\| + \cdots + \left\|\mathbf{h}_{Li}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{Li}^*, \mathbf{e}_{L,j}\rangle \mathbf{e}_{L,j}\right\|) s_i + n_i \\
&= \left(\sum_{k=1}^{L} \lambda_{k,i}\right) s_i + n_i
\end{aligned} \tag{4.15}$$

with $\lambda_{k,i}^2 = \left\| \mathbf{h}_{ki}^* - \sum_{j=1,j\neq i}^{N} \langle \mathbf{h}_{ki}^*, \mathbf{e}_{kj} \rangle \mathbf{e}_{k,j} \right\|^2 = |\langle \mathbf{h}_{ki}, \mathbf{e}_{k,i} \rangle|^2 + \sum_{j=N+1}^{R} |\langle \mathbf{h}_{ki}, \mathbf{e}_{k,j} \rangle|^2$.

4.2 Diversity order study

To obtain an accurate estimate of the potential diversity gain, we have to characterize the random variables $\lambda_{k,i}^2$. To do this, using the Gram-Schmidt process, we can write:

$$\mathbf{e}_{k,i} = \mu_{k,i} \left(\mathbf{h}_{ki} - \sum_{j=1}^{i-1} \langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle \mathbf{e}_{k,j} \right) \tag{4.16}$$

with:

$$\mu_{k,i} = \frac{1}{\left\| \mathbf{h}_{ki} - \sum_{j=1}^{i-1} \mathrm{proj}_{\,\mathbf{e}_{k,j}}(\mathbf{h}_{ki}) \right\|} \tag{4.17}$$

Then, using the orthogonal basis, we have $\mathbf{h}_{ki} = \sum_{j=1}^{R} \langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle \mathbf{e}_{k,j}$. Substituting this into (4.16) yields:

$$\mathbf{e}_{k,i} = \mu_{k,i} \left(\sum_{j=1}^{R} \langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle \mathbf{e}_{k,j} - \sum_{j=1}^{i-1} \langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle \mathbf{e}_{k,j} \right) = \mu_{k,i} \sum_{j=i}^{R} \langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle \mathbf{e}_{k,j} \tag{4.18}$$

from (4.18) we can see that for $j > i$, $\langle \mathbf{e}_{k,j}, \mathbf{h}_{k,i} \rangle = 0$. Using this property, $\lambda_{k,i}^2$ can now be simplified as:

$$\lambda_{k,i}^2 = \mu_{k,i}^2 \left\langle \mathbf{h}_{ki}, \mathbf{h}_{ki} - \sum_{j=1}^{i-1} \langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle \mathbf{e}_{k,j} \right\rangle = \mu_{k,i}^2 \left(\langle \mathbf{h}_{ki}, \mathbf{h}_{ki} \rangle - \sum_{j=1}^{i-1} |\langle \mathbf{e}_{k,j}, \mathbf{h}_{ki} \rangle|^2 \right) \tag{4.19}$$

Using (4.19) we can obtain the characterization of $\lambda_{k,i}^2$ by mathematical induction

- For $i = 1$, we have $\mathbf{e}_{k,1} = \mathbf{h}_{k1} / \|\mathbf{h}_{k1}\|$ thus:

$$\lambda_{k,1}^2 = |\langle \mathbf{h}_{k1}, \mathbf{e}_{k,1} \rangle|^2 = \left| \left\langle \mathbf{h}_{k1}, \frac{\mathbf{h}_{k1}}{\|\mathbf{h}_{k,1}\|} \right\rangle \right|^2 = \|\mathbf{h}_{k1}\|^2 \tag{4.20}$$

Since $\mathbf{h}_{k1}$ is a vector of R complex Gaussien random components each of which with zero mean and a variance equal to 0.5, the random variable $\lambda_{k,1}^2$ may be written as

$$\lambda_{k,1}^2 = \sum_{i=1}^{R} |h_{k1}(i)|^2 = \sum_{i=1}^{R} \left[\left(h_{k1}^R(i)\right)^2 + \left(h_{k1}^I(i)\right)^2 \right] \tag{4.21}$$

It is straightforward to conclude that $\lambda_{k,1}^2$ is a chi-square variable with $2R$ degrees of freedom.

- For $i = 2$, we have

$$\mathbf{e}_{k,2} = \frac{\mathbf{h}_{k2} - \langle \mathbf{e}_{k,1}, \mathbf{h}_{k2} \rangle \, \mathbf{e}_{k,1}}{\|\mathbf{h}_{k2} - \langle \mathbf{e}_{k,1}, \mathbf{h}_{k2} \rangle \, \mathbf{e}_{k,1}\|} \tag{4.22}$$

Hence we obtain

$$\begin{aligned} \lambda_{k,2}^2 &= |\langle \mathbf{h}_{k2}, \mathbf{e}_{k,2} \rangle|^2 \\ &= \left| \left\langle \mathbf{h}_{k2}, \frac{\mathbf{h}_{k2} - \langle \mathbf{e}_{k,1}, \mathbf{h}_{k2} \rangle \, \mathbf{e}_{k,1}}{\|\mathbf{h}_{k2} - \langle \mathbf{e}_{k,1}, \mathbf{h}_{k2} \rangle \, \mathbf{e}_{k,1}\|} \right\rangle \right|^2 \\ &= \mu_{k,2}^2 \left(\langle \mathbf{h}_{k2}, \mathbf{h}_{k2} \rangle - \left| \left\langle \frac{\mathbf{h}_{k1}}{\|\mathbf{h}_{k1}\|}, \mathbf{h}_{k2} \right\rangle \right|^2 \right) \\ &= \mu_{k,2}^2 \left(\langle \mathbf{h}_{k2}, \mathbf{h}_{k2} \rangle - \frac{|\langle \mathbf{h}_{k1}, \mathbf{h}_{k2} \rangle|^2}{\|\mathbf{h}_{k1}\|^2} \right) \end{aligned} \tag{4.23}$$

The term $|\langle \mathbf{h}_{k1}, \mathbf{h}_{k2} \rangle|^2$ can be calculated as:

$$\begin{aligned} \langle \mathbf{h}_{k1}, \mathbf{h}_{k2} \rangle &= \sum_{i=1}^{R} \left[h_{k1}^R(i) + j h_{k1}^I(i) \right] \left[h_{k2}^R(i) + j h_{k2}^I(i) \right] \\ &= \sum_{i=1}^{R} \left[h_{k1}^R(i) h_{k2}^R(i) - h_{k1}^I(i) h_{k2}^I(i) \right] \\ &\quad + j \sum_{i=1}^{R} \left[h_{k,1}^R(i).h_{k,2}^I(i) + h_{k,1}^I(i).h_{k,2}^R(i) \right] \end{aligned} \tag{4.24}$$

And this yields:

$$\frac{|\langle \mathbf{h}_{k1}, \mathbf{h}_{k2}\rangle|^2}{\|\mathbf{h}_{k1}\|^2} = \frac{\sum_{i=1}^{R} \left[h_{k1}^R(i)h_{k,2}^R(i) - h_{k1}^I(i)h_{k2}^I(i)\right]^2}{\sum_{i=1}^{R} \left[|h_{k1}^R(i)|^2 + |h_{k1}^I(i)|^2\right]} + \frac{\sum_{i=1}^{R} \left[h_{k1}^R(i)h_{k2}^I(i) + h_{k1}^I(i)h_{k2}^R(i)\right]^2}{\sum_{i=1}^{R} \left[|h_{k1}^R(i)|^2 + |h_{k1}^I(i)|^2\right]} \tag{4.25}$$

Taking the mean of this expression and using the same approximation as in [63]:

$$\mathbb{E}\left\{\frac{|\langle \mathbf{h}_{k1}, \mathbf{h}_{k2}\rangle|^2}{\|\mathbf{h}_{k1}\|^2}\right\} \approx \frac{\mathbb{E}\left\{|\langle \mathbf{h}_{k1}, \mathbf{h}_{k2}\rangle|^2\right\}}{\mathbb{E}\left\{\|\mathbf{h}_{k1}\|^2\right\}} \tag{4.26}$$

Using the approximation in (4.26) after some basic mathematical developments we obtain $\mathbb{E}\left\{\frac{|\langle \mathbf{h}_{k1}, \mathbf{h}_{k2}\rangle|^2}{\|\mathbf{h}_{k1}\|^2}\right\} = 1$. This entails that the random variable $\lambda_{k,2}^2$ is a chi-square variable with $2(R-1)$ degrees of freedom.

- Now let us assume that the random variable $\lambda_{k,i-1}^2$ is a chi-square random variable with $2\left[R-(i-2)\right]$ degrees of freedom. We search to obtain the distribution of $\lambda_{k,i}^2$. We have:

$$\begin{aligned}\lambda_{k,i}^2 &= \mu_{k,i}^2 \left[\langle \mathbf{h}_{ki}, \mathbf{h}_{ki}\rangle - \sum_{j=1}^{i-1} \left|\left\langle \mathbf{e}_{k,j}, \mathbf{h}_{ki}\right\rangle\right|^2\right] \\ &= \mu_{k,i}^2 \left[\langle \mathbf{h}_{ki}, \mathbf{h}_{ki}\rangle - \sum_{j=1}^{i-2} \left|\left\langle \mathbf{e}_{k,j}, \mathbf{h}_{ki}\right\rangle\right|^2 - \left|\left\langle \mathbf{e}_{k,i-1}, \mathbf{h}_{ki}\right\rangle\right|^2\right]\end{aligned} \tag{4.27}$$

Considering the hypothesis on $\lambda_{k,i-1}^2$ we know that $\langle \mathbf{h}_{ki}, \mathbf{h}_{ki}\rangle - \sum_{j=1}^{i-2} \left|\left\langle \mathbf{e}_{k,j}, \mathbf{h}_{ki}\right\rangle\right|^2$ is a chi-square variable with $2\left[R-(i-2)\right]$ degrees of freedom. Subtracting the quantity $\left|\left\langle \mathbf{e}_{k,i-1}, \mathbf{h}_{ki}\right\rangle\right|^2$ and using the same method as the case $i=2$, we can prove that $\lambda_{k,i}^2$ is a chi-square random variable with $2\left[R-(i-1)\right]$ degrees of freedom.

We proved that if the approximation in (4.26) is verified, the random variable $\lambda_{k,i}^2$ is a chi-square random variable with $2\,[R-(i-1)]$ degrees of freedom. Since the chi-square distribution is a special case of the gamma distribution, we will assume that the the $\lambda_{k,i}^2$ is a Gamma distributed random variable:

$$\lambda_{k,i}^2(x) \sim g(\alpha,\beta,x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \tag{4.28}$$

This result may be confirmed using the Expectation-Maximization (EM) algorithm[3] [64, 65]. As an example, we use the EM algorithm with $J = 1$ (a pure Gamma distribution) to test the behavior of random variables $\lambda_{k,i}^2$. We take $R = 10$ and $N = 5$. The probability distribution function (pdf) of $\lambda_{k,1}^2$ is depicted in Figure 4.4 and for $\lambda_{k,2}^2$ in Figure 4.5. Since the same channel statistics are assumed for different relays, the curves for different values of k are identical. The curves on the right are traced in log-log scale so that the slopes of the curves at the origin (determining the diversity order) would be easily seen. Using the Expectation-Maximization algorithm, we obtain $\alpha = 10$ and $\beta = 1$ for Figure 4.4 and $\alpha = 9$ and $\beta = 1$ for Figure 4.5. We can see that for the second MS ($\lambda_{k,2}^2$), the slope at the origin of the log-log curve is smaller than that of the first mobile station ($\lambda_{k,1}^2$). Note that the approximated distribution is a very tight evaluation of the simulated results, we have thus validated the fact that $\lambda_{k,i}^2$ has a chi-square distribution with $2R - 2i + 2$ degrees of freedom. Since each degree of freedom contributes 0.5 to the diversity order[4], the diversity order is equal to $D = R - (i-1) - 1$.

[3]For the use of EM algorithm see Section 3.3.3 on page 78

[4]By replacing $e^{-\beta x}$ in (4.28) by its Taylor series development, the smallest exponent of x in (4.28) is $\alpha - 1$. As a result the diversity order will be α

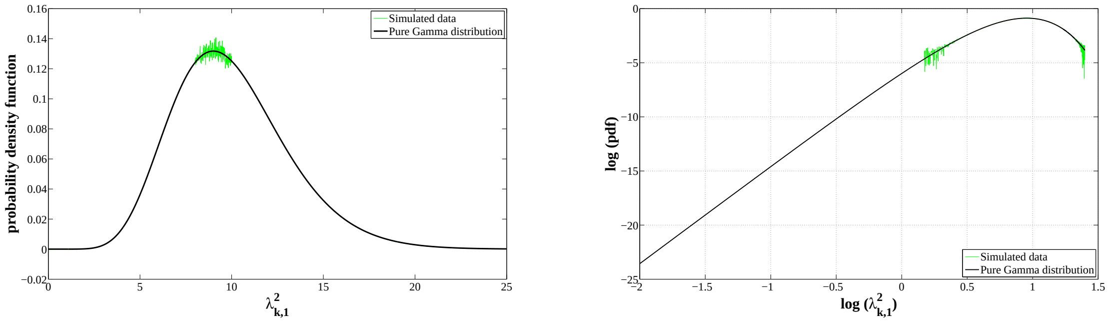


Figure 4.4: pdf of $\lambda_{k,1}^2$ for $R = 10$ and $N = 5$

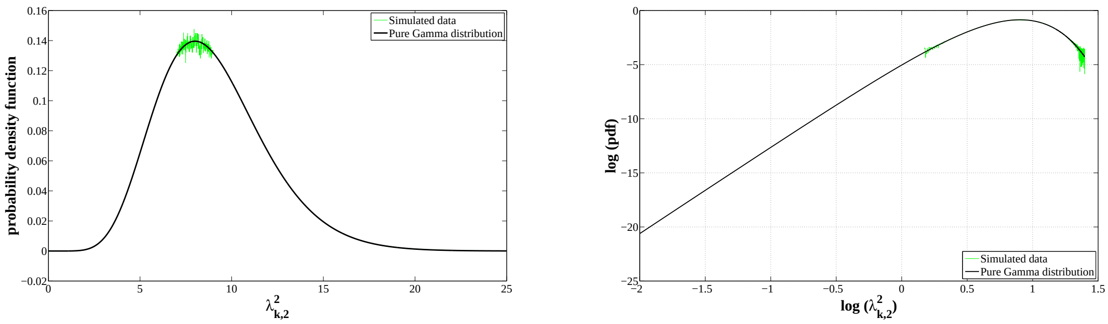
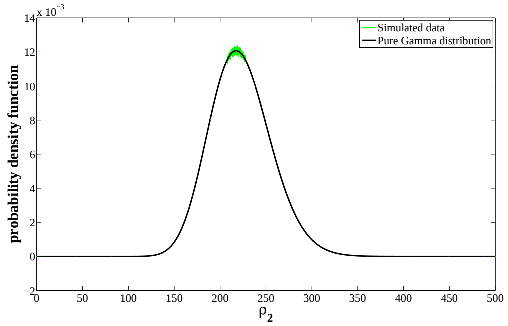


Figure 4.5: pdf of $\lambda_{k,2}^2$ for $R = 10$ and $N = 5$

The potential diversity gain at each MS can now be calculated. We have the relationship $y_i = \left[\sum_{k=1}^{L} \lambda_{k,i}\right] s_i + n_i$. The SNR is then defined as:

$$\rho_i = \frac{\left[\displaystyle\sum_{k=1}^{L} \lambda_{k,i}\right]^2}{\sigma_n^2} \tag{4.29}$$

It can be proved [66] by induction that ρ_i in (4.29) leads to a diversity order of $L\left[R-(i-1)\right]-1$. This result is verified by simulation, for $R = 10$, $N = 5$, and $L = 5$, we have $\rho_2 = \frac{\left[\sum_{k=1}^{L} \lambda_{k,2}\right]^2}{\sigma_n^2}$. As we can see in Figure 4.6, a diversity order of 43.5 is obtained for this system. This confirms that the diversity order is nearly equal



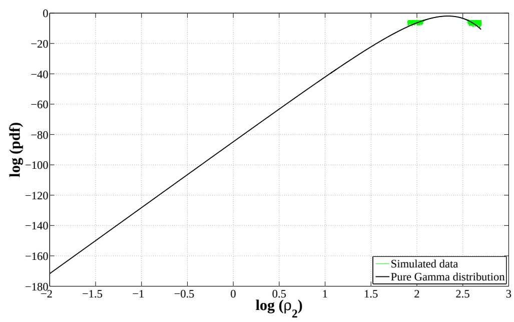


Figure 4.6: pdf of ρ_2 for $R = 10$, $N = 5$, and $L = 5$

to $L\left[R-(i-1)\right]-1 = 5\times(10-1)-1 = 44$.

4.3 Power Allocation optimization

We are now looking for power allocation algorithms in order to optimize the average SEP at each MS. Using the solution in (4.15), we can define the average transmitted power of each relay as:

$$\bar{P}_i = \sum_{k=1}^{N} \mathbb{E}\left\{s_k^* \mathbf{w}_k^{i\dagger}\mathbf{w}_k^i s_k\right\} = \sum_{k=1}^{N} \mathbb{E}\left\{\mathbf{w}_k^{i\dagger}\mathbf{w}_k^i\right\}\mathbb{E}\left\{s_k^* s_k\right\} = \sum_{k=1}^{N} \mathbb{E}\left\{\mathbf{w}_k^{i\dagger}\mathbf{w}_k^i\right\} = N \qquad (4.30)$$

By normalizing the transmitted power of each RS to one ($\bar{P}_i = 1$), the precoding vectors can be calculated from:

$$\mathbf{w}_k'^{l} = \frac{1}{\sqrt{N}}\mathbf{w}_k^l = \frac{1}{\sqrt{N}} \frac{\left[\mathbf{h}_{lk}^* - \displaystyle\sum_{j=1, j\neq k}^{N} \left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,j}\right\rangle \mathbf{e}_{l,j}\right]}{\left\|\mathbf{h}_{l,k}^* - \displaystyle\sum_{j=1, j\neq k}^{N} \left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,j}\right\rangle \mathbf{e}_{l,j}\right\|} \qquad (4.31)$$

We obtain $\lambda_{k,i}' = \frac{1}{\sqrt{N}}\lambda_{k,i}$. Denoting the transmit power of kth relay by P_k, we can calculate the received signal at a mobile station:

$$y_i = \left[\sum_{k=1}^{L} \sqrt{P_k} \lambda'_{k,i} \right] s_i + n_i \tag{4.32}$$

The signal to noise ratio is then:

$$\rho_i = \frac{\left(\sum_{k=1}^{L} \sqrt{P_k} \lambda'_{k,i} \right)^2}{\sigma_n^2} \tag{4.33}$$

We use this equation and the approximation of SEP given in [60]:

$$p_{e_i}(\rho_i) = 2\gamma \,\mathrm{erfc}(\sqrt{\xi \rho_i}) - \left[\gamma \,\mathrm{erfc}(\sqrt{\xi \rho_i}) \right]^2 \approx 2\gamma \mathrm{erfc}(\sqrt{\xi \rho_i}) \tag{4.34}$$

where $\gamma = 1 - 1/\sqrt{M}$ and $\xi = \frac{3}{2(M-1)}$ for a M-ary phase-shift keying (MPSK) constellation, and $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-t^2/2} dt$ being the complementary error function. The objective being to minimize the average SER for the complete set of mobile stations, the Power allocation problem can be described as:

$$\begin{cases} \text{minimize:} & \bar{p}_e = \frac{1}{N} \sum_{i=1}^{N} P_{e_i}(\rho_i) = \frac{2}{N} \gamma \sum_{i=1}^{N} \mathrm{erfc}(\sqrt{\xi \rho_i}) \\ \text{subject to :} & \sum_{k=1}^{L} P_k = P_T = C^{ste} \end{cases} \tag{4.35}$$

As a result, the cost function can be obtained as:

$$\begin{aligned} J(P_1, P_2, \cdots, P_L) &= \overline{p}_e + \eta \left(\sum_{k=1}^{L} P_k - P_T \right) \\ J(P_1, P_2, \cdots, P_L) &= (2\gamma/N) \sum_{i=1}^{N} \mathrm{erfc}(\sqrt{\xi \rho_i}) + \eta \left(\sum_{k=1}^{L} P_k - P_T \right) \end{aligned} \tag{4.36}$$

Calculating the derivative $\partial J / \partial P_k$, we obtain:

$$\frac{\partial J}{\partial P_k} = -2\gamma\frac{\sqrt{\xi}}{N}\sigma\sqrt{\pi P_k}\sum_{i=1}^{N}\lambda'_{k,i}\exp\left[-\xi(\sum_{k=1}^{L}\sqrt{P_k}\lambda'_{k,i})^2/\sigma^2\right] + \eta \tag{4.37}$$

Setting this derivative to zero, we obtain the equation:

$$\eta = 2\gamma\frac{\sqrt{\xi}}{N}\sigma\sqrt{\pi P_k}\sum_{i=1}^{N}\lambda'_{k,i}\exp\left[-\xi(\sum_{k=1}^{L}\sqrt{P_k}\lambda'_{k,i})^2/\sigma^2\right] \quad \forall k \in [1, L] \tag{4.38}$$

This yields:

$$\sqrt{P_k} \propto \sum_{i=1}^{N}\lambda'_{k,i}\exp[-\xi(\sum_{m=1}^{L}\sqrt{P_m}\lambda'_{m,i})^2/\sigma^2] \tag{4.39}$$

And we can write:

$$P_k = A^2\left[\sum_{i=1}^{N}\lambda'_{k,i}\exp[-\xi(\sum_{m=1}^{L}\sqrt{P_m}\lambda'_{m,i})^2/\sigma^2]\right]^2 \tag{4.40}$$

with A^2 is the positive proportionality constant. Rewriting (4.40) for each relay station and summing up all the different terms, we obtain:

$$\sum_{m=1}^{L}\sqrt{P_m}\lambda'_{m,i} = A\sum_{m=1}^{L}\sum_{i=1}^{N}\lambda'^2_{m,i}\exp[-\xi(\sum_{m=1}^{L}\sqrt{P_m}\lambda'_{m,i})^2/\sigma^2] \tag{4.41}$$

We will first calculate $X_i = \sum_{m=1}^{L}\sqrt{P_m}\lambda'_{m,i}$. From (4.41), X_i is the solution of the equation:

$$X_i = A\sum_{m=1}^{L}\sum_{i=1}^{N}\lambda'^2_{m,i}\exp\left[-\xi X_i^2/\sigma^2\right] \tag{4.42}$$

Using the power constraint $\sum_{k=1}^{L}P_k = P_T$, we have:

$$A = \frac{\sqrt{P_T}}{\sqrt{\sum_{k=1}^{L}\left[\sum_{i=1}^{N}\lambda'_{k,i}\exp\left[-\xi X_i^2/\sigma^2\right]\right]^2}} \tag{4.43}$$

Finally, we obtain:

$$X_i = \frac{\sqrt{P_T}\sum_{m=1}^{L}\sum_{i=1}^{N}\lambda_{m,i}^{'2}\exp[-\xi X_i^2/\sigma^2]}{\sqrt{\sum_{k=1}^{L}\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\exp[-\xi X_i^2/\sigma^2]\right]^2}} \quad i = 1, ..., N \tag{4.44}$$

It is possible to solve this set of non-linear equations using the MATLAB function `fsolve`. As soon as the quantities X_i are found, the optimum power P_k may be calculated as:

$$P_k = P_T \frac{\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\exp[-\xi X_i^2/\sigma^2]\right]^2}{\sum_{k=1}^{L}\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\exp[-\xi X_i^2/\sigma^2]\right]^2} \tag{4.45}$$

The equation presented in (4.44) is complicated to solve. We will simplify it using the low SNR approximation. Fortunately, the simulation results in section 4.5 show that the low SNR approximation can be also used for high SNRs. If we consider the low SNR regime we have $\exp[-\xi(\sum_{k=1}^{L}\sqrt{P_k}\lambda_{k,i}^{'})^2/\sigma^2] \approx 1$ and equation (4.40) reduces to:

$$P_k = A^2\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\right]^2 \tag{4.46}$$

The power constraint yields:

$$\sum_{k=1}^{L}P_k = A^2\sum_{k=1}^{L}\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\right]^2 = P_T \Rightarrow A^2 = \frac{P_T}{\sum_{k=1}^{L}[\sum_{i=1}^{N}\lambda_{k,i}^{'}]^2} \tag{4.47}$$

We can thus write:

$$P_k = A^2\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\right]^2 = \frac{P_T}{\sum_{k=1}^{L}[\sum_{i=1}^{N}\lambda_{k,i}^{'}]^2}\left[\sum_{i=1}^{N}\lambda_{k,i}^{'}\right]^2 \tag{4.48}$$

The received signal at the ith MS can be expressed as:

$$y_i = \sqrt{P_T} \frac{\left[\sum_{k=1}^{L} [\sum_{j=1}^{N} \lambda'_{k,j}] \lambda'_{k,i}\right]}{\sqrt{\sum_{k=1}^{L} \left[\sum_{j=1}^{N} \lambda'_{k,j}\right]^2}} s_i + n_i \tag{4.49}$$

4.4 Theoretical SER performance evaluation

In order to evaluate the system performance, we consider three cases: i) the case of uniform power allocation for which diversity gain has already been investigated at the end of Section 4.2, ii) the case of power allocation optimization introduced in the last section and iii) the case of non-uniform power allocation which is the same case as in Section 4.2 without normalizing all relay station powers to 1.

4.4.1 Uniform power allocation

For the case of uniform power allocation, the results of the Expectation-Maximization algorithm confirm that a gamma distribution with pdf $\gamma(x,\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ is a good approximation for the distribution of the random variable $[\sum_{k=1}^{L} \lambda'_{k,i}]^2$ with $\alpha = L\left(R-(i-1)\right)$ and $\beta = 1/L$. The SNR is then given by $\rho_i = \frac{[\sum_{k=1}^{L} \lambda'_{k,i}]^2}{\sigma^2}$ with $\sigma^2 = P_u.10^{-SNR_{\text{dB}}/10}$ and $P_u = \mathbb{E}\left\{(\sum_{k=1}^{L} \lambda'_{k,i})^2\right\}$. We have $P_u \approx L\alpha$. The pdf of the SNR can be approximated by:

$$h(x) = \sigma^2 \gamma(\sigma^2 x, \alpha, \beta) = \frac{\sigma^2.\beta^\alpha}{\Gamma(\alpha)} [\sigma^2 x]^{\alpha-1} e^{-\beta\sigma^2 x} = \frac{\sigma^{2\alpha}\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta\sigma^2 x} \tag{4.50}$$

The SEP for a given SNR or σ^2 is then given by:

$$\bar{p}_e = \int_0^{+\infty} p_e(u) h(u) du = \frac{2\gamma\sigma^{2\alpha}\beta^\alpha}{\Gamma(\alpha)} \int_0^{+\infty} u^{\alpha-1} e^{-\beta\sigma^2 u} \text{erfc}(\sqrt{\xi u}) du \tag{4.51}$$

To calculate (4.51) we notice that $\alpha - 1$ is an integer. We calculate:

$$I_{\alpha,\beta} = \int_0^{+\infty} u^{\alpha-1} e^{-\beta\sigma^2 u} \mathrm{erfc}(\sqrt{\xi u}) du \tag{4.52}$$

Using the variable substitution $t = \beta\sigma^2 u$, we obtain:

$$I_{\alpha,\beta} = \frac{1}{(\beta\sigma^2)^\alpha} \int_0^{+\infty} t^{\alpha-1} e^{-t} \mathrm{erfc}(\sqrt{\frac{\xi t}{\beta\sigma^2}}) dt \tag{4.53}$$

Integration by parts yields:

$$v = \mathrm{erfc}(\sqrt{\frac{\xi t}{\beta\sigma^2}}) \quad \Rightarrow \quad dv = \frac{-2}{\sqrt{\pi}} \sqrt{\frac{\xi}{\beta\sigma^2}} \frac{1}{2\sqrt{t}} e^{-\xi t/\beta\sigma^2} dt \tag{4.54}$$

$$dw = t^{\alpha-1} e^{-t} dt \quad \Rightarrow \quad w = -\sum_{k=0}^{\alpha-1} \frac{(\alpha-1)!}{k!} t^k e^{-t} \tag{4.55}$$

We obtain then:

$$I_{\alpha,\beta} = \frac{(\alpha-1)!}{(\beta.\sigma^2)^\alpha} \left[1 - \sum_{k=0}^{\alpha-1} \sqrt{\frac{\xi}{\beta\sigma^2}} \frac{1}{\sqrt{\pi} k!} \int_0^{+\infty} t^{k-1/2} e^{-t(1+\xi/\beta\sigma^2)} dt \right] \tag{4.56}$$

In order to calculate integral in (4.56), we set $x = (1 + \xi/\beta\sigma^2)t$:

$$\int_0^{+\infty} t^{k-1/2} e^{-t(1+\frac{\xi}{\beta\sigma^2})} dt = \frac{1}{(1+\frac{\xi}{\beta\sigma^2})^{k+\frac{1}{2}}} \Gamma(k + \frac{1}{2}) \tag{4.57}$$

with $\Gamma(z)$ being the Gamma function. Using the property $\Gamma(n+\frac{1}{2}) = \sqrt{\pi}\frac{(2n)!}{2^{2n} n!}$ we obtain:

$$I_{\alpha,\beta} = \frac{(\alpha-1)!}{(\beta\sigma^2)^\alpha} \left[1 - \sum_{k=0}^{\alpha-1} \sqrt{\frac{\xi}{\beta\sigma^2}} \frac{(2k)!}{2^{2k} k!^2 (1+\frac{\xi}{\beta\sigma^2})^{k+1/2}} \right] \tag{4.58}$$

And finally, replacing (4.58) in (4.51) we obtain:

$$\begin{aligned}\bar{p}_e &= \frac{2\gamma\sigma^{2\alpha}\beta^{\alpha}}{\Gamma(\alpha)} I_{\alpha,\beta} \\ &= 2\gamma\left[1-\sqrt{\frac{\xi}{\beta\sigma^2}}\frac{1}{\sqrt{\frac{\xi}{\beta\sigma^2}+1}}\sum_{k=0}^{\alpha-1}\frac{(2k)!}{2^{2k}k!^2(1+\frac{\xi}{\beta\sigma^2})^k}\right]\end{aligned} \tag{4.59}$$

It is possible to simplify this expression for high SNRs, we have:

$$\frac{1}{\sqrt{1+\sigma^2\beta/\xi}} \approx 1-\frac{\sigma^2\beta}{2\xi} \tag{4.60}$$

$$\frac{1}{(\xi/\beta\sigma^2+1)^k} \approx \frac{\beta^k\sigma^{2k}}{\xi^k} \tag{4.61}$$

This leads to:

$$\bar{p}_e = 2\gamma\left[1-(1-\frac{\sigma^2\beta}{2\xi})\sum_{k=0}^{\alpha-1}\frac{(2k)!\beta^k\sigma^{2k}}{2^{2k}\xi^k k!^2}\right] \tag{4.62}$$

4.4.2 Optimal power allocation

In this section, we will evaluate the SEP for the case where optimal power allocation is used. We have:

$$\rho_i = \frac{\left[\sum_{k=1}^{L}[\sum_{j=1}^{N}\lambda'_{k,j}]\lambda'_{k,i}\right]^2}{\sigma^2\sum_{k=1}^{L}[\sum_{j=1}^{N}\lambda'_{k,j}]^2} \tag{4.63}$$

Even though the pdf of ρ_i is difficult to obtain, we have demonstrated that $\delta_i = \sigma^2\rho_i$ leads to a diversity order of $[R-(i-1)]L-1$ [66]. In order to confirm this result, an EM based fitting algorithm may be used to find a tight approximation for the distribution of $\delta_k = \frac{\left[\sum_{k=1}^{L}[\sum_{j=1}^{N}\lambda'_{k,j}]\lambda'_{k,i}\right]^2}{\sum_{k=1}^{L}[\sum_{j=1}^{N}\lambda'_{k,j}]^2}$. Let us take the case where $L=2$, $N=3$ and $R=8$ as an example. Figure 4.7 shows the obtained pdf for δ_1 and Figure 4.8 shows the obtained pdf for δ_2. For the first mobile station (i.e. rank 1), the obtained pdf is tightly approximated by a pure Gamma distribution with $\alpha = 15.81$ and $\beta = 1$. The resulting diversity order is

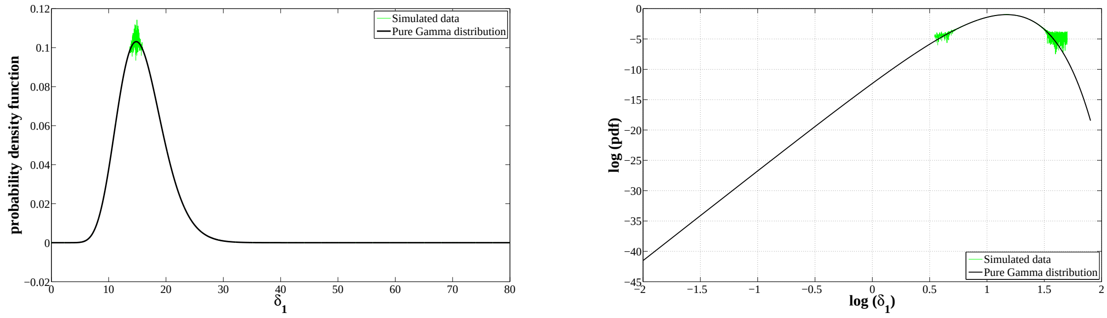


Figure 4.7: pdf of δ_1 for $L = 2$, $N = 3$, and $R = 8$

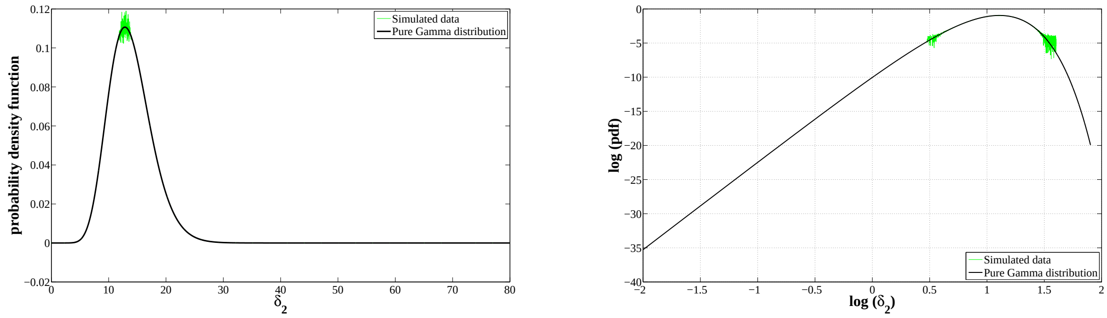


Figure 4.8: pdf of δ_2 for $L = 2$, $N = 3$, and $R = 8$

$D_1 \approx RL - 1$. For the second MS (i.e. rank 2), we obtain a pure Gamma distribution for δ_2 with $\alpha = 13.83$ and $\beta = 1$ leading to a diversity order of $D_2 \approx (R-1)L - 1$.

We can conclude experimentally that the variable δ_i is a Gamma distributed random variable leading to a diversity order of $D_i \approx [R - (i-1)]L - 1$. In this case, it is straightforward to obtain the theoretical SEP performance of this scheme:

$$\bar{p}_{e,optim} = 2\gamma \left[1 - \sqrt{\frac{\xi}{\sigma^2}} \frac{1}{\sqrt{\frac{\xi}{\sigma^2+1}}} \sum_{k=0}^{\alpha-1} \frac{(2k)!}{2^{2k} k!^2 (1 + \frac{\xi}{\sigma^2})^k} \right] \tag{4.64}$$

which is the same expression as in (4.56) with $\beta = 1$.

4.4.3 Non-uniform power allocation

We will now study the case where the precoding vectors are not normalized. In this case, the precoding vectors can be expressed as:

$$\mathbf{w}_k^l = \left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,k} \right\rangle \cdot \mathbf{e}_{l,k} \tag{4.65}$$

The average transmitted power of lth relay station is then written as:

$$\bar{P}_l = \sum_{k=1}^{N} \mathbb{E}\left\{ \mathbf{w}_k^{l\dagger} \mathbf{w}_k^l \right\} = \sum_{k=1}^{N} \mathbb{E}\left\{ \left| \left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,k} \right\rangle \right|^2 \right\} \tag{4.66}$$

Random variables $|\left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,k} \right\rangle|^2$ are chi-square random variables with $2\left[R-(k-1)\right]$ degrees of freedom[5], we can thus write:

$$\begin{aligned} \mathbb{E}\left\{ \left| \left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,k} \right\rangle \right|^2 \right\} &= \frac{1}{(q_k-1)!} \int_0^{+\infty} x.x^{q_k-1} e^{-x} dx \\ &= \frac{1}{(q_k-1)!} \int_0^{+\infty} x^{q_k} e^{-x} dx = \frac{q_k!}{(q_k-1)!} \\ &= q_k \end{aligned} \tag{4.67}$$

with $q_k = R-(k-1)$. Finally, the average transmitted power equals to:

$$\bar{P}_l = \psi(R,N) = \sum_{k=1}^{N} q_k = \sum_{k=1}^{N} [R-(k-1)] = N(R - \frac{N}{2} + \frac{1}{2}) \tag{4.68}$$

In this case, in order to have an average transmitted power of one, we have to use precoding vectors:

$$\mathbf{w}_k^l = \frac{\left\langle \mathbf{h}_{lk}^*, \mathbf{e}_{l,k} \right\rangle . \mathbf{e}_{l,k}}{\sqrt{\psi(R,N)}} \tag{4.69}$$

[5]Note that the chi-square distribution is a special case of the Gamma distribution

The received signal is then equal to:

$$y_i = \frac{1}{\sqrt{\psi(R,N)}}\left[\sqrt{P_1}\left|\left\langle \mathbf{h}_{1,i},\mathbf{e}_{1,i}\right\rangle\right|^2 + ... + \sqrt{P_k}\left|\left\langle \mathbf{h}_{k,i},\mathbf{e}_{k,i}\right\rangle\right|^2 \right.$$
$$\left. + ... + \sqrt{P_L}\left|\left\langle \mathbf{h}_{L,i},\mathbf{e}_{L,i}\right\rangle\right|^2\right] s_i + n_i \tag{4.70}$$

For simplicity, we will at first take the case $L = 2$, and then generalize the results for arbitrary L. For the case $L = 2$, we have to calculate the pdf of the random variable $Z = \sqrt{P_1'}\left|\left\langle \mathbf{h}_{1,i},\mathbf{e}_{1,i}\right\rangle\right|^2 + \sqrt{P_2'}\left|\left\langle \mathbf{h}_{2,i},\mathbf{e}_{2,i}\right\rangle\right|^2$ with $\sqrt{P_1'} = \sqrt{\frac{P_1}{\psi(R,N)}}$ and $\sqrt{P_2'} = \sqrt{\frac{P_2}{\psi(R,N)}}$. The resulting pdf is the convolution product of the two distributions of $\sqrt{P_1}\left|\left\langle \mathbf{h}_{1,i},\mathbf{e}_{1,i}\right\rangle\right|^2$ and $\sqrt{P_2}\left|\left\langle \mathbf{h}_{2,i},\mathbf{e}_{2,i}\right\rangle\right|^2$:

$$p_Z(x) = \frac{1}{\sqrt{P_1'}\Gamma(q_i)}\left(\frac{x}{\sqrt{P_1'}}\right)^{q_i-1} e^{-\frac{x}{\sqrt{P_1'}}} * \frac{1}{\sqrt{P_2'}\Gamma(q_i)}\left(\frac{x}{\sqrt{P_2'}}\right)^{q_i-1} e^{-\frac{x}{\sqrt{P_2'}}} \tag{4.71}$$

with $q_i = R - (i-1)$ and $*$ denoting the convolution product.

It is easier to use the Laplace Transform to evaluate (4.71). Setting $\Theta_1 = 1/\sqrt{P_1'}$ and $\Theta_2 = 1/\sqrt{P_2'}$ we can write:

$$\mathscr{L}\left\{p_Z(x)\right\} = P_Z(s) = (\Theta_1\Theta_2)^{q_i}\frac{1}{(\Theta_1+s)^{q_i}(\Theta_2+s)^{q_i}} \tag{4.72}$$

Note that $\mathscr{L}\left\{\frac{t^n}{n!}\right\} = \frac{1}{\Theta+s}^{n+1}$. If $\Theta_2 = \Theta_1 = \Theta$, we have directly $P_Z(s) = \Theta^{2q_i}\frac{1}{(\Theta+s)^{2q_i}}$ and $p_Z(x) = \Theta^{2q_i}\frac{x^{2q_i-1}e^{-\Theta x}}{(2q_i-1)!}$. Otherwise the decomposition of the partial fraction in (4.72) yields:

$$P_Z(s) = (\Theta_1\Theta_2)^{q_i}\left[\sum_{k=0}^{q_i-1}\frac{a_k}{(\Theta_1+s)^{q_i-k}} + \frac{b_k}{(\Theta_2+s)^{q_i-k}}\right] \tag{4.73}$$

with:

$$\begin{aligned} a_k &= \frac{(-1)^k \binom{q_i+k-1}{q_i-1}}{(\Theta_2-\Theta_1)^{q_i+k}} \\ \text{and} \qquad \mathrm{b_k} &= \frac{(-1)^k \binom{q_i+k-1}{q_i-1}}{(\Theta_1-\Theta_2)^{q_i+k}} \end{aligned} \tag{4.74}$$

Without the loss of generality we will assume that $\Theta_2 < \Theta_1$. The inverse Laplace transformation of (4.73) can be then calculated as:

$$p_Z(x) = (\Theta_1\Theta_2)^{q_i} \left[\sum_{k=0}^{q_i-1} (-1)^k \binom{q_i+k-1}{q_i-1} \frac{x^{q_i-k-1}}{(q_i-k-1)!} \frac{e^{-\Theta_1 x} + (-1)^{q_i+k} e^{-\Theta_2 x}}{(\Theta_2-\Theta_1)^{q_i+k}} \right] \tag{4.75}$$

The pdf of the signal to noise ratio $\rho = Z^2/\sigma^2$ is then given by:

$$p_\rho(u) = \frac{p_Z(\sigma\sqrt{u})\sigma}{2\sqrt{u}} \tag{4.76}$$

The average SEP is then calculated by averaging the $2\gamma\mathrm{erfc}(\sqrt{\xi\rho})$ over the pdf of the SNR:

$$\begin{aligned} \bar{p}_{e,nonuni} &= \int_0^{+\infty} 2\gamma\mathrm{erfc}(\sqrt{\xi u}) p_\rho(u) du \\ &= \sigma\gamma(\Theta_1\Theta_2)^{q_i} \sum_{k=0}^{q_i-1} (-1)^k \binom{q_i+k-1}{q_i-1} \frac{2\sigma^{q_i-k}}{(q_i-k-1)!} \left[\frac{J_{\sigma\Theta_1}^{q_i-k}}{(\Theta_2-\Theta_1)^{q_i+k}} \right. \\ &\quad \left. + \frac{J_{\sigma\Theta_2}^{q_i-k}}{(\Theta_1-\Theta_2)^{q_i+k}} \right] \end{aligned} \tag{4.77}$$

with

$$J_\lambda^m = \int_0^{+\infty} x^{m-1} e^{-\lambda x} \mathrm{erfc}(\sqrt{\xi} x) dx \tag{4.78}$$

The integrals J_λ^m may be integrated by parts setting:

$$dv = x^{m-1}e^{-\lambda x}dx \quad \Rightarrow \quad v = -\sum_{k=0}^{m-1}\frac{(m-1)!}{k!\lambda^{m-k}}x^k e^{-\lambda x} \tag{4.79}$$

$$u = \operatorname{erfc}(\sqrt{\xi}x) \quad \Rightarrow \quad du = \frac{-2\sqrt{\xi}}{\sqrt{\pi}}e^{-\xi x^2}dx \tag{4.80}$$

$$\begin{aligned} J_\lambda^m &= \int_0^{+\infty} x^{m-1}e^{-\lambda x}\operatorname{erfc}(\sqrt{\xi}x)dx \\ &= \frac{(m-1)!}{\lambda^m} - 2\sqrt{\frac{\xi}{\pi}}(m-1)!\sum_{k=0}^{m-1}\frac{e^{\lambda^2/4\xi}}{k!\lambda^{m-k}}\int_0^{+\infty} x^k e^{-\xi(x+\lambda/2\xi)^2}dx \end{aligned} \tag{4.81}$$

The last integral of (4.81) can be developed:

$$\begin{aligned} \int_0^{+\infty} x^k e^{-\xi(x+\lambda/2\xi)^2}dx &= \sum_{n=0}^{k}(-1)^n\binom{k}{n}\frac{\lambda^n}{2^n\xi^{(k+n+1)/2}}\int_{\lambda^2/4\xi}^{+\infty} x^{(k-n)/2}e^{-x}\frac{dx}{2\sqrt{x}} \\ &= \frac{1}{2}\sum_{n=0}^{k}(-1)^n\binom{k}{n}\frac{\lambda^n}{2^n\xi^{(k+n+1)/2}}\int_{\lambda^2/4\xi}^{+\infty} x^{(k-n-1)/2}e^{-x}dx \\ &= \frac{1}{2}\sum_{n=0}^{k}(-1)^n\binom{k}{n}\frac{\lambda^n}{2^n\xi^{\frac{k+n+1}{2}}}\left[\Gamma\left(\frac{k-n+1}{2}\right) - \Gamma_{\text{inc}}\left(\frac{k-n+1}{2}, \frac{\lambda^2}{4\xi}\right)\right] \end{aligned} \tag{4.82}$$

with $\Gamma(x)$ and $\Gamma_{\text{inc}}(n,x)$ respectively denoting Gamma and lower incomplete Gamma functions. $\Gamma_{\text{inc}}(n,x) = \int_0^x u^{n-1}e^{-u}du$. By substituting (4.82) in (4.81) we obtain:

$$\begin{aligned} J_\lambda^m = \frac{(m-1)!}{\lambda^m}\Bigg\{1 - \frac{1}{\sqrt{\pi}}\sum_{k=0}^{m-1}\frac{e^{\frac{\lambda^2}{4\xi}}\lambda^k}{k!}\sum_{n=0}^{k}(-1)^n\binom{k}{n}\frac{\lambda^n}{2^n\xi^{(k+n)/2}} \\ \times\left[\Gamma\Big(\frac{k-n+1}{2}\Big) - \Gamma_{inc}\Big(\frac{k-n+1}{2}, \frac{\lambda^2}{4\xi}\Big)\right]\Bigg\} \end{aligned} \tag{4.83}$$

And finally by substituting (4.83) in (4.77) the SEP can be calculated.

It is possible to generalize this result for an arbitrary number of relays. In the general case with L relays, we obtain:

$$\bar{p}_{e,nonuni} = 2\gamma(\Theta_1\Theta_2\cdots\Theta_L)^{q_i}\sum_{k=0}^{q_i-1}\frac{\sigma^{q_i-k}}{(q_i-1-k)!}\left[a_k^1 J_{\sigma\Theta_1}^{q_i-k} + a_k^2 J_{\sigma\Theta_2}^{q_i-k} + \cdots + a_k^L J_{\sigma\Theta_L}^{q_i-k}\right] \quad (4.84)$$

with J_λ^m given by (4.83) and:

$$\begin{aligned} a_k^i &= \frac{1}{k!}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^{k+1}q_i(q_i+1)\cdots(q_i+k) \\ &\times \frac{\sum\limits_{1\leq i_1<\cdots<i_k<\cdots<i_{L-n+k}\leq L, i_k\neq i}\cdots\sum \left[(\Theta_{i_1}-\Theta_i)\cdots(\Theta_{i_k}-\Theta_i)\cdots(\Theta_{i_{L-n+k}}-\Theta_i)\right]}{\left[(\Theta_1-\Theta_i)\cdots(\Theta_{i-1}-\Theta_i)(\Theta_{i+1}-\Theta_i)\cdots(\Theta_L-\Theta_i)\right]^{q_i+1+k}} \\ &= q_i\sum_{k=0}^{n-1}\binom{n-1}{k}\binom{q_i+k}{q_i}(-1)^{k+1} \\ &\times \frac{\sum\limits_{1\leq i_1<i_2<..i_k<...<i_{L-n+k}\leq L, i_k\neq i}\cdots\sum \left[(\Theta_{i_1}-\Theta_i)\cdots(\Theta_{i_k}-\Theta_i)\cdots(\Theta_{i_{L-n+k}}-\Theta_i)\right]}{\left[(\Theta_1-\Theta_i)\cdots(\Theta_{i-1}-\Theta_i)(\Theta_{i+1}-\Theta_i)\cdots(\Theta_L-\Theta_i)\right]^{q_i+1+k}} \end{aligned} \quad (4.85)$$

4.5 Simulation Results

In this section, simulation results are given to illustrate the accuracy of the proposed SEP estimations and to test the efficiency of the power allocation optimization algorithm given in Section 4.3.

Let us consider the simplest case where we have $N = 2$ mobile stations in the network. We assume that the total available power of the relays is 1 and that the relays have perfect CSI estimates. We suppose that we have $R = 2$ relays and each relay has $L = 2$ transmit antennas. The following cases are considered:

1. Uniform power allocation
2. Optimized power allocation (see Section 4.4.2) with two sub-cases:
   (a) Complete solution (see equations (4.43) and (4.44))

(b) Approximate solution based on low SNR approximation (see equations (4.47) and (4.48))

3. Zero forcing with complete CSI knowledge at each relay station and uniform relay power allocation (see Chapter 3)

4. Non-uniform power allocation (see 4.4.3)

The Monte-Carlo simulation results are compared to theoretical SEP approximations when available. We study the case of Quadrature Amplitude Modulation (QAM)-16 constellation. In this case, MS_1 (the first mobile station in the process of Gram-Schmidt) benefits from the maximum diversity order which is equal to $D_{GS_1} = L\,[R-(i-1)]-1 = 2\times 2-1 = 3$ while the zero forcing (ZF) precoding equalization discussed on the previous chapter exhibits a diversity order of $D_{ZF} = LR - N = 2\times 2 - 2 = 2$ [67]. These results are illustrated on Figure 4.9 and we can see that the proposed theoretical SER evaluations are always very close to the simulated results, especially for high SNR's. Moreover, in this case, where the maximum diversity remains moderate, the differences between the non-uniform and optimized power allocation strategies are considerable. For example, at SER $= 10^{-3}$ the optimized power allocation results in a 3.5 dB gain compared to the non-uniform case. At the same SER, the uniform power allocation scheme exhibits a loss of 2 dB when compared to the optimized allocation scheme.

Figure 4.10 depicts the same results for the second mobile station MS_2 (i.e. with ranking order two in the Gram-Schmidt orthonormalization process). The diversity order of this mobile is $D_{GS_2} = L\,[R-(i-1)]-1 = 2\times 1-1 = 1$ which is inferior to the diversity order of the ZF algorithm which remains equal to 2. It can be seen clearly that for high SNRs the slope of the curve concerning the ZF precoding equalization is superior to that of the Gram-Schmidt algorithm. The differences between the power allocation policies are more important than those corresponding to the first MS. For the second MS at SER $= 10^{-2}$

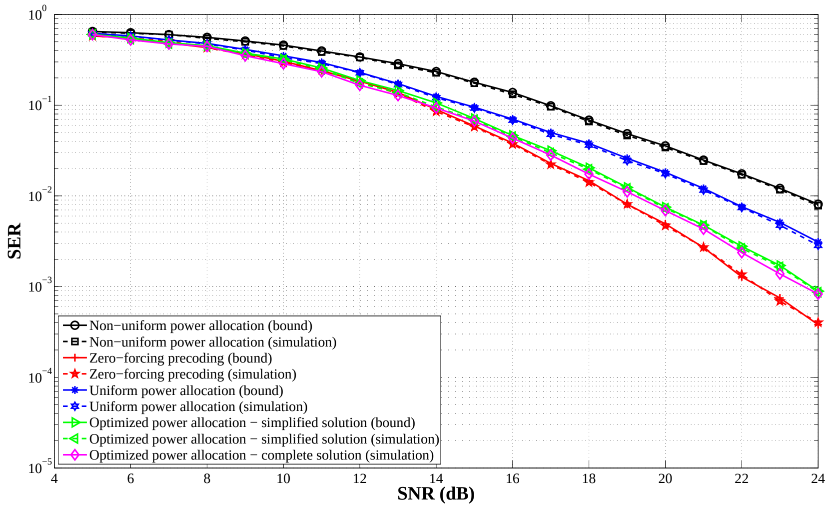


Figure 4.9: Symbol error rate for a cooperative communication system with $N = 2$, $R = 2$, $L = 2$ and $i = 1$ (MS_1)

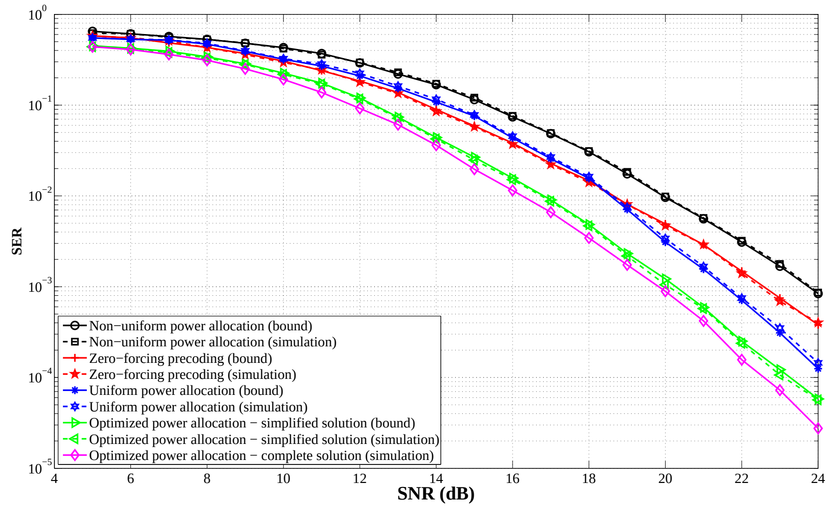


Figure 4.10: Symbol error rate for a cooperative communication system with $N = 2$, $R = 2$, $L = 2$ and $i = 2$ (MS_2)

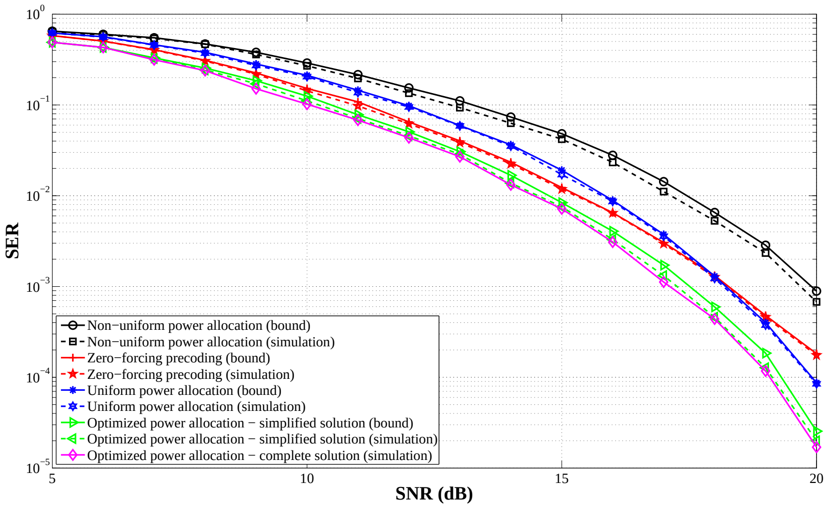


Figure 4.11: Symbol error rate for a cooperative communication system with $N = 4$, $R = 5$, $L = 6$ and $i = 1$ (1st mobile station)

there is more than 6 dB difference between the optimized power allocation process and the non-uniform power allocation, while there is more than 4 dB difference between the power allocation optimization scenario and the uniform power allocation. The results of the case $N = 2$, $R = 2$, and $L = 2$ show that:

- Power allocation optimization allows the MSs to benefit from the maximum coding gain.

- For the second mobile station MS_2,ZF precoding shows better performances at high SNRs.

These conclusions are completely different in the case of more elaborated systems with higher number of transmission/reception elements. On Figure 4.11 and Figure 4.12 we investigate the case where we have $N = 4$ MSs, $R = 5$ relay stations, and each RS is equipped with $L = 6$ transmit antennas. Figure 4.11 shows the obtained results for the first MS ($i = 1$). The potential diversity order for MS with ranking one in the Gram-Schmidt

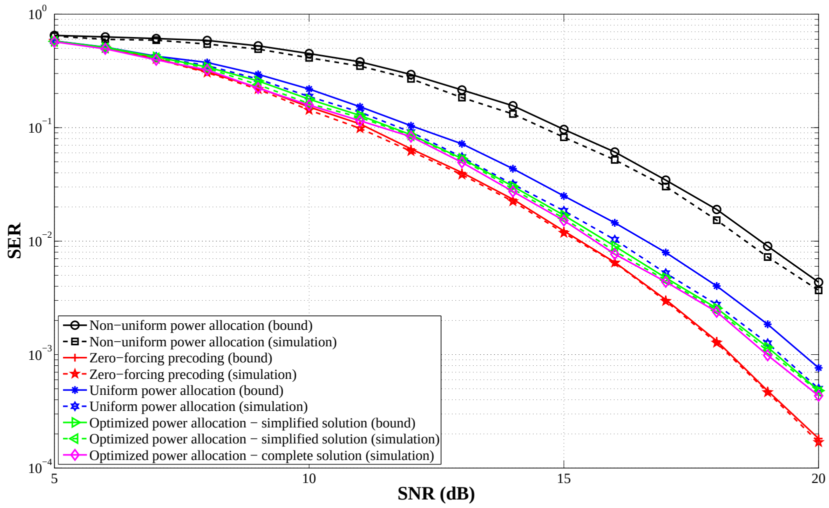


Figure 4.12: Symbol error rate for a cooperative communication system with $N = 4$, $R = 5$, $L = 6$ and $i = 3$ (3rd mobile station)

orthonormalization process is equal to $D_{GS_1} = L\,[R - (i-1)] - 1 = 6 \times 5 - 1 = 29$ while the diversity order for the ZF precoding policy is $D_{ZF} = LR - N = 6 \times 5 - 4 = 26$. In this case, the difference between the uniform power allocation and the optimal power allocation is not very considerable. This result is intuitively correct; since there are many independent paths between the relays and mobile stations, the average channel quality tends to be the same for all the MSs, and the optimum power allocation algorithm will attribute approximately same SNR for all mobile stations. As a result the most cost-effective power allocation policy is to allocate the same power to each relay. The non-uniform power allocation scheme shows a poor performance in this case.

Figure 4.12 shows the results for the third mobile station MS_3 in the network ($i = 3$), the diversity order obtained by the Gram-Schmidt algorithm is $D_{GS_3} = L\,[R - (i-1)] - 1 = 6 \times (5-2) - 1 = 17$ which is inferior of the diversity order of the ZF which remains at 26. The difference between the optimized power allocation policy and the uniform power allocation is smaller for the third mobile than for the first mobile (less than 1.8 dB at SER

$= 10^{-3}$). This entails the following conclusions:

- For a system with a high potential diversity uniform power allocation between RSs is the best strategy.
- Only the first few MSs in the network will benefit from a higher diversity, while the other ones will encounter a loss compared to the ZF precoding.

4.6 Conclusion

In this chapter, we proposed a new precoding algorithm for multi-user communications using cooperative relay stations. This algorithm concentrates the complexity on the relay station side where the precoding vectors are calculated in order to cancel out MAI at MS's side. This is done using the well known Gram-Schmidt orthonormalization process. The strength of this method is that it only requires the knowledge of CIR's from a given relay to all the mobile stations. In other words, to compute its precoding vectors, a relay does not need to know the CIR's of the other relays. We analyzed the potential diversity gain at the receivers and we showed that the diversity gain is not the same for all MS's at the receiver side and depends on the ranking of the mobile station used in the orthogonalization process. The diversity gain for the first MS can be equal to the number of transmit antennas of each relay multiplied by the number of relays in the network, which is superior to the value obtained with a centralized Z.F precoding policy. Of course, this is obtained at the price of a loss in the diversity order for the last MS's included in the algorithm. We also investigated the power allocation between the relays. We developed a mathematical analysis for the Symbol Error Rate (SER) at each MS in both cases: when each MS benefits from optimized power, and when the allocated power is chosen randomly. The obtained results clearly demonstrate the advantage of optimizing the allocated power at the cost of a centralized strategy for the cases where the potential diversity gain remains moderate, i.e when the product $R \times L$ is roughly inferior to 10. For the systems with a high potential diversity, the uniform power allocation is sufficient to exploit the diversity of the system.

Further studies should include the case where MS's benefits from multiple receive antennas and the influence of C.I.R estimation algorithms.

In this work, the complexity is placed on the relay station side where the precoding vectors are calculated in order to cancel out MAI at MS's. Mitigating MAI, the goal is clearly to maximize the number of potential MS'c in a given cell. This is done using the well known Gram-Schmidt orthonormalization process. The strength of this method is that it only requires the knowledge of CIR's from a given relay to all the mobile stations. In other words, to compute its precoding vectors, a given relay does not need to know the CIR's of the other relays to MS's. Using this algorithm, we carefully study the potential diversity gain at the receivers and we show that, unlike in [22], the diversity gain is not the same for all receivers and depends on the ranking of the mobile station used in the orthogonalization process. However, the diversity gain for the first MS can be equal to the number of transmit antennas per relay multiplied by the number of relays in the network, in other words some MS's will benefit from a higher diversity gain compared to the reference system. Obviously this is obtained at the price of a loss in the diversity gain for the lower end MS's.

Furthermore, since we demonstrate that the obtained SNR at each MS is closely related to the average transmitted power of each RS, we examine the problem of optimizing the power allocation to the relays. We obtain a mathematical analysis on the Symbol Error Rate (SER) at each MS in both cases where all MS's benefit from optimized power and where the allocated power is chosen randomly. This mathematical derivation is obtained thanks to a tight approximation of the p.d.f of the optimized SNR. We use an iterative

Expectation-Maximization (EM) algorithm to obtain it. For the case of a low number of relaying stations, the obtained results clearly demonstrate the advantage of optimizing the allocated power at the cost of the increased complexity of a centralized strategy. This means that, in the case of optimized power allocation, each relay requires the knowledge of the CIR from all of the relays in the network. However in the case of a high number of relaying stations, a uniform power allocation strategy shows to be the best solution. This strategy also preserves the advantage of only requiring the knowledge of CIR's from the concerned relay.

Chapter 5

Beamforming Technique Using Second Order Statistics

This chapter covers the same problem as the previous chapters, except that in this chapter we will assume that only the second order statistics information of the channels is provided to the relays. The other difference is that in this chapter we consider an amplify-and-forward (AF) protocol whereas in the previous chapters a decode-and-forward (DF) strategy was addressed.

The remainder of this chapter is organized as follows. The system model is discussed under Section5.1. The optimization process is covered by Section 5.2. We consider two kinds of power constraints: individual relay power constraints and a total (source and relay) power constraint. Section 5.3 provides a comparing base for the results by a system having the perfect channel state information (CSI) knowledge and using a zero forcing (ZF) algorithm. Numerical and simulation results are given in Section 5.4.

5.1 System model

We consider a system with R relays and M mobile stations as depicted in Figure 5.1. The source is equipped with M antennas and sends a message vector of data $\mathbf{s} = [s_1, s_2, ..., s_M]$ towards mobile stations where s_j is a unitary symbol ($\mathbb{E}\left\{|s_j|^2\right\} = 1$) destinated to MS$_j$.

Each relay is equipped with only one antenna. The channel between the base station (BS) and the ith relay is denoted by vector $\mathbf{h}_i = \left[\begin{array}{cccc} h_i(1) & h_i(2) & \cdots & h_i(M) \end{array} \right]$ of size $1 \times M$ with $h_i(k) \sim \mathcal{CN}(0, 1)$. The channel between the ith single-antenna relay and the jth mobile station is denotes by $g_{ij} \sim \mathcal{CN}(0, 1)$. Furthermore we define:

$$\mathbf{g}_j = \left[\begin{array}{cccc} g_{1j} & g_{2j} & \cdots & g_{Rj} \end{array} \right] \tag{5.1}$$

denoting the vector of channel coefficients between the relays and the jth mobile station

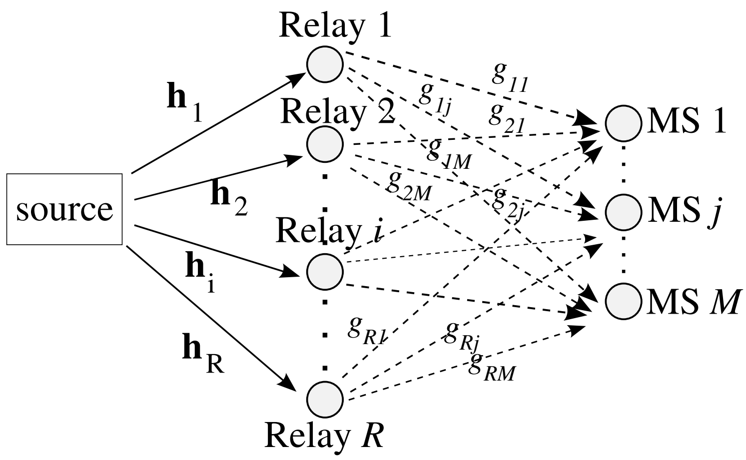


Figure 5.1: System model for the multi-relay multi user scheme

and

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \vdots \\ \mathbf{g}_M \end{bmatrix}_{M\times R} \tag{5.2}$$

We also define:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_R \end{bmatrix}_{R\times M} = \begin{bmatrix} \mathbf{h}^{(1)} & \mathbf{h}^{(2)} & \cdots & \mathbf{h}^{(M)} \end{bmatrix} \tag{5.3}$$

with $\mathbf{h}^{(j)}$ of size $R \times 1$ being the vector consisting the channel coefficients between the jth antenna of the BS and all relays.

If the transmitted signal by the source is $\mathbf{t} = \sqrt{P_S/M}\mathbf{s}^T_{|M\times 1}$ where where P_S denotes the source average transmit power, the received signal at the ith relay can be written as:

$$z_{i|1\times 1} = \sqrt{P_s/M}\mathbf{h}_{i|_{1\times M}}\mathbf{s}^T_{|_{M\times 1}} + v_i \qquad i = 1, \cdots, R \tag{5.4}$$

where v_i is an additive white Gaussian noise sample with zero mean and variance σ_{SR}^2.

Note that the transmitted power of the source can be easily verified: $P_{\text{trans}} = \mathbb{E}\left\{\mathbf{t}^\dagger \mathbf{t}\right\} = \frac{P_s}{M}\mathbb{E}\left\{|s_1|^2 + |s_2|^2 + \cdots + |s_M|^2\right\} = \frac{P_s}{M} \cdot M = P_s$.

Each relay weights its received signal z_i with a scalar coefficient w_i and retransmits the signal $x_i = w_i z_i$ to the mobile stations. The received signal at the jth mobile station can be thus written as:

$$
\begin{aligned}
u_j &= \sum_{k=1}^{R} g_{kj} w_k z_k + \eta_j = \sum_{k=1}^{R} g_{kj} w_k \left[\sqrt{P_s/M}\mathbf{h}_{k|1\times M}\mathbf{s}^T_{|M\times 1} + v_k\right] + \eta_j \\
&= \sqrt{P_s/M}\sum_{k=1}^{R} g_{kj} w_k \mathbf{h}_i \cdot \mathbf{s}^T + \sum_{k=1}^{R} g_{kj} w_k v_k + \eta_j \\
&= \sqrt{P_s/M}\sum_{k=1}^{R} g_{kj} w_k \cdot [h_k(1)s_1 + h_k(2)s_2 + \cdots + h_k(M)s_M] + \sum_{k=1}^{R} g_{kj} w_k v_k + \eta_j \\
&= \sqrt{P_s/M}\sum_{k=1}^{R} g_{kj} w_k h_k(j) s_j + \sqrt{P_s/M}\sum_{k=1}^{R} g_{kj} w_k \sum_{\substack{n=1\\ n\neq j}}^{M} h_k(n) s_n + \sum_{k=1}^{R} g_{kj} w_k v_k + \eta_j \\
&= \sqrt{P_s/M}\mathbf{w}_{|1\times R} \cdot \operatorname{diag}(\mathbf{g}_j)_{|R\times R} \cdot \mathbf{h}^{(j)T}_{|R\times 1} s_j \\
&\quad + \sqrt{P_s/M}\sum_{k=1}^{R} g_{kj} w_k \sum_{\substack{n=1\\ n\neq j}}^{M} h_k(n) s_n + \sum_{k=1}^{R} g_{kj} w_k v_k + \eta_j
\end{aligned}
\tag{5.5}
$$

where $\mathbf{w} = [w_1, w_2, ..., w_R]$ and η_j is the receiver noise with zero mean and variance σ_{RD}^2. In (5.5) the first summation represents the information part of the received signal, the second summation is the interference caused by other users, and the two last terms represent the noise.

We define:

$$
\begin{aligned}
\mathbf{s}_{-j}^{T} &= \begin{bmatrix} s_1 & s_2 & \cdots & s_{j-1} & s_{j+1} & \cdots & s_M \end{bmatrix}_{(M-1)\times 1}^{T} \\
\mathbf{h}_{k,-j} &= \begin{bmatrix} h_k(1) & h_k(2) & \cdots & h_k(j-1) & h_k(j+1) & \cdots & h_k(M) \end{bmatrix}_{1\times(M-1)} \\
\mathbf{H}_{-j} &= \begin{bmatrix} \mathbf{h}_{1,-j}^{T} & \mathbf{h}_{2,-j}^{T} & \cdots & \mathbf{h}_{R,-j}^{T} \end{bmatrix}_{R\times M-1}^{T}
\end{aligned}
\tag{5.6}
$$

Now the interference term in (5.5) can be rewritten as:

$$
\begin{aligned}
\sqrt{P_s/M}\sum_{k=1}^{R} g_{kj} w_k \sum_{\substack{n=1\\ n\neq j}}^{M} h_k(n) s_n &= \sum_{k=1}^{R} g_{kj} w_k \mathbf{h}_{k,-j}\mathbf{s}_{-j}^{T} \\
&= \mathbf{w}_{|1\times R}\cdot \operatorname{diag}(\mathbf{g}_j)_{|R\times R}\cdot \mathbf{H}_{-j|R\times M-1}\cdot \mathbf{s}_{-j|M-1\times 1}^{T}
\end{aligned}
\tag{5.7}
$$

Now (5.5) can be rewritten:

$$
u_j = \sqrt{P_s/M}\mathbf{w}\cdot \operatorname{diag}(\mathbf{g}_j)\cdot \mathbf{h}^{(j)T} s_j + \sqrt{P_s/M}\mathbf{w}\cdot \operatorname{diag}(\mathbf{g}_j)\cdot \mathbf{H}_{-j}\cdot \mathbf{s}_{-j}^{T} + \mathbf{w}\cdot \operatorname{diag}(\mathbf{g}_j)\cdot \mathbf{v} + \eta_j \tag{5.8}
$$

where $\mathbf{v}$ is a column vector of size $R \times 1$ consisting the receiver noises of all relays.

By defining:

$$
\begin{aligned}
\mathbf{p}_{j|R\times 1} &= \operatorname{diag}(\mathbf{g}_j)_{|R\times R}\cdot \mathbf{h}_{|R\times 1}^{(j)T} = \begin{bmatrix} g_{1j}H(1,j) & g_{2j}H(2,j) & \cdots & g_{Rj}H(R,j) \end{bmatrix}^{T} \\
\mathbf{U}_{j|R\times(M-1)} &= \operatorname{diag}(\mathbf{g}_j)_{R\times R}\cdot \mathbf{H}_{-j|R\times M-1} \\
\mathbf{t}_{j|R\times 1} &= \operatorname{diag}(\mathbf{g}_j)_{R\times R}\cdot \mathbf{v}_{R\times 1}
\end{aligned}
\tag{5.9}
$$

We can rewrite (5.8) as follows:

$$
u_j = \sqrt{P_s/M}\mathbf{w}\cdot \mathbf{p}_j s_j + \sqrt{P_s/M}\mathbf{w}\cdot \mathbf{U}_j\cdot \mathbf{s}_{-j}^{T} + \mathbf{w}\cdot \mathbf{t}_j + \eta_j \tag{5.10}
$$

Once again in (5.10), the first term represents the message information, the second term is the multiple access interference (MAI), and the last two terms correspond to the system noise. The signal power of the received signal can be calculated as:

$$\begin{aligned} P_d &= \mathbb{E}\left\{\left|\sqrt{P_s/M}\mathbf{w}\cdot\mathbf{p}_j s_j\right|^2\right\} = \frac{P_s}{M}\mathbb{E}\left\{(\mathbf{w}\cdot\mathbf{p}_j s_j)\cdot(\mathbf{w}\cdot\mathbf{p}_j s_j)^\dagger\right\} \\ &= \frac{P_s}{M}\mathbb{E}\left\{\mathbf{w}\cdot\mathbf{p}_j s_j s_j^*\mathbf{p}_j^\dagger\cdot\mathbf{w}^\dagger\right\} = \frac{P_s}{M}\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger \end{aligned} \tag{5.11}$$

where $\mathbf{P}_j$ is an $R\times R$ matrix defined by $\mathbf{P}_j = \mathbb{E}\left\{\mathbf{p}_j\mathbf{p}_j^\dagger\right\}$.

The total interference plus noise power in (5.10) can be calculated as:

$$\begin{aligned} P_n &= \mathbb{E}\left\{\left|\sqrt{P_s/M}\mathbf{w}\cdot\mathbf{U}_j\cdot\mathbf{s}_{-j}^T+\mathbf{w}\cdot\mathbf{t}_j+\eta_j\right|^2\right\} \\ &= \mathbb{E}\left\{\left|\sqrt{P_s/M}\mathbf{w}\cdot\mathbf{U}_j\cdot\mathbf{s}_{-j}^T\right|^2\right\}+\mathbb{E}\left\{|\mathbf{w}\cdot\mathbf{t}_j|^2\right\}+\mathbb{E}\left\{|\eta_j|^2\right\} \\ &= \mathbb{E}\left\{\frac{P_s}{M}\mathbf{w}\mathbf{U}_j\mathbf{s}_{-j}^T(\mathbf{s}_{-j}^T)^\dagger\mathbf{U}_j^\dagger\mathbf{w}^\dagger\right\}+\mathbb{E}\left\{\mathbf{w}\mathbf{t}_j\mathbf{t}_j^\dagger\mathbf{w}^\dagger\right\}+\sigma_{RD}^2 \\ &= \mathbb{E}\left\{\frac{P_s}{M}\mathbf{w}\mathbf{U}_j\mathbf{s}_{-j}^T(\mathbf{s}_{-j}^T)^\dagger\mathbf{U}_j^\dagger\mathbf{w}^\dagger\right\}+\mathbb{E}\left\{\mathbf{w}\,\mathrm{diag}(\mathbf{g}_j)\mathbf{v}\cdot\mathbf{v}^\dagger\,\mathrm{diag}(\mathbf{g}_j)^\dagger\mathbf{w}^\dagger\right\}+\sigma_{RD}^2 \\ &= \frac{P_s}{M}\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger+\sigma_{SR}^2\mathbf{w}\mathbf{G}\mathbf{w}^\dagger+\sigma_{RD}^2 \end{aligned} \tag{5.12}$$

where $\mathbf{G}_j = \mathbb{E}\left\{\mathrm{diag}(\mathbf{g}_j)\cdot\mathrm{diag}(\mathbf{g}_j)^\dagger\right\} = \mathrm{diag}\left(\mathbb{E}\left\{|g_{1j}|^2\right\},\mathbb{E}\left\{|g_{2,j}|^2\right\},...,\mathbb{E}\left\{|g_{R,j}|^2\right\}\right)$, and $\mathbb{E}\left\{\mathbf{Q}_j\right\} = \mathbf{U}_j\cdot\mathbf{U}_j^\dagger$ The signal to noise plus interference ratio (SINR) at the jth mobile station is then equal to:

$$\Gamma_d^{(j)} = \frac{P_d}{P_n} = \frac{P_s/M\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s/M\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger+\sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger+\sigma_{RD}^2} \tag{5.13}$$

Now we can calculate the total relay transmit power and transmit power at the ith relay:

- **total relay transmit power:**

$$
\begin{aligned}
P_r &= \sum_{i=1}^{R} \mathbb{E}\left\{|z_i|^2\right\} = \sum_{i=1}^{R} \mathbb{E}\left\{\left|w_i(\sqrt{P_s/M}\mathbf{H}_{i|1\times M} \cdot \mathbf{s}^T_{|M\times 1} + v_i)\right|^2\right\} \\
&= \sum_{i=1}^{R} \mathbb{E}\left\{\left|w_i(\sqrt{P_s/M}\sum_{p=1}^{M} H_i(p)s_i + v_i)\right|^2\right\} \\
&= \frac{P_s}{M}\mathbf{w}\cdot\mathbf{D}\cdot\mathbf{w}^\dagger + \sigma^2_{SR}\mathbf{w}\cdot\mathbf{w}^\dagger
\end{aligned}
\tag{5.14}
$$

with $\mathbf{D} = \mathbb{E}\left\{\mathbf{d}^\dagger\mathbf{d}\right\}$ and $\mathbf{d} = \left[\sum_{p=1}^{M} H_1(p), \sum_{p=1}^{M} H_2(p), \cdots, \sum_{p=1}^{M} H_R(p)\right]_{1\times R}$.

- **Power of ith relay :**

$$
P_{r,i} = \mathbb{E}\left\{|z_i|^2\right\} = (P_s D_{ii} + \sigma^2_{SR})\,|w_i|^2 \quad i = 1, 2, ...R \tag{5.15}
$$

where D_{ii} represents the (i,i)th element of matrix $\mathbf{D}$.

5.2 Mathematical optimization

In this section we will determine the beamforming weights w_i in order to maximize the SINR at a given mobile station while respecting power constraints. two scenarios may be envisaged, the first scenario is when the total transmission power (from source and relays) is maintained below a given value:

$$
P_s + P_r \leq P_0 \tag{5.16}
$$

Where P_0 is the maximum allowable total transmit power of the source and all relays. The second scenario is when the individual relay power of each relay node is restricted:

$$P_{r,i} \leq P_i \tag{5.17}$$

where P_i is the maximum allowable transmit power at the ith relay.

5.2.1 SNR Optimization under total power constraints

We have to solve the following optimization problem:

$$\begin{aligned} \max_{P_s,\mathbf{w}} \Gamma_d^{(j)} &= \frac{P_d}{P_n} = \frac{P_s/M\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s/M\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2} \\ \text{s.t.} \quad & P_s + \frac{P_s}{M}\mathbf{w}\mathbf{D}\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{w}^\dagger \leq P_0 \end{aligned} \tag{5.18}$$

If we denote the solution of (5.18) by $(\mathbf{w}^{\text{opt}}, P_s^{\text{opt}})$, the first step is to prove that $P_s^{\text{opt}} + \frac{P_s^{\text{opt}}}{M}\mathbf{w}^{\text{opt}}\mathbf{D}\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\mathbf{w}^{\text{opt}}\mathbf{w}^{\text{opt}\dagger} = P_0$.

Proof. Otherwise, if we suppose that $P_s^{\text{opt}} + \frac{P_s^{\text{opt}}}{M}\mathbf{w}^{\text{opt}}\mathbf{D}.\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\mathbf{w}^{\text{opt}}\mathbf{w}^{\text{opt}\dagger} < P_0$, let $\alpha = \frac{P_0 - P_s^{\text{opt}}}{\frac{P_s^{\text{opt}}}{M}.\mathbf{w}^{\text{opt}}.\mathbf{D}.\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2.\mathbf{w}^{\text{opt}}.\mathbf{w}^{\text{opt}\dagger}}$, with $\alpha > 1$. We will verify that $(\sqrt{\alpha}\mathbf{w}^{\text{opt}}, P_s^{\text{opt}})$ also satisfies the constraint but results in a larger objective value [68]. This is in contradiction with the optimality of $(\mathbf{w}^{\text{opt}}, P_s^{\text{opt}})$. In order to verify that $(\sqrt{\alpha}\mathbf{w}^{\text{opt}}, P_s^{\text{opt}})$ verifies the constraint we have:

$$\begin{aligned} & P_s^{\text{opt}} + \frac{P_s^{\text{opt}}}{M}\alpha\mathbf{w}^{\text{opt}}\mathbf{D}\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\alpha\mathbf{w}^{\text{opt}}\mathbf{w}^{\text{opt}\dagger} \\ &= P_s^{\text{opt}} + \alpha\left(\frac{P_s^{\text{opt}}}{M}\mathbf{w}^{\text{opt}}\mathbf{D}\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\mathbf{w}^{\text{opt}}\mathbf{w}^{\text{opt}\dagger}\right) \\ &= P_s^{\text{opt}} + P_0 - P_s^{\text{opt}} \\ &= P_0 \end{aligned} \tag{5.19}$$

Hence, the constraint is satisfied. Now we will calculate the new SINR:

$$\begin{aligned}\Gamma_d^{\text{new}(j)} &= \frac{P_d}{P_n} = \frac{P_s^{\text{opt}}/M\alpha\mathbf{w}^{\text{opt}}\mathbf{P}_j\mathbf{w}^{\text{opt}\dagger}}{P_s^{\text{opt}}/M\alpha\mathbf{w}^{\text{opt}}\mathbf{Q}_j\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\alpha.\mathbf{w}^{\text{opt}}\mathbf{G}_j\mathbf{w}^{\text{opt}\dagger} + \sigma_{RD}^2} \\ &= \frac{P_s^{\text{opt}}/M\mathbf{w}^{\text{opt}}\mathbf{P}_j\mathbf{w}^{\text{opt}\dagger}}{P_s^{\text{opt}}/M\mathbf{w}^{\text{opt}}\mathbf{Q}_j\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\mathbf{w}^{\text{opt}}\mathbf{G}_j\mathbf{w}^{\text{opt}\dagger} + \sigma_{RD}^2/\alpha}\end{aligned} \tag{5.20}$$

Since, $a > 1$, we have $\sigma_{RD}^2/\alpha < \sigma_{RD}^2$ which yields:

$$\Gamma_d^{\text{new}(j)} > \Gamma_d^{\text{opt}(j)} = \frac{P_s^{\text{opt}}/M\mathbf{w}^{\text{opt}}\mathbf{P}_j\mathbf{w}^{\text{opt}\dagger}}{P_s^{\text{opt}}/M\mathbf{w}^{\text{opt}}\mathbf{Q}_j\mathbf{w}^{\text{opt}\dagger} + \sigma_{SR}^2\mathbf{w}^{\text{opt}}\mathbf{G}_j\mathbf{w}^{\text{opt}\dagger} + \sigma_{RD}^2} \tag{5.21}$$

This completes the proof. □

now, the problem of (5.18) is equivalent to:

$$\begin{aligned}\max_{P_s,\mathbf{w}} \Gamma_d^{(j)} &= \frac{P_d}{P_n} = \frac{P_s/M\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s/M\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2} \\ \text{s.t.} \quad & P_s + \frac{P_s}{M}\mathbf{w}\mathbf{D}\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{w}^\dagger = P_0\end{aligned} \tag{5.22}$$

From the constraint of (5.22) we have:

$$\frac{\mathbf{w}\left[(P_s/M)\mathbf{D} + \sigma_{SR}^2\mathbf{I}\right]\mathbf{w}^\dagger}{P_0 - P_s} = 1 \tag{5.23}$$

Using (5.23) into the definition of $\Gamma_d^{(j)}$, we obtain:

$$\begin{aligned}
\Gamma_d^{(j)} = \frac{P_d}{P_n} &= \frac{P_s/M\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s/M\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2} \\
&= \frac{P_s/M\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s/M\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2\mathbf{w}\frac{P_s}{M}\mathbf{D} + \sigma_{SR}^2\mathbf{I})\mathbf{w}^\dagger/(P_0 - P_s)} \\
&= \frac{P_s(P_0 - P_s)}{M} \\
&\times \frac{\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s\frac{P_0-P_s}{M}\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + (P_0 - P_s)\sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2\mathbf{w}(\frac{P_s}{M}\mathbf{D} + \sigma_{SR}^2\mathbf{I})\mathbf{w}^\dagger} \\
&= \frac{P_s(P_0 - P_s)}{M} \\
&\times \frac{\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{\mathbf{w}\left[P_s\frac{P_0-P_s}{M}\mathbf{Q}_j + (P_0 - P_s)\sigma_{SR}^2\mathbf{G}_j + \sigma_{RD}^2(\frac{P_s}{M}\mathbf{D} + \sigma_{SR}^2\mathbf{I})\right]\mathbf{w}^\dagger}
\end{aligned} \tag{5.24}$$

It is possible to solve (5.24) using the following lemma.

Lemma 1. For two definite semi-positive Hermitian matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ i.e. $\mathbf{C}_1 \succeq 0$ and $\mathbf{C}_2 \succeq 0$, we have the following result [68]:

$$\max_{\mathbf{x}\neq 0} \frac{\mathbf{x}.\mathbf{C}_1.\mathbf{x}^\dagger}{\mathbf{x}.\mathbf{C}_2.\mathbf{x}^\dagger} = \frac{1}{\lambda_{\min}(\mathbf{C}_1^{-1/2}.\mathbf{C}_2.\mathbf{C}_1^{-1/2})} \tag{5.25}$$

with $\lambda_{\min}$ being the smallest eigenvalue of matrix $\mathbf{C}_1^{-1/2}\mathbf{C}_2\mathbf{C}_1^{-1/2}$ and the vector $\mathbf{x}^{\text{opt}}$ being the eigenvector associated with $\lambda_{\min}$. □

In order to use (5.25) to solve (5.24), we use:

$$\begin{aligned}
\mathbf{C}_1 &= \mathbf{P}_j \\
\mathbf{C}_2 &= P_s\frac{P_0 - P_s}{M}\mathbf{Q}_j + (P_0 - P_s)\sigma_{SR}^2\mathbf{G}_j + \sigma_{RD}^2\left(\frac{P_s}{M}\mathbf{D} + \sigma_{SR}^2\mathbf{I}\right)
\end{aligned} \tag{5.26}$$

and we have:

$$
\begin{aligned}
&\mathbf{C}_1{}^{-1/2}\mathbf{C}_2\mathbf{C}_1^{-1/2} \\
&\quad = \mathbf{P}_j^{-1/2}\left[P_s\frac{P_0-P_s}{M}\mathbf{Q}_j + (P_0-P_s)\sigma_{SR}^2\mathbf{G}_j + \sigma_{RD}^2(\frac{P_s}{M}\mathbf{D}+\sigma_{SR}^2\mathbf{I})\right]\mathbf{P}_j^{-1/2} \\
&\quad = P_s\frac{P_0-P_s}{M}\mathbf{P}_j^{-1/2}\mathbf{Q}_j\mathbf{P}_j^{-1/2} + (P_0-P_s)\sigma_{SR}^2\mathbf{P}_j^{-1/2}\mathbf{G}_j\mathbf{P}_j^{-1/2} \\
&\qquad + \sigma_{RD}^2\frac{P_s}{M}\mathbf{P}_j^{-1/2}\mathbf{D}\mathbf{P}_j^{-1/2} + \sigma_{RD}^2\sigma_{SR}^2\mathbf{P}_j^{-1} \\
&\quad = P_s\frac{P_0-P_s}{M}\mathbf{A}_1 + (P_0-P_s)\sigma_{SR}^2\mathbf{A}_2 + \sigma_{RD}^2\frac{P_s}{M}\mathbf{A}_3 + \sigma_{RD}^2\sigma_{SR}^2\mathbf{P}_j^{-1}
\end{aligned} \tag{5.27}
$$

with $\mathbf{A}_1 = \mathbf{P}_j^{-1/2}\mathbf{Q}_j\mathbf{P}_j^{-1/2}$, $\mathbf{A}_2 = \mathbf{P}_j^{-1/2}\mathbf{G}_j\mathbf{P}_j^{-1/2}$, and $\mathbf{A}_3 = \mathbf{P}_j^{-1/2}\mathbf{D}\mathbf{P}_j^{-1/2}$.

The vector $\mathbf{w}^{\text{opt}}$ corresponds to the eigenvector associated with the smallest eigenvalue of matrix $\mathbf{C}_1^{-1/2}\mathbf{C}_2\mathbf{C}_1^{-1/2}$.

The last step is to find P_s^{opt}, we have the following optimization problem:

$$
\begin{aligned}
&P_s^{\text{opt}} = \max_{P_s}\frac{P_s(P_0-P_s)}{M\lambda_{\min}(P_s\frac{P_0-P_s}{M}\mathbf{A}_1 + (P_0-P_s)\sigma_{SR}^2\mathbf{A}_2 + \sigma_{RD}^2\frac{P_s}{M}\mathbf{A}_3 + \sigma_{RD}^2\sigma_{SR}^2\mathbf{P}_j^{-1})} \\
&\quad \text{s.t.} \quad 0 \le P_s \le P_0
\end{aligned} \tag{5.28}
$$

Using the new variable $x = P_s/P_0 \le 1$, we can write:

$$
\begin{aligned}
x^{\text{opt}} &= \max_x \frac{xP_0(P_0-xP_0)}{M\lambda_{\min}(xP_0\frac{P_0-xP_0}{M}\mathbf{A}_1 + (P_0-xP_0)\sigma_{SR}^2\mathbf{A}_2 + \sigma_{RD}^2\frac{xP_0}{M}\mathbf{A}_3 + \sigma_{RD}^2\sigma_{SR}^2\mathbf{P}_j^{-1})} \\
&= \max_x \frac{P_0^2x(1-x)}{M\lambda_{\min}(P_0^2x\frac{1-x}{M}\mathbf{A}_1 + P_0(1-x)\sigma_{SR}^2\mathbf{A}_2 + \sigma_{RD}^2\frac{P_0}{M}x\mathbf{A}_3 + \sigma_{RD}^2\sigma_{SR}^2\mathbf{P}_j^{-1})}
\end{aligned} \tag{5.29}
$$

Two cases may occur based of orthogonality of matrices.

Case 1:$\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3$ and $\mathbf{P}_j^{-1}$ are diagonal matrices

If $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3$ and $\mathbf{P}_j^{-1}$ are diagonal matrices we simplify (5.29) into (5.30):

$$x^{\mathrm{opt}} = \min_{k=1,\cdots,R} \max_{0<x<1} \frac{P_0^2 x(1-x)}{M(P_0^2 x \frac{1-x}{M} a_k + P_0(1-x)\sigma_{SR}^2 b_k + \sigma_{RD}^2 \frac{P_0}{M} x c_k + \sigma_{RD}^2 \sigma_{SR}^2 d_k)} \tag{5.30}$$

We have:

$$\begin{aligned} &\frac{\partial}{\partial x}\frac{x(1-x)}{(P_0^2 x \frac{1-x}{M} a_k + P_0(1-x)\sigma_{SR}^2 b_k + \sigma_{RD}^2 \frac{P_0}{M} x c_k + \sigma_{RD}^2 \sigma_{SR}^2 d_k)} \\ &= \frac{\partial}{\partial x}\frac{x(1-x)}{(\lambda_k x(1-x) + \mu_k(1-x) + \rho_k x + \omega_k)} \\ &= \frac{\partial}{\partial x}[\frac{x(1-x)}{(\lambda_k x(1-x) + (\rho_k - \mu_k)x + \omega_k + \mu_k)} \end{aligned} \tag{5.31}$$

with $\lambda_k = \frac{P_0^2}{M} a_k$, $\mu_k = P_0 \sigma_{SR}^2 b_k$, $\rho_k = \sigma_{RD}^2 (P_0/M) c_k$, and $\omega_k = \sigma_{RD}^2 \sigma_{SR}^2 d_k$. We solve the optimization problem:

$$\begin{aligned} &\frac{\partial}{\partial x}\left[\frac{x(1-x)}{(\lambda_k x(1-x) + (\rho_k - \mu_k)x + \omega_k + \mu_k)}\right] = 0 \\ &(1-2x)\left[\lambda_k x(1-x) + \mu_k(1-x) + \rho_k x + \omega_k\right] - x(1-x)\left[\lambda_k(1-2x) - \mu_k + \rho_k\right] = 0 \\ &(1-2x)\left[\lambda_k x(1-x) + \mu_k(1-x) + \rho_k x + \omega_k - \lambda_k x(1-x)\right] + x(1-x)(\mu_k - \rho_k) = 0 \\ &(1-2x)\left[x(\rho_k - \mu_k) + \mu_k + \omega_k\right] + x(1-x)(\mu_k - \rho_k) = 0 \\ &\left[x(\rho_k - \mu_k) + \mu_k + \omega_k\right] - 2x^2(\rho_k - \mu_k) - 2x(\mu_k + \omega_k) + x(\mu_k - \rho_k) - x^2(\mu_k - \rho_k) = 0 \\ &x^2(\mu_k - \rho_k) + x(\rho_k - \mu_k - 2\mu_k - 2\omega_k + \mu_k - \rho_k) + \mu_k + \omega_k = 0 \\ &x^2(\mu_k - \rho_k) - 2x(\mu_k + \omega_k) + \mu_k + \omega_k = 0 \\ &\Longrightarrow x^{\mathrm{opt}} = \min_{k=1,\ldots,R} \frac{\mu_k + \omega_k + \sqrt{\omega_k^2 + \mu_k\omega_k + \rho_k\mu_k + \rho_k\omega_k}}{\mu_k - \rho_k} \end{aligned} \tag{5.32}$$

Case 2: $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, or $\mathbf{P}_j^{-1}$ is not diagonal

In the case that at least one matrix in the set is not diagonal, we proceed in the same way as in [68] to obtain x^{opt}. The optimization problem (5.29) is equivalent to:

$$x^{\text{opt}} = \min_{0<x<1} \lambda_{\min} \left(P_0^2 \frac{\mathbf{A}_1}{M} + P_0 \sigma_{SR}^2 \frac{\mathbf{A}_2}{x} + \frac{P_0}{M} \sigma_{RD}^2 \frac{\mathbf{A}_3}{1-x} + \sigma_{RD}^2 \sigma_{SR}^2 \frac{\mathbf{P}_j^{-1}}{x(1-x)} \right) \tag{5.33}$$

Since the objective in (5.33) is not always a convex function it is convenient to use Newton's method to search for the stationary points. However, it is important to set the starting point of the algorithm to the best approximation of the location of optimal x. To do this we can safely assume that at practical signal to noise ratios, we have the following approximations:

$$\sigma_{SR}^2 \frac{\|\mathbf{P}_j^{-1}\|}{x(1-x)} << (P_0/M) \frac{\|\mathbf{A}_3\|}{1-x} \tag{5.34}$$

$$\sigma_{RD}^2 \frac{\|\mathbf{P}_j^{-1}\|}{x(1-x)} << P_0 \frac{\|\mathbf{A}_2\|}{x} \tag{5.35}$$

In this case we have:

$$x^{\text{opt}} \approx \min_{0<x<1} \lambda_{\min} \left(P_0^2 \frac{\mathbf{A}_1}{M} + P_0 \sigma_{SR}^2 \frac{\mathbf{A}_2}{x} + \sigma_{RD}^2 \frac{P_0}{M} \frac{\mathbf{A}_3}{1-x} \right) \tag{5.36}$$

Since $P_0^2 \frac{\mathbf{A}_1}{M}$ does not depend on x, we have:

$$x^{\text{opt}} \approx \min_{0<x<1} \lambda_{\min} \left(P_0 \sigma_{SR}^2 \frac{\mathbf{A}_2}{x} + \sigma_{RD}^2 \frac{P_0}{M} \frac{\mathbf{A}_3}{1-x} \right) \tag{5.37}$$

We use the following Lemma:

Lemma 2. The optimal value of x for the optimization problem [68]:

$$x^{\text{opt}} \approx \min_{0<x<1} \lambda_{\min}\left(\frac{S_1}{1-x} + \frac{S_2}{x}\right) \tag{5.38}$$

lies in the interval $[x_l, x_u]$ where:

$$x_l = \frac{\sqrt{c}}{1+\sqrt{c}} \tag{5.39}$$

$$x_u = \frac{\sqrt{d}}{1+\sqrt{d}} \tag{5.40}$$

$$c = \lambda_{\min}(\mathbf{S}_1^{-1/2}\mathbf{S}_2\mathbf{S}_1^{-1/2}) \tag{5.41}$$

$$d = \lambda_{\max}(\mathbf{S}_1^{-1/2}\mathbf{S}_2\mathbf{S}_1^{-1/2}) \tag{5.42}$$

$\square$

In order to use this Lemma to solve (5.37), we set $\mathbf{S}_1 = \sigma_{RD}^2 \frac{P_0/M}{\mathbf{A}}{}_3$ and $\mathbf{S}_2 = P_0\sigma_{SR}^2\mathbf{A}_2$. As a result (5.37) is equivalent to:

$$x^{\text{opt}} \approx \min_{x_l<x<x_u} \lambda_{\min}\left(P_0\sigma_{SR}^2\frac{\mathbf{A}_2}{x} + \sigma_{RD}^2\frac{P_0}{M}\frac{\mathbf{A}_3}{1-x}\right) \tag{5.43}$$

At this stage, we can incorporate all the terms involved in the optimization problem in (5.33) and we use the Newton's method to search for the stationary points. We have:

$$x^{\text{opt}} \approx \min_{x_l\le x\le x_u} \lambda_{\min}\left(P_0\sigma_{SR}^2\frac{\mathbf{A}_2}{x} + \sigma_{RD}^2(P_0/M)\frac{\mathbf{A}_3}{1-x} + \sigma_{RD}^2\sigma_{SR}^2\frac{\mathbf{P}_j^{-1}}{x(1-x)}\right) \tag{5.44}$$

We set:

$$\mathbf{H}(x) = P_0\sigma_{SR}^2\frac{\mathbf{A}_2}{x} + \sigma_{RD}^2(P_0/M)\frac{\mathbf{A}_3}{1-x} + \sigma_{RD}^2\sigma_{SR}^2\frac{\mathbf{P}_j^{-1}}{x(1-x)} \quad x \in [x_l, x_u] \tag{5.45}$$

We assume as in [69] that $\mathbf{H}(x)$ depends smoothly on $x \in]0,1[$ since any order derivative of $\mathbf{H}(x)$ exists. The necessary conditions on the first and second order derivatives of $\mathbf{H}(x)$ for x to be a local minimizer are:

$$\frac{d}{dx}\lambda_{\min}(\mathbf{H}(x)) = 0 \quad \text{and} \quad \frac{d^2}{dx^2}\lambda_{\min}(\mathbf{H}(x)) \geq 0 \tag{5.46}$$

In Newton's method, the $(k+1)$th iteration is given by:

$$x_{k+1} = x_k - \alpha_k \frac{\frac{d}{dx}\lambda_{\min}(\mathbf{H}(x))}{\frac{d^2}{dx^2}\lambda_{\min}(\mathbf{H}(x))} \quad k = 0, 1, \cdots \tag{5.47}$$

where $\alpha_k > 0$ is chosen such that $x_{k+1} \in [x_l, x_u]$, otherwise $\alpha_k \leftarrow \alpha_k/2$. In the iteration expression in (5.47), the first and second order derivatives of $\lambda_{\min}(\mathbf{H}(x))$ must be calculated. Let $\mathbf{u}_0(x)$ be the eigenvector of $\mathbf{H}(x)$ associated with $\lambda_{\min}$ and $\mathbf{u}_i(x)$, $i = 1, 2, ..., R-1$ be the eigenvectors associated with the other eigenvalues with $\lambda_1(x) > \cdots > \lambda_i \cdots \lambda_{R-1}(x) > \lambda_{\min}$. The first and second order derivatives of $\lambda_{\min}(\mathbf{H}(x))$ are then respectively given by:

$$\begin{aligned} \frac{d}{dt}\lambda_{\min}(\mathbf{H}(x)) &= \mathbf{u}_0(x)^\dagger \frac{d\mathbf{H}(x)}{dx}\mathbf{u}_0(x) \\ \frac{d^2}{dt^2}\lambda_{\min}(\mathbf{H}(x)) &= \mathbf{u}_0(x)^\dagger \frac{d^2\mathbf{H}(x)}{dx^2}\mathbf{u}_0(x) - \sum_{i=1}^{R-1} \frac{2\left|\mathbf{u}_i(x)^\dagger d\mathbf{H}(x)/dx \mathbf{u}_0(x)\right|^2}{\lambda_i(x) - \lambda_{\min}(\mathbf{H}(x))} \end{aligned} \tag{5.48}$$

where

$$\begin{aligned} \frac{d\mathbf{H}(x)}{dx} &= \frac{-\mathbf{S}_2}{x^2} + \frac{\mathbf{S}_1}{(1-x)^2} + \mathbf{S}_3(\frac{-1}{x^2} + \frac{1}{(1-x)^2}) \\ \frac{d\mathbf{H}(x)}{dx} &= \frac{-\mathbf{S}_2}{x^2} + \frac{\mathbf{S}_1}{(1-x)^2} + \frac{\mathbf{S}_3(2x-1)}{x^2(1-x)^2} \end{aligned} \tag{5.49}$$

and

$$
\begin{aligned}
\frac{d^2\mathbf{H}(x)}{dx^2} &= \frac{\mathbf{S}_2}{x^3} - \frac{\mathbf{S}_1}{(1-x)^3} + \mathbf{S}_3(\frac{2}{x^3} + \frac{2}{(1-x)^3}) \\
\frac{d^2\mathbf{H}(x)}{dx^2} &= \frac{\mathbf{S}_2}{x^3} - \frac{\mathbf{S}_1}{(1-x)^3} + 2\mathbf{S}_3(\frac{1-3x-3x^2}{x^3(1-x)^3})
\end{aligned}
\tag{5.50}
$$

with $\mathbf{S}_3 = \sigma_{SR}^2\sigma_{RD}^2\mathbf{P}_j^{-1}$.

5.2.2 SINR Optimization under individual relay power constraint

From (5.13) and (5.15), the SINR maximization problem subject to individual relay power constraints is expressed as:

$$
\begin{aligned}
\max_{P_s,\mathbf{w}} \Gamma_d^{(j)} &= \frac{P_s/M\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{P_s/M\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2} \\
\text{s.t.} \quad & (P_sD_{ii} + \sigma_{SR}^2)\left|w_i\right|^2 \leq P_k \quad k \in \{1, 2, ..., R\}
\end{aligned}
\tag{5.51}
$$

The problem in (5.51) belongs is a quadratically constrained fractional programs. In the case of uncorrelated Rayleigh fading channels it is possible as in [68] to obtain a closed form solution as demonstrated below.

Case1: $\mathbf{P}_j$, $\mathbf{Q}_j$, and $\mathbf{G}_j$ are all diagonal matrices

In case of uncorrelated Rayleigh channels, matrices $\mathbf{P}_j$,$\mathbf{Q}_j$ and $\mathbf{G}_j$ are diagonal matrices. Using the Dinkelbach-type method in [69], we introduce the following function:

$$
\begin{aligned}
G(t) = &\max_{\mathbf{w}} \left[g(t,\mathbf{w}) = \frac{P_s}{M}\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger - t\left(\frac{P_s}{M}\mathbf{w}\mathbf{Q}_j\mathbf{w}^\dagger + \sigma_{SR}^2\mathbf{w}\mathbf{G}_j\mathbf{w}^\dagger + \sigma_{RD}^2\right)\right] \\
&\text{s.t.} \quad P_{r,i} = (P_sD_{ii} + \sigma_{SR}^2)\left|w_i\right|^2 \leq P_i \quad i = 1, 2, \cdots, R
\end{aligned}
\tag{5.52}
$$

The relation between $G(t)$ and the problem of (5.51) is given in the following property

[69].

Property 1.

1. $G(t)$ is strictly decreasing and $G(t) = 0$ has a unique root, say t^*;

2. Let $\mathbf{w}^*$ be the solution of (5.52) corresponding to t^*. Then $\mathbf{w}^*$ is also the solution of (5.51) with the largest objective value t^* exactly.

□

According to property 1, we want to find t^* and the associated $\mathbf{w}^*$, which is also the solution of (5.51). To this end, by denoting the (k,k)th entry of $\mathbf{P}_j$,$\mathbf{Q}_j$ and $\mathbf{G}_j$ as $p_{j,k}$,$q_{j,k}$ and $g_{j,k}$, respectively, we can rewrite the objective function $g(t,\mathbf{w})$ as:

$$g(t,\mathbf{w}) = -t\sigma_{RD}^2 + \sum_{n=1}^{R}\left[\frac{P_s}{M}p_{j,n} - t(\frac{P_s}{M}q_{j,n} + \sigma_{SR}^2 g_{j,n})\right] . |w_n|^2 \tag{5.53}$$

to get that:

$$G(t) = -t\sigma_{RD}^2 + \sum_{n=1}^{R}\frac{P_n}{P_s D_{nn} + \sigma_{SR}^2}\varphi\left[\frac{P_s}{M}p_{j,n} - t\left(\frac{P_s}{M}q_{j,n} + \sigma_{SR}^2 \cdot g_{j,n}\right)\right] \tag{5.54}$$

associated with the optimal:

$$|w_n|^2 = \begin{cases} \dfrac{P_n}{P_s D_{nn} + \sigma_{SR}^2} & \text{if } \dfrac{P_s}{M}p_{j,n} - t(\dfrac{P_s}{M}q_{j,n} + \sigma_{SR}^2 g_{j,n}) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.55}$$

with

$$\varphi(x) = \begin{cases} x & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.56}$$

To find the root of $G(t) = 0$, let us denote:

$$t_n = \frac{P_s p_{j,n}}{M(\frac{P_s}{M} q_{j,n} + \sigma_{SR}^2 g_{j,n})}\ , \qquad n = 1, 2, ..., R \tag{5.57}$$

and their rearrangement $\tilde{t}_1 < \tilde{t}_2 < \cdots < \tilde{t}_R$ corresponding to $\tilde{p}_{j,n}, \tilde{q}_{j,n}, \tilde{g}_{j,n}, \tilde{P}_n$ and $\tilde{D}_{nn}$ respectively. With these, we can rewrite (5.54) in the following way:

$$G(t) = -t\sigma_{RD}^2 + \sum_{n=1}^{R} \frac{\tilde{P}_n}{P_s D_{nn} + \sigma_{SR}^2} \varphi \left[\frac{P_s}{M} \tilde{p}_{j,n} - t\left(\frac{P_s}{M} .\tilde{q}_{j,n} + \sigma_{SR}^2 \tilde{g}_{j,n} \right) \right] \tag{5.58}$$

Note that $G(0) > 0$ and $G(\tilde{t}_R) = -\tilde{t}_R < 0$. Thus, it follows from Property 1 that $0 < t^* < \tilde{t}_R$. The root t^* is then determined based on the following theorem:

Theorem 2. If $G(\tilde{t}_{k_0}) = 0$ for an integer k_0, then $t^* = \tilde{t}_{k_0}$. Otherwise, let k_0 be the smallest integer such that $G(\tilde{t}_{k_0}) < 0$, then

$$t^* = \left[\sigma_{RD}^2 + \frac{P_s}{M} \sum_{n=k_0}^{R} \frac{\tilde{P}_n}{P_s \tilde{D}_{nn} + \sigma_{SR}^2} (\frac{P_s}{M} \tilde{q}_{j,n} + \sigma_{SR}^2 \tilde{g}_{j,n}) \right]^{-1} \sum_{n=k_0}^{R} \frac{\tilde{P}_n P_s}{M(P_s \tilde{D}_{nn} + \sigma_{SR}^2)} \tilde{p}_{j,n} \tag{5.59}$$

Proof. If $k_0 = 1$, then $0 < t^* < \tilde{t}_1$ and

$$\frac{P_s}{M} \tilde{p}_{j,n} - t(\frac{P_s}{M} \tilde{q}_{j,n} + \sigma_{SR}^2 \tilde{g}_{j,n}) > 0 \qquad j = 1, \cdots, R \tag{5.60}$$

Thus $G(t) = 0$ in (5.58) leads to:

$$
\begin{aligned}
&-t^*\sigma_{RD}^2 + \sum_{n=1}^{R} \frac{\tilde{P}_n}{P_s\tilde{D}_{nn} + \sigma_{SR}^2} \left[\frac{P_s}{M}\tilde{p}_{j,n} - t^*(\frac{P_s}{M}\tilde{q}_{j,n} + \sigma_{SR}^2\tilde{g}_{j,n}) \right] = 0 \\
&t^* \left[\sigma_{RD}^2 + \frac{P_s}{M} \sum_{n=1}^{R} \frac{\tilde{P}_n}{P_s\tilde{D}_{nn} + \sigma_{SR}^2} (\frac{P_s}{M}\tilde{q}_{j,n} + \sigma_{SR}^2\tilde{g}_{j,n}) \right] = \sum_{n=1}^{R} \frac{\tilde{P}_n}{P_s\tilde{D}_{nn} + \sigma_{SR}^2} \frac{P_s}{M}\tilde{p}_{j,n} \\
&t^* = \left[\sigma_{RD}^2 + \frac{P_s}{M} \sum_{n=1}^{R} \frac{\tilde{P}_n}{P_s\tilde{D}_{nn} + \sigma_{SR}^2} (\frac{P_s}{M}\tilde{q}_{j,n} + \sigma_{SR}^2\tilde{g}_{j,n}) \right]^{-1} \sum_{n=1}^{R} \frac{\tilde{P}_n P_s}{M(P_s\tilde{D}_{nn} + \sigma_{SR}^2)} \tilde{p}_{j,n}
\end{aligned} \tag{5.61}
$$

If $k_0 > 1$, then $\tilde{t}_{n_0-1} < t^* < \tilde{t}_{n_0}$ and

$$
\frac{P_s}{M}\tilde{p}_{j,n} - t^*(\frac{P_s}{M}\tilde{q}_{j,n} + \sigma_{SR}^2\tilde{g}_{j,n}) \begin{cases} > 0 & n = k_0, ..., R \\ < 0 & n = 1, ..., k_0 - 1 \end{cases} \tag{5.62}
$$

In this case $G(t) = 0$ in (5.58) leads to:

$$
\begin{aligned}
&-t^*\sigma_{RD}^2 + \sum_{n=k_0}^{R} \frac{\tilde{P}_n}{P_s\tilde{D}_{nn} + \sigma_{SR}^2} (\frac{P_s}{M}\tilde{p}_{j,n} - t^*(\frac{P_s}{M}\tilde{q}_{j,n} + \sigma_{SR}^2\tilde{g}_{j,n})) = 0 \\
&t^* = \left[\sigma_{RD}^2 + \frac{P_s}{M} \sum_{n=k_0}^{R} \frac{\tilde{P}_n}{P_s\tilde{D}_{nn} + \sigma_{SR}^2} (\frac{P_s}{M}\tilde{q}_{j,n} + \sigma_{SR}^2\tilde{g}_{j,n}) \right]^{-1} \sum_{n=k_0}^{R} \frac{\tilde{P}_n.P_s}{M(P_s\tilde{D}_{nn} + \sigma_{SR}^2)} \tilde{p}_{j,n}
\end{aligned} \tag{5.63}
$$

□

Once $t^*$is obtained, we can obtain $\mathbf{w}^*$ from (5.55).

Case 2: $\mathbf{P}_j$, $\mathbf{Q}_j$ or $\mathbf{G}_j$ is not diagonal

In the case where at least one matrix among the set: $\mathbf{P}_j$, $\mathbf{Q}_j$ and $\mathbf{G}_j$ is not diagonal, we can rewrite the optimization problem as:

$$\begin{aligned} &\max_{P_s,\mathbf{w}} \left[\Gamma_d^{(j)} = \frac{\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{1+\mathbf{w}\left[P_s/(M\sigma_{RD}^2)\mathbf{Q}_j + (\sigma_{SR}^2/\sigma_{RD}^2)\mathbf{G}_j\right]\mathbf{w}^\dagger} \right] \\ &\text{s.t.} \quad (P_s D_{ii} + \sigma_{SR}^2)\left|w_i\right|^2 \leq P_k \quad k \in \{1,2,...,R\} \end{aligned} \tag{5.64}$$

Denoting $T_j = P_s/(M\sigma_{RD}^2)\mathbf{Q}_j + (\sigma_{SR}^2/\sigma_{RD}^2)\mathbf{G}_j$ we obtain the same formulation as in [69] equation (30):

$$\begin{aligned} &\max_{P_s,\mathbf{w}} \left[\Gamma_d^{(j)} = \frac{\mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger}{1+\mathbf{w}\,T_j\mathbf{w}^\dagger} \right] \\ &\text{s.t.} \quad (P_s.D_{ii} + \sigma_{SR}^2)\left|w_i\right|^2 \leq P_k \quad k \in \{1,2,...,R\} \end{aligned} \tag{5.65}$$

We will use the following Lemma

Lemma 3. Let $\mathbf{w}^o$ be the solution of the following Quadratically Constrained Quadratic Programming (QCQP) problem:

$$\begin{aligned} &\max_{\mathbf{w}} \quad \mathbf{w}\mathbf{P}_j\mathbf{w}^\dagger \\ &\text{s.t.} \quad \mathbf{w}\mathbf{A}_{k,j}\mathbf{w}^\dagger \leq 1 \quad k \in I = \{1,2,...,R\} \end{aligned} \tag{5.66}$$

where

$$\mathbf{A}_{k,j} = \frac{P_s D_{kk} + \sigma_{SR}^2}{P_k}\mathbf{J}_k + \mathbf{T}_j \tag{5.67}$$

and $\mathbf{J}_k$ is a matrix with all zero entries except for the (k,k)th entry. Let:

$$\eta = \max_{k\in I} \frac{P_s D_{k,k} + \sigma_{SR}^2}{P_k}\mathbf{w}^o\mathbf{J}_k\mathbf{w}^{o\dagger} \tag{5.68}$$

Then, $\frac{1}{\sqrt{\eta}}\mathbf{w}^{\mathrm{o}}$ is the solution to the problem (5.64). $\square$

We will now find the solution of (5.66). The semi definite programming (SDP) relaxation is a popular method for QCQP problems [68]. Let $\mathbf{X} = \mathbf{w}^{\dagger}\mathbf{w}$, we can write: $\mathbf{w}\mathbf{P}_j\mathbf{w}^{\dagger} = \mathrm{Trace}(\mathrm{RP}_j\mathbf{X})$ and $\mathbf{w}\mathbf{A}_{k,j}\mathbf{w}^{\dagger} = \mathrm{Trace}(\mathbf{A}_{k,j}\mathbf{X})$. With this, we can rewrite the problem of (5.66) as:

$$
\begin{aligned}
\min_{\mathbf{X}} \quad & -\mathrm{Trace}(\mathbf{P}_j\mathbf{X}) \\
\text{s.t.} \quad & \mathrm{Trace}(\mathbf{A}_{k,j}\mathbf{X}) \leq 1, \quad k \in I \\
& \mathbf{X} \succeq 0 \\
& \mathrm{rank}(\mathbf{X}) = 1
\end{aligned}
\tag{5.69}
$$

Dropping the non-convex constraint $\mathrm{rank}(\mathbf{X}) = 1$, we obtain the SDP relaxation:

$$
\begin{aligned}
\min_{\mathbf{X}} \quad & -\mathrm{Trace}(\mathbf{P}_j\mathbf{X}) \\
\text{s.t.} \quad & \mathrm{Trace}(\mathbf{A}_{k,j}\mathbf{X}) \leq 1, \quad k \in I \\
& \mathbf{X} \succeq 0
\end{aligned}
\tag{5.70}
$$

The problem of (5.70) is a convex problem than can be effectively solved by Matlab Software for Disciplined Convex Programming (CVX), which is a open-source Matlab-based modeling system for convex optimization [70]. However, in the general case, the solution $\mathbf{X}^*$ from CVX software does not necessarily have rank one. For the case in which the solution $\mathbf{X}^*$ from the CVX software has a rank greater than one, a search technique may be used to obtain the suboptimal solution of the original problem. The Gaussian random procedure (GRP) is such method [31, 71]. However the Gaussian random procedure it is in general time-consuming and sometimes ineffective. In the following, we give another effective methods for that case based on Coordinate descent method [68, 72].

Coordinate descent method If the solution $\mathbf{X}$ from CVX software has rank greater than one we can use the coordinate descent method to directly deal with the original problem of (5.51). The main idea behind the coordinate descent method is the following. At each iteration, the objective is minimized with respect to one element of $\mathbf{w}$ while keeping the other elements fixed. The method is particularly attractive when the sub-problem is easy to solve and also satisfies certain condition for convergence. The coordinate descent algorithm applied to our problem is as follows.

Algorithm 1.

1. Set $\varepsilon = 10^{-3}$, choose an initial point $\mathbf{w}^0$; set $k = 0$
2. For $p = 1 : R$ determine the optimal pth element while keeping the other elements fixed. This gives $\mathbf{w}_p^k$
3. $\mathbf{w}^{k+1} = \mathbf{w}_R^k$
4. If $\frac{\|\mathbf{w}^{k+1} - \mathbf{w}^k\|}{\|\mathbf{w}^k\|} < \varepsilon$ then stop.
5. $k \to k+1$, goto 2

It is possible to demonstrate that the subproblem in Step 2 has a closed form solution. In fact, it is easy to verify that minimizing the objective with respect to the kth element of $\mathbf{w}$ while keeping the other elements fixed, leads to the following optimization problem:

$$\begin{aligned} &\max_t \quad \frac{a_1\left|t\right|^2 + b_1 t + b_1^* t^* + c_1}{a_2\left|t\right|^2 + b_2 t + b_2^* t^* + c_2} \\ &\text{s.t.} \quad |t| \leq \beta \end{aligned} \tag{5.71}$$

with $\beta = \sqrt{P_k/(P_s D_{k,k} + \sigma_{SR}^2)}$, $a_1 = RP_j(k,k)$, $a_2 = T_j(k,k)$, and b_1, b_2, c_1, c_2 can be deduced from (5.65).

For the solution of (5.71) we have the following theorem [71].

Theorem 3. If $a_1/a_2 = b_1/b_2 = c_1/c_2$, the objective in (5.71) is a constant and the optimum t, denoted t^*, is any value satisfying $|t| \leq \beta$. Otherwise: if the equation $(a_1 - ta_2)\beta^2 + 2\left|b_1 - tb_2\right|\beta + c_1 - tc_2 = 0$ has a real root t_1, such that: $|b_1 - t_1 b_2| \geq (t_1 a_2 - a_1)\beta$, then the optimal t is given by:

$$t^* = \beta e^{-j\theta_1} \tag{5.72}$$

where $\theta_1 \in]-\pi, \pi]$ is the argument of $b_1 - t_1 b_2$; Else, let t_2 be the root of $\left|b_1 - tb_2\right|^2 = (a_1 - ta_2)(c_1 - tc_2)$ such that $|b_1 - t_2 b_2| < (t_2 a_2 - a_1)\beta$, then the optimal t is given by

$$t^* = \frac{|b_1 - t_2 b_2|}{t_2 a_2 - a_1} e^{-j\theta_2} \tag{5.73}$$

where θ_2 is the argument of $b_1 - t_2 b_2$. $\square$

It can be easily proved that the sequence $\left\{\mathbf{w}^k\right\}$ generated by the algorithm converges globally to a stationary point [68].

5.3 The ZF solution when perfect CSI is available

In the simulation results part, we will compare the results of second-order statistics based algorithms with the case of perfect CSI with ZF equalization at the relay side. To obtain this solution, we begin to rewrite the received signal at mobile station j:

$$
\begin{aligned}
u_j &= \sum_{k=1}^{R} g_{kj}\mathbf{w}_k\mathbf{z}_k + \eta_j \\
&= \sum_{k=1}^{R} g_{kj}\mathbf{w}_{k|1\times 1}\left[\sqrt{P_s/M}\, H_{k|_{1\times M}}\,\mathbf{s}^T_{|_{M\times 1}} + \mathbf{v}_k\right] + \eta_j \\
&= \sqrt{P_s/M}\sum_{k=1}^{R} g_{kj}\mathbf{w}_k\left[h_k(1)\cdots h_k(j)\cdots h_k(M)\right]\begin{bmatrix} s_1 \\ \vdots \\ s_j \\ \vdots \\ s_M \end{bmatrix} + \sum_{k=1}^{R} g_{kj}\mathbf{w}_k\mathbf{v}_k + \eta_j
\end{aligned}
\tag{5.74}
$$

Writing the corresponding equations for all mobile stations and stacking them into a column vector, we obtain the following matrix form:

$$
\mathbf{u}_{|M\times 1} = \mathbf{G}_{|M\times R}\,(\mathrm{diag}\,\mathbf{w})_{|R\times R}\,\mathbf{H}_{|R\times M}\,\mathbf{s}_{|M\times 1} \tag{5.75}
$$

with $\mathbf{G}$ and $\mathbf{H}$ defined in (5.2) and (5.3) respectively.

In order to avoid MAI, we must assure that $\mathbf{G}\,\mathrm{diag}(\mathbf{w})\mathbf{H}$ is a diagonal matrix. supposing that $R > M$, we have:

$$
\mathbf{A} = \mathbf{G}\,\mathrm{diag}(\mathbf{w})\mathbf{H} = \mathrm{diag}[\rho_1, \rho_2, ..., \rho_M] \tag{5.76}
$$

Denoting C_{ij} the $(i,j)th$ element of the matrix $\mathbf{c}$, the elements of $\mathbf{A}$ in (5.76) can be calculated as:

$$
A_{ij} = G_{i1}w_1H_{1j} + G_{i2}w_2H_{2j} + \cdots + G_{ik}w_kH_{kj} + \cdots + G_{iR}w_RH_{Rj} \tag{5.77}
$$

Now (5.76) can be summarized:

$$\sum_{k=1}^{R} G_{ik} H_{kj} w_k = \rho_i \delta_{ij} \quad 1 \leq i \leq M, 1 \leq j \leq M \tag{5.78}$$

with $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ otherwise. We have M^2 equations with $M + R$ unknowns ρ_1 to ρ_M and w_1 to w_R.

Let us take the case $R = 4$ and $M = 2$ as an example. In this case (5.78) leads to 2^2 equations with $2 + 4$ unknowns:

$$\begin{gathered}
G_{11}H_{11}w_1 + G_{12}H_{21}w_2 + G_{13}H_{31}w_3 + G_{14}H_{41}w_4 = \rho_1 \\
G_{11}H_{12}w_1 + G_{12}H_{22}w_2 + G_{13}H_{32}w_3 + G_{14}H_{42}w_4 = 0 \\
G_{21}H_{11}w_1 + G_{22}H_{21}w_2 + G_{23}H_{31}w_3 + G_{24}H_{41}w_4 = 0 \\
G_{21}H_{12}w_1 + G_{22}H_{22}w_2 + G_{23}H_{32}w_3 + G_{24}H_{42}w_4 = \rho_2
\end{gathered} \tag{5.79}$$

And this results in:

$$\begin{bmatrix} G_{11}H_{11} & G_{12}H_{21} & G_{13}H_{31} & G_{14}H_{41} \\ G_{11}H_{12} & G_{12}H_{22} & G_{13}H_{32} & G_{14}H_{42} \\ G_{21}H_{11} & G_{22}H_{21} & G_{23}H_{31} & G_{24}H_{41} \\ G_{21}H_{12} & G_{22}H_{22} & G_{23}H_{32} & G_{24}H_{42} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} \rho_1 \\ 0 \\ 0 \\ \rho_2 \end{bmatrix} \tag{5.80}$$

Or:

$$\begin{bmatrix} G_{11}H_{11} & G_{12}H_{21} & G_{13}H_{31} & G_{14}H_{41} & -1 & 0 \\ G_{11}H_{12} & G_{12}H_{22} & G_{13}H_{32} & G_{14}H_{42} & 0 & 0 \\ G_{21}H_{11} & G_{22}H_{21} & G_{23}H_{31} & G_{24}H_{41} & 0 & 0 \\ G_{21}H_{12} & G_{22}H_{22} & G_{23}H_{32} & G_{24}H_{42} & 0 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.81}$$

$$\Longrightarrow \mathbf{\Psi}. \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.82}$$

We use singular value decomposition of matrix $\mathbf{\Psi}$:

$$\mathbf{\Psi} = \mathbf{U}.\mathbf{\Sigma}.\mathbf{V}^{\dagger} \tag{5.83}$$

To ensure that (5.83) has a solution, $\mathbf{\Sigma}$ must be rank deficient (i.e. not full rank). In this case, we can take $[w_1, w_2, w_3, w_4, \rho_1, \rho_2]^T$ in the kernel of matrix Σ. So we can take:

$$[w_1, w_2, w_3, w_4, \rho_1, \rho_2]^T = \mathbf{V}^{\dagger}(:, 5) \quad \text{or} \quad [w_1, w_2, w_3, w_4, \rho_1, \rho_2]^T = \mathbf{V}^{\dagger}(:, 6) \tag{5.84}$$

In the general case matrix $\mathbf{\Sigma}$ is written as:

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \Sigma_2 & 0 & \ddots & \vdots & \vdots \\ 0 & 0 & \Sigma_3 & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \Sigma_4 & 0 & 0 \end{bmatrix} \tag{5.85}$$

In the general case the matrix $\mathbf{\Psi}$ is built in the following way:

$$\mathbf{\Psi} = \begin{bmatrix} G_{11}H_{11} & G_{12}H_{2M} & \cdots & G_{1R}H_{RM} & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots \\ G_{11}H_{1M} & G_{12}H_{2M} & \cdots & G_{1R}H_{RM} & 0 & 0 & \cdots & 0 & 0 \\ G_{21}H_{11} & G_{22}H_{21} & \cdots & G_{2R}H_{R1} & 0 & 0 & \cdots & 0 & 0 \\ G_{21}H_{12} & G_{22}H_{22} & \cdots & G_{2R}H_{R2} & 0 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & \cdots & \cdots & 0 \\ G_{21}H_{1M} & G_{22}H_{2M} & \cdots & G_{2R}H_{RM} & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \cdots & \cdots & \vdots \\ G_{M1}H_{1M} & G_{M2}H_{2M} & \cdots & G_{MR}H_{RM} & 0 & 0 & \cdots & \cdots & -1 \end{bmatrix}_{M^2 \times (R+M)} \tag{5.86}$$

and we have:

$$\mathbf{\Psi} \cdot \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_R \\ \rho_1 \\ \rho_2 \\ \vdots \\ \rho_M \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{5.87}$$

The equation in (5.87) can be solved provided that matrix $\mathbf{\Psi}$ is rank deficient i.e. if and only if:

$$M^2 < R + M \Longleftrightarrow M(M-1) < R \tag{5.88}$$

In this case, using the singular value decomposition of matrix $\mathbf{\Psi}$ as $\mathbf{\Psi} = \mathbf{U\Sigma V}^{\dagger}$, we can choose the vector $[w_1, w_2, ..., w_R, \rho_1, \rho_2, ..., \rho_M]^T$ in the kernel of $\mathbf{\Psi}$.

5.4 Simulation results

We consider at first different system configurations with one source and a variable number of mobile stations. This number varies from 4 to 7 mobile stations. We use each time the minimum number of relay stations (i.e. $N_{\min} = M(M-1)+1$) to guarantee the existence of the ZF equalization solution. The transmitted symbols are modulated using QPSK. We evaluate in Figure 5.2 the average outage probability at a mobile station as a function of the total available maximum power (source plus relays) $10.\log_{10}(P_0)$ for the two different studied contexts: beamforming with second order statistics and ZF with perfect CSI. The target SINR is equal to 10 dB. The channel coefficients for each link (Source to Relays and Relays to Mobile Stations) are modelled as independent identically distributed Gaussian random variables with mean zero and variance 0.5 per dimension. We consider that the channel parameters remain constant over 50 consecutive transmitted packets. We suppose in our simulation runs that $\sigma_{SR}^2 = \sigma_{RD}^2$ and they correspond to an average SNR equal to 10 dB (i.e. when we ignore the contribution of interfering signals). We consider two series of simulation results depending on the kind of power optimization constraints we choose: optimization under total power constraint or optimization under individual relay power constraint.

5.4.1 Optimization under total power constraint

We can see on Figure 5.2 that the ZF always outperforms the second order statistics based algorithm and the more important is the number of mobile stations the larger is the gap between the two kinds of beamforming algorithms. For example, at a considered target

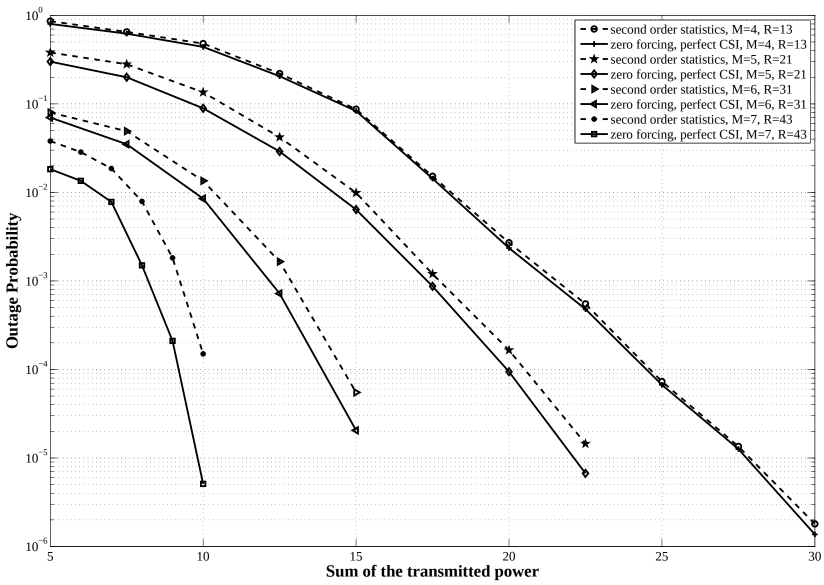


Figure 5.2: Outage Probability as a function of transmitted power (dB), Target SINR = 10 dB

outage probability of 10^{-3}, the gap is not distinguishable between ZF and the second order statistics based algorithm for the case of $M = 4$ mobile stations whilst it becomes equal to 0.5 dB in the case $M = 6$ and nearly equal to 1 dB in the case $M = 7$.

In Figure 5.3 we plot the average relay power versus the source power needed to meet the SINR requirement of each destination when they are in non-outage transmission periods. Once again the target SINR at a mobile station is 10 dB and we suppose that $\sigma_{SR}^2 = \sigma_{RD}^2$ with each of them corresponding to an average SNR equal to 10 dB.

We can see on Figure 5.3 that the ZF yields the smallest required transmit power at the relays. However, for a small number of mobile stations, the difference between the second order statistic method and ZF is small. The gap between the two methods increases as the number of mobile stations increases too.

The effect of the number of relays on the optimal transmit power for a fixed number

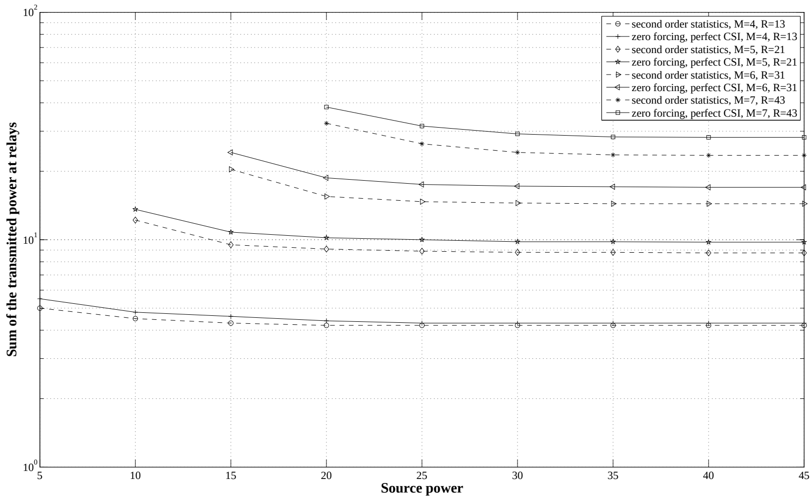


Figure 5.3: Sum of the transmitted power at relays as a function of source Power (dB), Target SINR = 10 dB

of mobile stations is presented in Figure 5.4. We fix $M = 4$ for our simulation set-up and we choose a target SINR equal to 10 dB. The source power is 15 dB. It is clear that when: $R < M.(M-1)+1 = 13$, the ZF does not work because there is no kernel for the matrix equation (5.87). This results in the curb on Figure 5.4 in prohibitive sum transmitted power at relays for the small values of R. The great merit of the second order statistics method is that it always provides a valuable solution, even in the case where ZF is non efficient. With more relays, the ZF becomes more efficient (the threshold over which ZF outperforms second order statistics is equal to 14) and outperforms the second order statistics method. It is worth mentioning that in all cases, provided that we have a number of relays greater than 13, the sum transmit power of all relays is all under 30 dB for providing 10 dB SINR at the mobile stations. This is an affordable power consumption and this is an evidence of the faculty of the multiuser relaying system to support a high number of parallel data streams.

In conclusion, in the case of power optimization under total power constraint, the second order statistics method is an attractive alternative solution to the ZF when the number of relays is moderate and particularly, of course, when this number is less than $M.(M-1)+1$ where M denotes the number of mobile stations. For a high number of mobile stations, ZF clearly outperforms the second order statistics algorithm and the more important is the number of mobile stations the larger is the gap between the two kinds of beamforming algorithms in favour of the ZF method.

5.4.2 Optimization under individual relay power constraint

The results are plotted on Figure 5.4 for the outage probability vs the source power in dB. We consider the same simulation set-up as those given just before except for the individual power relay constraint which is equal here to 5 dB. The results are clearly worse than those presented on Figure 5.2. This can be explained by the fact that there are more constraints on the power assignment to relays and this reduces the number of freedom degrees to enable power exchange between relays.

For comparison purposes and to better illustrate the differences between the two kinds of constraints, we have plotted on Figure 5.6 the results of Figure 5.2 and Figure 5.5, only for the cases $M = 4$, $R = 13$ and $M = 5$, $R = 21$. We can see for example that for an outage probability equal to 10^{-2} and in the case $M = 4$, $R = 13$, the sum transmitted power is equal to 19 dB for the total power constraint and 22 dB for the individual relay power constraint. For an outage probability equal to 10^{-3} in the case $M = 4$, $R = 13$, the sum transmitted power is equal to 22 dB for the total power constraint and 25 dB for the individual relay power constraint. For the case $M = 5$, $R = 21$, we have the following result: for an outage probability of 10^{-2} the sum transmitted power is equal to 15 dB for the total power constraint and 17 dB for the individual relay power constraint. For an outage probability equal to 10^{-3} in the case $M = 5$, $R = 21$, the sum transmitted power is

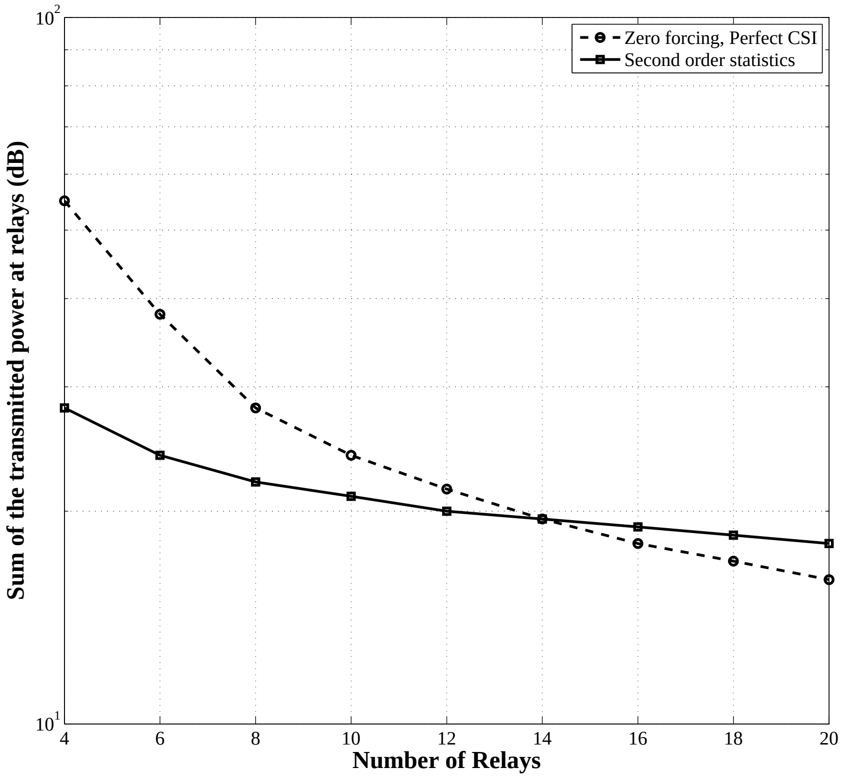


Figure 5.4: Relay power v.s. number of relays R, target SINR $=$ 10 dB, Source power $=$ 15 dB

equal to 18.5 dB for the total power constraint and 20.5 dB for the individual relay power constraint.

We can see on these simple examples how the increasing number of constraints influences on the overall outage performance of the system.

In Figure 5.7 we plot the average relay power versus the source power needed to meet the SINR requirement of each destination when they are in non-outage transmission periods. Once again the target SINR at a mobile station is 10 dB and we suppose that $\sigma_{SR}^2 = \sigma_{RD}^2$ with each of them corresponding to an average SNR equal to 10 dB. We can see that the powers are slightly increased when compared to Figure 5.3, illustrating the advantage of

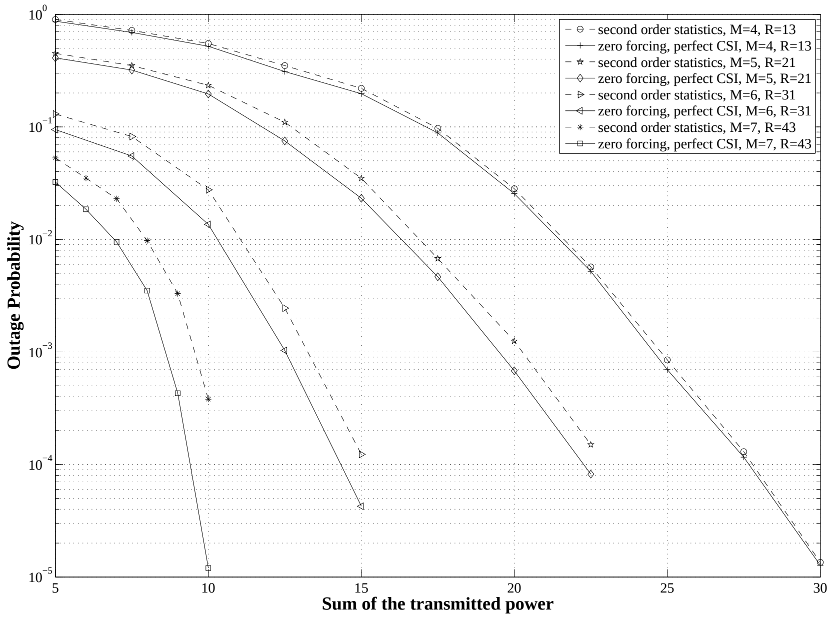


Figure 5.5: Outage Probability v.s. Sum Transmitted Power (dB), Target SINR = 10 dB

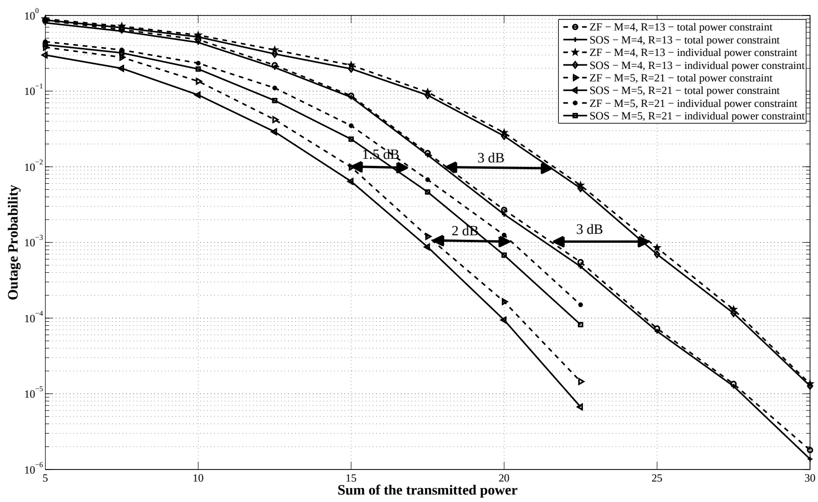


Figure 5.6: Comparison between total power constraint and individual relay power constraint, Target SINR = 10 dB

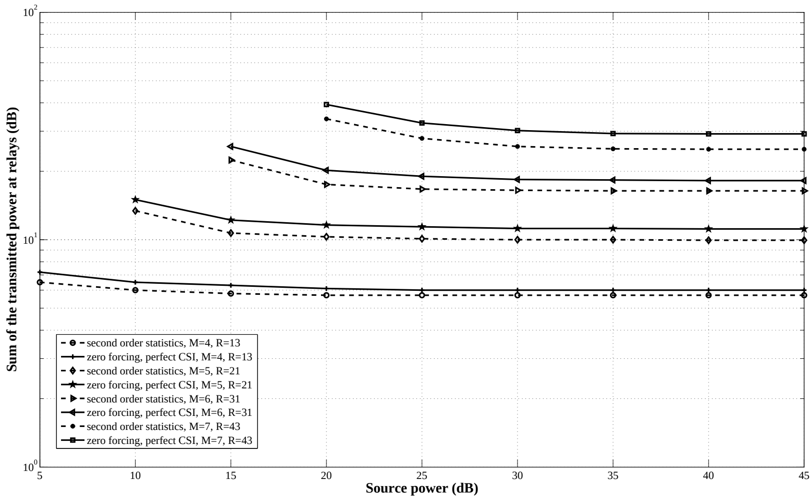


Figure 5.7: Sum Transmitted Power at Relays v.s. Source Power (dB), Target SINR = 10 dB

the first kind of constraint: total power constraint.

For example, when we consider the case $M = 4$, $R = 13$ we have for a source power equal to 10 dB, a sum transmitted power equal to 6 dB in the case of individual power relay constraint with second order statistics based algorithm and equal to 4.5 dB in the case of total power constraint with second order statistics, this enables a power saving of 1.5 dB. For all the other points, the power saving remains equal between 1.5 and 2 dB for all the different simulation contexts.

The effect of the number of relays on the optimal transmit power for a fixed number of mobile stations is presented in Figure 5.8. We fix $M = 4$ for our simulation set-up and we choose a target SINR equal to 10 dB. The source power is 15 dB. It is clear that when: $R < M.(M-1)+1 = 13$, the ZF does not work because there is no kernel for the matrix equation (5.87). This results in the curb on Figure 5.8 in prohibitive sum transmitted power at relays for the small values of R. The great merit of the second order statistics method is that it always provides a valuable solution, even in the case where ZF is non efficient.

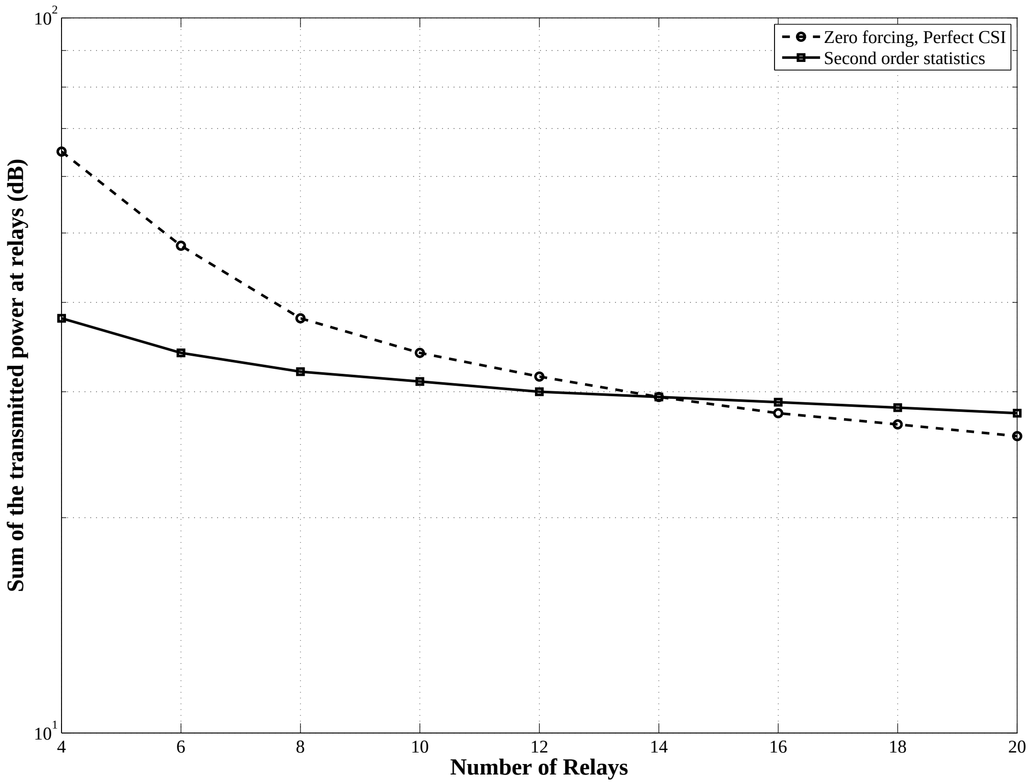


Figure 5.8: Relay power v.s. number of relays R, target SINR $=$ 10 dB, Source power $=$ 15 dB

With more relays, the ZF becomes more efficient (the threshold over which ZF outperforms second order statistics is equal to 14) and outperforms the second order statistics method. It is worth mentioning that in all cases, provided that we have a number of relays greater than 13, the sum transmit power of all relays is all under 30 dB for providing 10 dB SINR at the mobile stations.

In conclusion, in the case of power optimization under individual relay power constraints, the second order statistics method remains an attractive alternative solution to the ZF when the number of relays is moderate and particularly when this number is less than $M.(M-1)+1$ where M denotes the number of mobile stations. Due to the higher number of constraints when compared to the former case of total power constraint, we encounter some losses but this remains small enough to preserve a good overall performance of the system in terms of outage probability.

5.5 Conclusion

In this chapter, we have proposed linear beamforming techniques based on channel second order statistics for one source transmitting towards several mobile stations with the help of multiple relay stations in amplify and forward mode. A comparison of the results with the Zero Forcing technique which needs the perfect channel state information at the relay side was effectued. We have processed the optimization with two kinds of power constraints: total (relay plus source) power constraint and individual relay power constraint. Despite some losses compared to the ZF technique particularly when the number of relays is high enough, the obtained results clearly show that second order statistics based beamforming algorithms are good candidates to reliably support multiple parallel data streams with SINR requirements in a multiuser multi-relay scheme.

Chapter 6

General conclusions

This dissertation addresses different aspects of multi-user multi-relay cooperative communications. A large variety of mathematical tools and techniques are investigated in this dissertation. Here are a few examples:

- Lagrange multipliers method was modified to address the problems involving vectors and matrices.
- Moore-Penrose pseudoinverse matrix was used to cancel out multiple access interference (MAI).
- Expectation-Maximization (EM) algorithm was used to approximate the distribution of a random variable with a mixture of known distributions. The case of Nakagami and Gamma distribution mixtures were studied in this dissertation.
- The Gram-Schmidt process was used for orthonormalising a set of vectors in a vector space. This process was used to cancel out the MAI in a cooperative scheme.
- Coordinate descent method was used to iteratively solve an optimization solution with many variables by optimizing one variable at a time.

The first three chapter of this dissertation deal with the system depicted in Figure 6.1. This is a multi-user multi-relay scheme with multiple antennas at each relay.

In chapter 2, a modified version of Lagrange multipliers method was used to optimize the system performance. It is shown that if the system constraints satisfy certain criteria, the optimization problem will be reduced to a simple matrix inversion. System constraints and objective functions remain largely flexible. For example the objective function can be set to maximize *any* linear combination of the signal to noise ratio (SNR) of received signals at individual mobile stations. Due to the flexible nature of system constraints and objectives, no theoretical analysis can be produced.

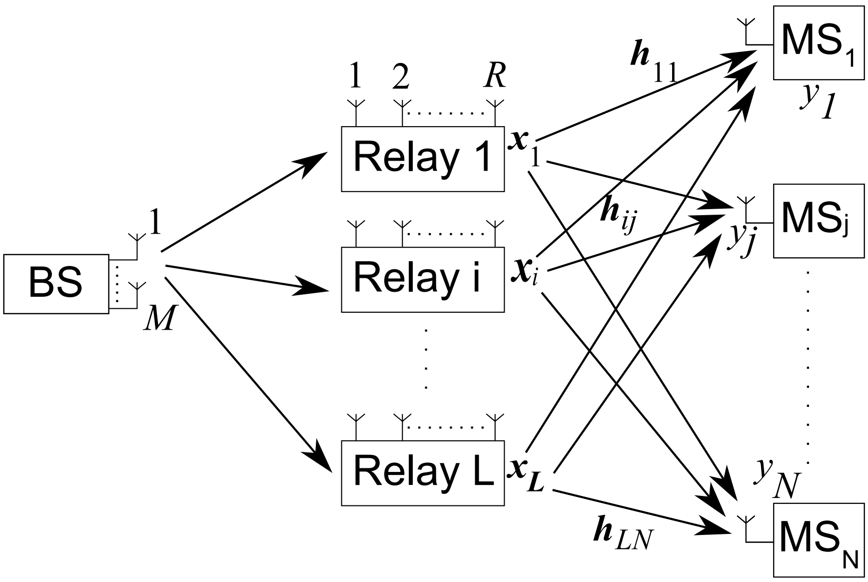


Figure 6.1: System model of the first 3 chapters

Chapter 3 covers a special case of the problem when the same SNR is desired at all relays. In this case the Moore-Pensore pseudoinverse may be used to diagonalize the equivalent channel plus precoding vectors matrix from the relays to mobile stations and consequently canceling out MAI. Two different strategies are considered. In the first strategy all relays have the channel state information (CSI) of all relays to the mobile station (MS)s. In the second strategy each relay only accesses its own channel to the mobiles. The former clearly outperforms the latter at the cost of more complicated system and signaling protocols due to centralized precoding calculations. Theoretical system performances are also analyzed and confirmed by simulations. For the case of two mobile stations and arbitrary relay number and structure, the system diversity is analytically derived. For arbitrary number of MSs symbol error probability (SEP) is semi-analytically calculated. The EM algorithm is used to approximate the equivalent SNR by a mixture of Nakagami distributions and system performance is derived from that approximation.

In chapter 4, the same system is optimized using the Gram-Schmidt orthonormalization process. This process is used to maximize the SNR at mobile stations by optimizing the relays transmit power while mitigating the MAI. This scenario permits different levels of user privilege to be defined. It is shown that for higher number of relays a uniform relay power assignment is the best choice while for the smaller number of relays, an optimized power assignment, even at the cost of centralized system, may be justified. Diversity order of the system is derived analytically. The system SEP is calculated semi-analytically using the EM algorithm.

The main inconveniences of the system of 6.1 are that it requires the exact CSI at the transmitter side; due to the multi-antenna relay structures, the system is complex and as a result expensive; and we assumed perfect source-relay links which may not be realistic. In the last chapter a multi-relay distributed cooperative relaying scheme was addressed in which: i) only second order statistics of the channels are known, ii) the relays are equipped with only one relay, and iii) the source-relay link imperfections are taken into account. In this chapter the precoding vectors are optimized to maximize the signal to noise plus interference ratio (SINR) at the MSs. Two type of power constraints are discussed. The first case is when the total source plus relay transmission power is constrained while in the second scenario the individual relay powers are constrained.

The following may be envisaged for the continuation of this work:

- Several numerical optimization processes may be used in order to allocate the relay powers. One promising solution is the use of Particle Swarm Optimization (PSO) [73,74] to maximize the overall capacity or to minimize the SEP or outage probability while maintaining the sum of the transmission power of all relays below a given threshold. In fact a slightly modified version of PSO dealing with optimizing a goal function over a spheric surface has been proposed in [48]. Since our system constraint (i.e. $\sum P_i = \sum x_i^2 = C^{ste}$) is the mathematical expression of a sphere, this modified version of PSO is well adapted to our system.

- Another interesting continuation would be to take into account the imperfection of the first hop where the source sends the information towards the relays.

- We can try to use relay assignment algorithms [75] in order to find the best L relays out of available L' relays. The selection criterion may be to maximize overall capacity or to minimize the outage probability.

- The estimation error of the channel coefficients at the relays can be taken into account and the effect of such error on the SEP at the destination can be evaluated.

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List of Publications

[1] M. Zahabi, V. Meghdadi, H. Meghdadi, and J.-P. Cances, "Analog decoding of tail-biting convolutional codes on graphs," in *Wireless Communication Systems. 2008. ISWCS '08. IEEE International Symposium on*, pp. 533 –537, oct. 2008.

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Formation de faisceaux coopératifs pour transmissions multiutilisateurs par relais

Résumé :
La demande en moyens de communications toujours plus rapides et plus fiables est en augmentation permanente. Pour répondre à cela, les chercheurs doivent faire face à de nombreux défis. Les systèmes coopératifs sont reconnus comme une solution performante pour améliorer la qualité de transmission des systèmes existants. Parmi les différentes stratégies de transmission coopérative, l'utilisation conjointe de plusieurs relais est considérée comme une voie prometteuse. Ce document de thèse traite du calcul des précodeurs aux relais pour améliorer la performance de systèmes multi-relais multiutilisateurs. Le précodage est utilisé pour annuler le plus possible les interférences multiutilisateurs, maximiser le rapport signal à bruit à la réception, et optimiser l'allocation de puissance aux relais.

Cooperative Beamforming for Multiuser Relaying Schemes

Abstract:
The demand for high speed reliable communication systems will never stop increasing. Many challenges face researchers trying to provide such systems and schemes. Cooperative networks have been successfully used to enhance the performance of telecommunication systems. Among different cooperative strategies, distributed cooperative relaying have shown to be a promising scheme. This dissertation addresses the problem of optimizing the precoding vectors in order to improve the system performance of multi-user multi-relay cooperative networks. Precoding vectors are used to cancel out the multiple access interference, maximize the signal to noise ratio at the destination, and optimize the power allocation at relaying stations.

Disipline : Electronique des Hautes Fréquences, Photonique et Systèmes

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Mots clés :

Réseaux coopératifs
Procédé d'orthonormalisation Gram Schmidt
Annulation d'interférences multiutilisateurs
Optimisation du rapport signal à bruit
Algorithme de Maximisation de l'Espérance

Cooperative networks
Gram Schmidt Orthonormalazation
Multi-user interference cancellation
Signal to noise ratio optimization
Expectation Maximization algorithm

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